Bank resolution and public backstop in an asymmetric banking union

by
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Abstract

This paper characterizes the optimal banking union with endogenous participation in a two-country economy in which domestic bank failures may be contemporaneous to sovereign crises, giving rise to risk-sharing motives to mutualize the funding of bail-outs. Raising public funds to conduct a bail-out entails the deadweight loss of distortionary taxation. Bank bail-ins create disruption costs in the economy. When country asymmetry is large, resolution policies exhibit reduced contributions to the public backstop and forbearance in early bank intervention in the fiscally stronger country, facilitating bail-outs in this country.

JEL Classification: G01, G21, G28
Keywords: banking union, bail-in, bailout, public backstop, mechanism design
1 Introduction

At the peak of the Euro Area sovereign debt crisis in June 2012, the members of the eurozone agreed on the creation of a banking union.\textsuperscript{1} State members have undertaken considerable steps towards its completion since then. The supervision and resolution of big banks in the Euro Area are now conducted at a supranational level by the European Central Bank (ECB) and the Single Resolution Board (SRB), respectively. Besides, the transposition into national law of the Bank Recovery and Resolution Directive (BRRD) provides a common framework for the resolution of failing banks and confers authorities more powers to impose losses on private stakeholders, the so-called bail-in tools.\textsuperscript{2}

Although the norm in the BRRD is to resolve banks through bail-ins, this directive allows public support to failing institutions when authorities consider that private burden sharing would endanger financial stability.\textsuperscript{3} In that case, sovereigns in financial distress can obtain a fraction of the required public funds from the European Stability Mechanism (ESM).\textsuperscript{4} However, the International Monetary Fund (IMF) has warned that the banking union currently falls short of providing sufficient risk-sharing across countries.\textsuperscript{5} The European Commission (EC), in turn, has recently issued a communication that identifies “a last resort common fiscal backstop for the single resolution mechanism” as a critical missing element to complete the banking union.\textsuperscript{6} The June 2018 Euro Summit agreed that by the end of the year the Eurogroup would have prepared the terms of reference for the use of the ESM as a common fiscal backstop.

How should a common fiscal backstop be articulated? How should bank resolution decisions be conducted? How much risk-sharing can countries achieve through a banking union?

\textsuperscript{3}Even in this case, the BRRD establishes strong conditions before resorting to public funds: The bank’s shareholders and creditors must bear losses of at least 8% of all liabilities, and the privately funded Single Resolution Fund must contribute 5% of all liabilities to the resolution of the bank.
\textsuperscript{4}The European Stability Mechanism (ESM) is a legal entity backed by euro area members that provides financial assistance to sovereigns experiencing or threatened by severe financing problems.
\textsuperscript{5}IMF, 2016. 2016 article iv. consultation press release; Staff report; and statement by the executive director for the euro area. IMF Country Report No. 16/219.
\textsuperscript{6}EC, 2017. Communication to the European Parliament, the Council, the European Central Bank, the European Economic and Social Committee and the Committee of the Regions on completing the banking union. European Commission 11/10/2017.
We address these questions in a model that takes into account two asymmetries that are prevalent in the Euro Area context. First, the newly created supranational resolution authority is required to operate in strict cooperation with more established domestic resolution authorities which, as put forward by Navaretti, Calzolari and Pozzolo (2015), may have informational advantages about the local banking sector. Second, there exists a significant degree of heterogeneity on the fiscal strength of member states. Our stylized model shows how informational and fiscal asymmetries shape the way a banking union should provide risk-sharing across countries. This analysis highlights the need for imposing laxer conditions on countries with initially stronger fiscal positions to access the common fiscal backstop, which increase the probability of bailing out their banks. Besides, the optimal early intervention policy exhibits forbearance with the bank located in the stronger country and tightness with the bank in the weaker country.

The environment consists of a two-date and two-country economy with one bank and a resolution authority in each country. The bank funds its assets issuing debt from local investors. If its assets do not deliver the promised return to debtholders, the bank is resolved through either a bail-out or a bail-in. Raising public funds to conduct a bail-out entails the deadweight loss of distortionary taxation. A bail-in, which imposes losses on debtholders, generates a costly disruption in the economy where the bank resides. The resolution policy must trade off these costs. When a country undergoing a sovereign crisis faces a bank failure, the recourse to the foreign country’s funds may reduce the overall cost of funding a bail-out, because the cost of public funds is exceptionally high during a crisis. This risk-sharing motive constitutes the rationale for the creation of a banking union.

Precisely, a banking union consists of a set of policies to resolve a failing bank that are contingent on both the state of the sovereigns at the time of resolving the bank (i.e., whether countries are fiscally stable or undergoing a crisis) and the ex-ante standpoint of the sov-

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7The Single Resolution Mechanism (SRM), whose purpose is to “ensure an orderly resolution of failing banks,” is composed by the Single Resolution Board (SRB) and the National Resolution Authorities (NRAs) of participating Member States. The Regulation (EU) No 806/2014 of the European Parliament, which establishes the procedures for the resolution of banks in the framework of the SRM, states that “national resolution authorities should assist the Board in resolution planning and the preparation of resolution decisions.”

8See Section 2.1 for a further discussion of the informational asymmetries between domestic and supranational authorities.

9Our analysis yields the same qualitative results if we assume that countries differ instead in the strength of their banks.
ereigns (i.e., the relative likelihood that countries undergo a crisis). The paper characterizes the resolution policies that minimize the aggregate resolution cost subject to the participation constraints of both countries. A standard mechanism-design approach to elicit the private information about the bail-in disruption costs allows us to focus on resolution mechanisms that establish the home and foreign country’s contribution to a potential bail-out and delegate the resolution decision to the home authority.

The paper first characterizes the *ex-post optimal banking union*, namely the resolution policies that minimize the aggregate resolution cost without considering the countries’ participation constraints. In the event in which the home country is undergoing a crisis and the foreign country is fiscally stable, the ex-post optimal resolution policy establishes a co-funding of bail-outs. The home country copayment is a consequence of the informational asymmetry between the supranational and the national authorities. Requiring the home country to partially finance the bail-out increases the overall resolution cost. However, it curbs the home authority’s willingness to overstate the magnitude of the bail-in contagion cost to persuade the central authority of the need for a bailout, which would be partially funded by the foreign country.\(^\text{10}\) In the rest of contingencies (e.g., when both countries are simultaneously stable) there is no cost gain from co-funding bail-outs. Consequently, the ex-post optimal resolution rules prescribe that the home sovereigns fully fund their own bail-outs.

The ex-post optimal banking union minimizes the aggregate resolution cost but induces an expected positive net transfer from the stronger country to the weaker country, since the latter is more likely to experience a sovereign crisis. As a result, the ex-post optimal banking union is feasible only when countries’ fiscal strength asymmetry is not too large. When countries are sufficiently asymmetric,\(^\text{11}\) the *feasible optimal banking union* distorts the ex-post optimal resolution policy in two dimensions to ensure the voluntary participation of the fiscally stronger country. First, the stronger country reduces its contribution to bailing out a foreign bank when fiscally stable and receives a larger aid to bail-out its bank if it

\(^{10}\)The structure of this copayment scheme resembles the out-of-pocket deductible displayed by several insurance contracts to reduce moral hazard. In this framework, however, the role of the copayment is to prevent the home country from overstating the bail-in costs to conduct a bail-out partially funded with the foreign country’s funds.

\(^{11}\)In particular, this is the case when the strong country is always expected to be stable, a case that we address in the main body of the paper (Section 4.3, “Optimal banking union in stylized economies,” item ii.).
undergoes a crisis. Second, the stronger country receives partial funding of its bail-outs even when there is no cost advantage (e.g., when both countries are concurrently fiscally stable). These country-specific contributions to the funding of bail-outs effectively imply laxer conditions to accessing the common fiscal backstop for the stronger country. The optimal resolution framework thus increases the likelihood that banks in fiscally stronger countries be bailed-out, which suggests that the minimization of the aggregate resolution cost may not be compatible with leveling the playing field among banks within a banking union with heterogeneous countries.

One of the obstacles towards the completion of the Euro Area banking union has been the core countries’ concern that it could lead to net transfers of funds towards the periphery. On the contrary, the optimal banking union characterized in the paper features a net expected transfer from the fiscally weaker country towards the fiscally stronger when the country asymmetry is sufficiently large. In this case, while the stronger country benefits from an expected positive net transfer inflow, the weak country obtains the strong country’s aid when it is most valuable, that is, when it concurrently experiences a sovereign crisis and a bank failure. This result highlights that imposing zero net expected cross-country transfers constitutes a suboptimal way of ensuring each country’s legitimate demand that the creation of a banking union not be detrimental to its constituency.

In addition to the resolution of failed banks, the banking union is completed by the prerogative of the supranational authority to early intervene banks upon the observation of a signal of the quality of their assets. An intervention ahead of potential deterioration

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12The distortions of the ex-post optimal resolution policies imposed by the country heterogeneity arise because we restrict all transfers across countries within the banking union to be employed in the resolution of banks. Ex-post efficiency could be re-established with unconditional transfers from the weak country to the strong country. Effectively, these transfers would amount to an insurance premium paid by the fiscally weaker country. The paper does not consider this type of transfers in the framework of a banking union, as these would likely confront political economy constraints in their practical implementation.

13Notice also that although these conditions seem to privilege the stronger country, the weaker country appropriates all the expected welfare gains that the banking union creates. Besides, although entering the banking union does not affect the expected aggregate welfare of the agents of the fiscally stronger country, it leads to a redistribution of welfare from the public to the bank owners.

14The following excerpt in the context of the establishment of a common fiscal capacity at the EU level provides an example: “The common fiscal capacity must not represent a disguised form of permanent transfers from one part of the union to another. Without effective safeguards in the design of the mechanism, which will ensure its fiscal neutrality amongst the Member States in the medium term, it could prove difficult to address the concerns of some Member States that they will become overall net contributors.” (Interparliamentary conference on stability, economic coordination and governance in the European Union, Bratislava 16-18 October, 2016).
of the bank assets allows repaying the bank’s creditors without resorting to public funds. When countries asymmetry is sufficiently large, the optimal early intervention policy features forbearance with the stronger country’s bank, allowing the continuation of the bank under some contingencies in which its early intervention would be efficient. On the contrary, the bank in the weaker country faces tighter conditions for continuation. The rationale for these ex-post suboptimal early intervention policies is that they soften the fiscally stronger country’s participation constraint by reducing its expected net transfer outflow in the resolution phase.

Enlarging the set of policy tools to include the possibility of granting a bridge loan does not change the essential aspects of the optimal resolution mechanism. The optimal funding mix of bail-outs at a country undergoing a crisis combines a positive direct transfer and a loan from the stable foreign sovereign. Since loans have to be paid back at a future date, once the country undergoing a crisis may have regained its fiscal stability, loans constitute a less costly form of copayment than contemporaneous home contributions. However, direct transfers from a stable foreign country should be used as well, as these funds constitute the least costly source of funding available within the banking union. In contrast, the current agreements among EU members establish that access to the Direct Recapitalization Instrument (DRI), by which countries receive funds without a counterpart, can only be granted if access to the Indirect Recapitalization Instrument (IRI) (i.e., through loans) is not possible, and after a sufficient home contribution has been realized (Arts. 15-20 of the ESM-Treaty). The results in this extension suggest that the DRI of the ESM should not be considered a last resource option, but a tool to complement the IRI.

Thus far, the only rationale for the creation of the common fiscal backstop is the mutualization of bail-out funding across countries exposed to potentially simultaneous bank failures and sovereign crises. Many economists have also emphasized the need to coordinate supervision and resolution actions for banks with significant cross-border liabilities as an essential motivation for the Euro Area banking union.\textsuperscript{15} Consistent with this view, introducing cross-border liabilities in our framework partially realigns the home and the foreign

\textsuperscript{15}For instance, see van Claessens and van Horen (2013) for a survey on cross-border banking, Bolton and Oehmke (2018) on the resolution of cross-national banks, and Schoenmaker (2011) for a critical assessment of the impossibility of simultaneously achieving financial stability, financial integration and national financial policies.
countries’ objectives, increasing the welfare gains through the creation of a banking union.

1.1 Related Literature

Our paper contributes to several strands of the literature. First, it belongs to an emerging literature on the architecture of banking unions. Carletti, Dell’Ariccia and Marquez (2016) build a model of a “hub-and-spokes” banking union in which a central authority takes decisions based on information revealed by local supervisors. They find that a tough central authority may weaken the local authority’s incentives to collect information, thereby increasing risk-taking by regulated banks. In Colliard (2017), the central authority internalizes all the externalities associated with bank failures, but has an informational disadvantage relative to the local authority. The paper identifies a complementarity between the centralization of supervision and the degree of financial integration that may lead to inefficient self-fulfilling equilibria with local supervision and low financial integration. Calzolari, Colliard and Loarath (2017) show that a supranational supervisor solves a coordination problem between local authorities in the supervision of multinational banks. The authors also find that banks may respond to the more intense supervision of a central authority by reducing foreign expansion. A common feature in these papers, which is also central to our analysis, is that there is a misalignment between national and supranational authorities that distorts information transmission, as in Holthausen and Rønde (2004). Relative to these papers our work is, to the best of our knowledge, the first one to analyze how the mutualization of a public backstop affects national authorities’ incentives to reveal all their relevant information to the central resolution authority, and its implications for risk-sharing among the members of the banking union.

A second strand of the literature analyzes how heterogeneity across jurisdictions affects the feasibility and efficiency of supranational regulation, supervision or resolution. In Dell’Ariccia and Marquez (2006), competing domestic regulators do not internalize cross-border externalities, which leads to inefficiently low regulatory standards. Centralized regulation with common standards increases overall welfare, but is only feasible when jurisdictions are sufficiently homogeneous. Similarly, in Beck and Wagner (2016) the establishment of centralized supervision with common standards reduces cross-border default externalities, but is not feasible when countries are very heterogeneous. Our work departs from these papers
in that we analyze the country specific standards (in both bank resolution and access to the common fiscal backstop) that are both feasible and maximize aggregate welfare when country asymmetry is large. In this respect, we are closer to the analysis of the choice of either a single-point-of-entry (SPOE) or a multiple-point-of-entry (MPOE) for the resolution of global banks in Bolton and Oehmke (2017). The paper finds that SPOE resolution is efficient, but not implementable, when the expected transfers across jurisdictions are too asymmetric. In those cases, an MPOE resolution with jurisdiction specific loss-absorbing capital is optimal. Finally, in a context of mutual insurance with respect to sovereign crises that abstracts from banking sector considerations, Tirole (2015) analyzes how the optimal pact across countries depends on their asymmetry. In a similar vein to that paper, we establish the feasible agreements that can be reached among countries depending on their degree of heterogeneity.

Our paper is also related to a recent theoretical literature that assesses the effects of different approaches to bank resolution. Dell’Ariccia et al. (2018) examine the costs and benefits of bail-ins and bail-outs, hinging on the trade-off between the moral hazard costs associated with bail-outs and the potential spillovers arising from bail-ins. Colliard and Gromb (2017) show that the use of bail-in tools in bank resolution may reduce bank stakeholders incentives to restructure bank debt.Walther and White (2017) find that a commitment to impose bail-ins based on private information often lacks credibility, because regulators must avoid conveying bad news that may spread over to the financial system. Sandri (2015) addresses the optimal bank resolution policy in a model in which bail-outs are fiscally costly and bail-ins entail disruptions in the economy. In his setup an International Finance Institution may alleviate the financial constraints faced by a country in distress, but it requires the fiscal consolidation of the country. Our paper differs from these mainly in that our focus is the optimal design of a voluntary risk-sharing banking union in which fiscally strong countries require an asymmetric treatment to avoid subsidizing weaker countries.

Finally, we also contribute to the literature linking fiscal capacity and bank regulation, supervision and resolution. Stavrakeva (2017) shows that capital requirements should be lower in countries with larger fiscal capacity in a framework in which banks can be bailed out with resources levied from consumers through distortionary taxation. Our paper mainly differs from hers in that our focus is on the optimal ex-post resolution mechanism instead of
on ex-ante regulation.

The rest of the paper is organized as follows. Section 2 presents the model set-up. Section 3 describes the resolution decisions in the absence of a banking union. Section 4, which constitutes the core of the paper, characterizes the optimal design of the banking union and show how it depends on the degree of country asymmetry. Section 5 analyzes the direction of the expected net transfers across countries in the optimal feasible banking union. Section 6 extends the model to allow for early intervention by the central authority at an interim date and analyzes how country asymmetry may lead to a differential treatment of banks across countries in these interventions. Section 7.1 conducts other extensions of the baseline model. Section 8 concludes. The proofs of the formal results of the paper are contained in the Appendix.

2 The baseline model

In this section we describe the model setup. In Section 2.1 we further discuss some of the main assumptions of the model. We consider a two-country–i ∈ {1, 2}– and two-date–t ∈ {0, 1}–economy where all agents are risk-neutral and have a zero discount rate. We refer to country 1 as the strong country and we label country 2 the weak country. As we formalize below, this distinction reflects the higher likelihood of sovereign crises in the weak country. In each country there is a bank, a continuum of competitive investors, and a local bank resolution authority. We refer to the bank and local authority in country i as bank i and authority i, respectively. We also say that authority i is the home authority of bank i and we use the notation i to refer to the other country, or to its agents.

Resolution authorities aim at maximizing the aggregate expected utility in their respective jurisdictions. They have resolution prerogatives over their country’s bank and the capability to raise public funds to implement their initiatives. In addition, these domestic resolution authorities may agree at the initial date on the creation of a banking union to confer their prerogatives to a central resolution authority, as we specify below.

At t = 0, each bank has access to a project that requires an outlay of 1 unit. Banks raise these funds from local investors, issuing debt claims with face value Di. The project’s payoff at t = 1 is R > 0, if successful, and 0 otherwise. The probability that the project succeeds is p.
If a bank’s project fails at \( t = 1 \), the bank cannot repay its debt and the competent resolution authority must resolve the bank. There are two types of resolution: A \textit{bail-out} and a \textit{bail-in}.

**Bank bail-out** A bank bail-out consists of the repayment of the unit of principal investment to the bank’s debtholders.\(^{16}\) Bailouts are funded with public resources. We assume that the net social cost of public funds is \( \lambda_S \) if the country is stable (\( S \)) at \( t = 1 \), and \( \lambda_C \) if the country is undergoing a sovereign crisis (\( C \)), with \( 0 < \lambda_S < \lambda_C \). For each pair \( \sigma \in \{ SS, SC, CS, CC \} \) representing the state of the public finances in both countries at \( t = 1 \), we denote by \( q^\sigma \) the probability as of \( t = 0 \) that state \( \sigma \) realizes. We assume that the state of both countries public finances is common knowledge at \( t = 1 \). We denote \( q^S \) and \( q^C \) the probability that country \( i \) is stable and undergoing a crisis at \( t = 1 \), respectively. Hence, we have that \( q^S = q^{SS} + q^{SC} \) and \( q^C = q^{CS} + q^{CC} \). We assume that the strong country \((i = 1)\) is more likely to be stable than the weak country \((i = 2)\), that is:

**Assumption 1 (Strong country less likely to experience a crisis)** \( q^S > q^C \).

Notice that Assumption 1 implies that \( q^{SS} + q^{CC} < 1 \), as well as \( q^{SC} > 0 \), so that countries’ public finances are not perfectly correlated.\(^{17}\)

**Bank bail-in** A bank bail-in imposes the full burden of bank losses on debtholders. We assume that losses suffered by bank debtholders may disrupt the rest of the financial sector, and therefore the economy at large, and generate a local deadweight contagion cost of \( k \). As of \( t = 0 \), \( k \) is a random variable, which we assume to be uniformly distributed in the interval \([0, k_{\text{max}}]\). For notational simplicity, we write its probability density function and cumulative distribution function as \( g(k) \) and \( G(k) \), respectively. The bail-in cost \( k \) is realized at \( t = 1 \), after the bank’s project fails, but before the resolution decision is taken. We assume that its realization is only observed by the home authority. We make the following assumption:

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\(^{16}\) We assume that debtholders only get the principal \( 1 \) for simplicity, as it renders the resolution decisions and the expected social costs of resolution independent of the promised repayment \( D_i \). Alternatively, we could consider that a bailout consists of the repayment of the debt face value \( D_i \). This assumption would introduce a dependence of the cost of bailouts on bailout expectations, which would complicate the analysis without adding further insight.

\(^{17}\) Observe that Assumption 1 does not rule out the particular case that the strong country is always stable, that is, \( q^S = 1 \) or \( q^{SS} = q^{CC} = 0 \). Indeed, we devote a special subsection within Section 4.3 to analyzing the characteristics of the optimal banking union in the case \( q^S = 1 \).
Assumption 2 (Inefficient unconditional bail-outs) \( k_{\text{max}} > \lambda_C > \lambda_S > \int_0^{k_{\text{max}}} kdG(k) \).

The assumption that \( \lambda_S > \int_0^{k_{\text{max}}} kdG(k) \) implies that unconditionally bailing out a bank is inefficient, even if the funding of the bail-out is entirely provided by a stable country. Put differently, this assumption asserts that in the absence of information on the realized value of \( k \), the alternative is to bail in a failing bank, regardless of the cost of public funds. This assumption is consistent with the stated objective of the BRRD in the EU, which attempts to make bail-ins the norm in bank resolutions, bail-outs being the exception (European Parliament, 2014). The assumption that \( k_{\text{max}} > \lambda_C \) is inessential and only imposed to reduce the number of cases to consider.\(^\text{18}\) It implies that bail-outs are preferable to very disruptive bail-ins, even if the net social cost of public funds is high.

**Banking Union** A bank failure may occur when its home country is experiencing a crisis and the foreign country is stable. Consequently, domestic authorities may find it convenient to provide each other with some insurance by agreeing at the initial date to mutualize (to some extent) the funding of public support to failing banks. More precisely, we assume that at \( t = 0 \) local authorities may create a banking union conferring their prerogatives (both the decision on the resolution of failing banks and the capability to levy resources from sovereigns) to a central authority. This central authority operates without information on the realized bail-in contagion costs, so that it must resort to the local authorities to elicit such information. The banking union agreement establishes a protocol of bank resolution decisions and sovereign contributions to bail-outs in each contingency that may arise.

Formally, a banking union is characterized by a resolution mechanism \( \left\{ b_{i,\sigma}(\hat{k}), x_{i,\sigma}(\hat{k}) \right\} \), where the bank failure state \((i, \sigma)\) represents the location \( i \in \{1, 2\} \) of the failing bank and the state \( \sigma \in \{SS, SC, CS, CC\} \) of the public finances in both countries at \( t = 1 \). For each disruption cost \( \hat{k} \) reported by the failing bank’s home authority, the resolution mechanism establishes whether the bank is bailed out \( (b_{i,\sigma}(\hat{k}) = 1) \) or bailed in \( (b_{i,\sigma}(\hat{k}) = 0) \), as well as the home country contribution \( x_{i,\sigma}(\hat{k}) \) in case a bail-out is prescribed, the remaining funding needs \( 1 - x_{i,\sigma}(\hat{k}) \) being contributed by the foreign country.

By the virtue of the revelation principle, we can focus on truth-telling resolution mechanisms for each bank failure state \((i, \sigma)\). Using standard mechanism design arguments, we

\(^{18}\)It suffices to have \( k_{\text{max}} > \lambda_S \) so that bail-ins are not always preferable to bailouts.
show in Appendix A that any relevant truth-telling mechanism that elicits the cost $k$ and establishes a contingent bail-out/bail-in decision can be implemented as follows: The central authority sets the home contribution $x_i, i$ to the funding of a bail-out and delegates the decision of whether to bail out or bail in the bank to the home authority. Consequently, we can characterize a banking union by a vector $x = (x_i, i \in \{1, 2\}, i \in \{SS, SC, CS, CC\})$, establishing the home contribution to bail-outs in each of the eight bank failure states $(i, \sigma)$. Note that the degenerate “banking union” described by the vector of home contributions $1 = (1, ..., 1)$ corresponds to the autarky case, in which each country resolves its bank failures on its own.

From now on, we will always refer to a banking union $x$ as to a proper banking union, that is, one in which $x \neq 1$.

Finally, we say that a banking union $x$ is feasible if each country attains at least as much expected utility as in autarky. The banking union problem we address in the paper is that of analyzing the optimal feasible banking union, that is, the aggregate welfare maximizing banking union that ensures the endogenous participation of the two countries.

2.1 Discussion of assumptions

Reduced form for bail-in disruption costs

The establishment of a new bank resolution framework within the Euro Area, which includes the so-called bail-in tools, is paving the way towards minimizing the negative spillovers of bank bail-ins. However, several authors have warned that in some situations the use of bail-in tools may still cause severe disruptions in the rest of the economy (e.g., Dewatripont, 2014; Goodhart and Avgouleas, 2016). Systemic spillovers may manifest in different forms. For instance, bank bail-ins may undermine the investors’ confidence, leading to roll-over freezes or deposit withdrawals, which may compromise financial stability (see Brown, Evangelou, and Stix, 2018, for evidence on this phenomenon following a bail-in event in Cyprus in 2013). Besides, as put forward by Dell’Ariccia et al. (2018), bail-in spillovers may be mechanically transmitted through the balance sheet of the bank, endangering the solvency of its debtholders and precipitating chains of bankruptcies. As evidenced by Beck, Da Rocha Lopes, and Silva (2017), which analyze the response of financial institutions to a bail-in in Portugal in 2014, the banks holding the bailed-in debt may tighten the access to credit to its borrowers, leading to further deterioration of the economic outlook. We model all such
potential spillovers in reduced form, subsuming these costs in the parameter $k$.

**Informational advantage of national authorities**

We assume that national authorities possess better information than the supranational authority about the potential consequences that a bank bail-in may inflict on the country’s economy. This information advantage may arise from several sources.

First, the SRB is a newly created authority, while many national resolution authorities have been operating for a long time. Some of these national resolution authorities form an integral part of national central banks and have accumulated knowledge about the domestic banking sector as part of their long-standing existence. Such expertise may allow the national authorities to better assess the relevant determinants of the bail-in spillovers, such as the interconnections of the domestic financial system, the potential reaction of the bank’s debtholders or other investors, or the extent of the reliance of other financial institutions on the failing bank.

Second, and relatedly, the SRB is a resolution authority that is formally independent of the ECB, which is the supranational supervisory authority in the banking union. The SRB can only gather information on banks through the ECB or the relevant national supervisory authorities through a Memorandum of Understanding between the two institutions. Instead, in several countries the resolution and the supervision authorities, although organically different, reside in the central bank, which has access to bank data to conduct its supervisory function. This privileged access to information on the entire domestic banking system might prove especially useful in the short time windows under which bank resolution processes are typically conducted.

Third, in the similar context of the bank-supervisor relationship, several empirical analyses show that information acquisition is better the closer the geographical and cultural proximity between banks and their supervisors. Gopalan, Kalda, and Manela (2017) find that a larger geographical distance reduces the supervisor’s effectiveness in reducing the bank’s risk-taking behavior. Along these lines, Behn, Haselmann, Kick, and Vig (2015) state that “[...] proximity [between supervisors and banks] has the potential to improve the decision-making process, as it provides politicians with good information about banks that get into distress.”

Fourth, although the transposition into national law of the BRRD has constituted an
important step towards the harmonization of the legal framework governing bank resolutions in the EU, “the divergence of national insolvency laws is a major obstacle towards a fully-fledged banking union”, as recently acknowledged by the Chair of the ESRB.\textsuperscript{19} The particularities of national insolvency laws do in fact play a key role in determining the counterfactual of “no-creditor-worse-off” required for an application of the bail-in tools that is not exposed to potentially very costly litigation actions from affected bank stakeholders. In this respect, national authorities are likely to have a deeper knowledge of the intricacies of the domestic insolvency law due to the expertise of their human capital and the experience built throughout time operating under this legal framework.

Finally, as argued by Ferrarini (2015), the institutional set-up of the Euro Area banking union exhibits “semi-strong” centralization, consisting on supranational authorities with prerogatives on banks supervision and resolution, but sharing their tasks and operation in coordination with national authorities. This architecture may well constitute the optimal institutional response to the informational advantages of national authorities, at least in the current juncture.

**Transfers across countries only for bail-out funding**

The mechanism analyzed in the paper restricts transfers across countries within the banking union to resolve failing banks. As we shall see below, ex-post efficient resolution policies lead to net positive transfers from the strong to the weak country, which precludes the voluntary participation of the strong country in the banking union if countries are sufficiently asymmetric. In this case, the optimal feasible banking union is not ex-post efficient, featuring reduced contributions of the strong country to the common fiscal backstop.

Allowing transfers unrelated to the funding of bail-outs from the weak to the strong country—either at the initial date or at the final date if the weak country is stable—would restore ex-post efficiency. Such transfers would be equivalent to an insurance fee paid by the weak country to compensate the strong country for its expected larger contribution to the funding of bail-outs abroad. However, this sort of transfers are likely to confront important political economy constraints that may make its practical implementation unfeasible.\textsuperscript{20}


\textsuperscript{20}In fact, as evidenced by several agreements among Euro Area countries, there exists a strong political opposition to build on financial capabilities to address uncertain episodes of financial distress. Common public funds to support a sovereign in distress are typically only mobilized once the dreadful event has
Nevertheless, the central message stemming from the paper is that the optimal banking union requires country-specific policies that recognize the heterogeneity of the initial fiscal standpoint of the countries (Proposition 3). This feature of the optimal resolution policy would remain even if we allow for unconditional transfers. The only departure from the current analysis would be that the optimal resolution policies would feature an insurance premium paid by the weak to the strong country, instead of a larger contribution to fund bail-outs both at home and abroad by the weak country.

**National authorities’ preferences**

As noted by Navaretti, Calzolari and Pozzolo (2015), the assistance of national resolution authorities to the SRB on the preparation of resolution plans, potentially creates a conflict of interest or coordination problems between national and central authorities, since their objectives are likely not aligned entirely. Along these lines, we assume that each domestic supervisor is concerned only with the welfare of the agents within its constituency, as in Colliard (2017) and Calzolari, Colliard and Lorantz (2016), where local supervisors do not take into account the cross-border externalities that banks impose on foreign countries, but only their impact on the local economy.\(^{21}\) This view is consistent with the long-standing political economy view that national agents neglect the impact of their actions on foreign countries, recently put forward by Faia and Weder di Maduro (2015).

Our assumption that national authorities neglect the impact of resolution policies on the welfare of the foreign country’s agents is extreme. In fact, the set of national central banks within the Eurosystem has evolved towards operating as a single multinational institution aiming at enhancing the welfare of the Euro Area beyond national borders. However, this assumption is essentially a simplifying one: All of our qualitative results hold if we assume that national authorities also care for the well-being of the agents of the banking union at large, as long as they attribute a higher weight to their respective constituencies.

**Cost of bank bail-in independent of the state of the sovereigns**

We assume for the sake of simplicity that the distribution of the bail-in disruption cost is independent of the state of the sovereigns. We could instead adopt the arguably more realistic assumption that the bail-in cost distribution is shifted to the right when the failing

---

\(^{21}\)Other related papers that assume that there is a misalignment between national and supranational authorities are Holthausen and Ronde (2004) and Carletti, Dell’Ariccia and Marquez (2017).
bank’s home country is undergoing a crisis. As long as the bail-in distribution is common knowledge, as in our current framework, this alternative assumption would only entail a notational change, but no qualitative implications.

**Cross-border liabilities**

The main rationale for the creation of a banking union within the framework of this paper is risk-sharing across countries with independent banking systems. Many commentators have argued that the banking union should also serve the purpose of providing a common space for an increasingly integrated banking system. We take on this issue in Section 7.2 and show that the main insights from the baseline analysis generalize in set-up in which there are cross-border debtholdings. Moreover, we find that introducing cross-border liabilities in our framework partially realigns the home and the foreign countries’ objectives, increasing the welfare gains through the creation of a banking union.

### 3 Bank resolution in autarky

In this section we characterize the optimal resolution policy of a country that addresses its bank failures on its own. The associated expected utility of its agents constitutes the outside option for the country when deciding whether or not to participate in a banking union.

Consider first the situation in which the failure of a bank at $t = 1$ occurs when the home country is stable. A bail-out amounts to a redistribution of one unit of funds within local agents at a (net) social cost of $S_1 \cdot 1$. Hence, the home authority bails out the bank when the bail-in contagion cost exceeds the social cost of the bail-out, that is, when $k \geq \lambda_S$. The expected social cost of a bank resolution when its home country is stable is thus given by:

$$\Pi^S = \int_{\text{Bail-in cost}}^{\text{Bail-out cost}} [kdG(k) + (1 - G(\lambda_S))] \lambda_S.$$  \hfill (1)

If the bank’s home country is undergoing a crisis the social cost of funds is $\lambda_C > \lambda_S$, which increases the cost of bail-outs and, consequently, elevates the bail-out region to $k \geq \lambda_C$. The expected social cost $\Pi^C$ of a bank resolution when its home country is undergoing a crisis is then given by an expression analogous to (1) in which $\lambda_S$ is replaced by $\lambda_C$. The expected
cost difference of a bank resolution in these two states is therefore given by:

$$
\Pi^C - \Pi^S = \int_{k_S}^{k_C} (k - \lambda_S) dG(k) + \left[1 - G(\lambda_C)\right] (\lambda_C - \lambda_S) > 0.
$$

When the bail-in cost is small ($k < \lambda_S$), banks are bailed in regardless of the social cost of funds. Therefore the resolution cost does not depend on whether the country is stable or undergoing a sovereign crisis. The first term in Expression (2) captures the cost difference between a bail-in and a bail-out financed by a stable country, in the range of bail-in cost in which bail-outs are conducted if and only if the country is stable (i.e., for $\lambda_S \leq k < \lambda_C$). The last term in Expression (2) accounts for the difference in the social cost of funds between a country undergoing a crisis and a stable country in the range in which the optimal policy is to bail out the bank regardless of the state of the country (i.e., for $k \geq \lambda_C$).

Taking into account that banks’ debt is priced competitively,\(^{22}\) the expected utility as of $t = 0$ of the agents in country $i$, which we denote $U_i$, is given by:

$$
U_i = pR - \frac{1}{1 - p} (1 - p) \Pi^S - (1 - p) \Pi^C (\Pi^C - \Pi^S).
$$

The first term in this expression accounts for the expected value of the bank’s project, while the second and third terms capture the expected resolution cost. The second term stands for the expected resolution cost for a country that is always stable, and constitutes a lower bound on resolution costs. The third term, in turn, accounts for the additional resolution cost in the event in which the country is undergoing a crisis when its bank fails. Observe that the strong country’s expected utility is higher than the weak country’s (i.e., $U_1 > U_2$), since the probability that the former experiences a sovereign crisis is lower.

\(^{22}\)Notice that the equilibrium promised repayment $D_i^*$ required by the competitive debtholders anticipates the probability of a bailout, so that it satisfies:

$$
pD_i^* + (1 - p) \left[ \eta_i^* (1 - G(\lambda_S)) + \eta_i^* (1 - G(\lambda_C)) \right] \cdot 1 = 1.
$$

The particular value of such promised repayment does not have welfare implications. We henceforth omit it from our computations.
4 Optimal feasible banking union

In this section we characterize the optimal feasible banking union, that is, the aggregate welfare maximizing banking union within the class of agreements that ensure the participation of the two countries at \( t = 0 \). In order to do so, we first describe the ex-post optimal resolution policies in a banking union. Then, we show that these are feasible if and only if the degree of asymmetry between the two countries is not too large. Finally, we analyze how resolution policies in the optimal feasible banking union with large country asymmetry differ from ex-post optimal ones to ensure the participation of the strong country.

4.1 Ex-post optimal resolution policies

In this section, we describe the ex-post optimal resolution policies in a banking union for each bank failure state. These are defined as the aggregate welfare maximizing resolution policies that can be designed at \( t = 1 \) by a central authority with the prerogative of raising funds from the two countries, but without information on the realization of the bail-in contagion costs.

We start with the analysis of the case in which a bank fails at a country that is undergoing a sovereign crisis, while the other country is stable, which corresponds to the bank failure states \((1, CS)\) and \((2, SC)\). A resolution policy is characterized by the home contribution \( x \) to the funding of a bail-out. Observe that, by construction, the ex-post optimal resolution policy is independent of whether the home country is strong or weak.

We first address the effect of the required bail-out funding home contribution \( x \) on the home authority decision to ask for either a bail-out or for a bail-in after observing that the bail-in disruption cost amounts to \( k \). Bailing out the bank entails a utility for the agents in the home country of \( 1 - (1 + \lambda_C)x \). The first term corresponds to the transfer to the failing bank’s debtholders associated with the bail-out. The second stands for the gross social cost of the resources that the home country is required to contribute to the bail-out. Bailing in the bank, in turn, entails a utility of \(-k\). Comparing the two, we have that the home authority prefers to ask for a bail-out whenever the bail-in disruption cost \( k \) is above a threshold \( \overline{F}(x) \) given by:

\[
\overline{F}(x) = \max \{(1 + \lambda_C)x - 1, 0\}. 
\]
Note that the bail-out threshold is $\bar{k}(x) = 0$ for all $x \leq \frac{1}{1+C}$. The reason is that when the home contribution is small, the transfer that local debtholders receive upon a bail-out exceeds the social cost of the required home contribution. Hence, the home country authority always calls for a bail-out regardless of the observed $k$. In addition, the bail-out threshold is increasing in $x$ because higher home contributions make bail-outs more costly for the home country. Finally, we have that $\bar{k}(1) = \lambda_C$, since when the home country is required to fund the bail-out in its entirety (as in autarky) it fully internalizes the cost of bail-outs.

The expected aggregate social resolution cost associated with a resolution policy with home contribution $x$ to bail-outs is therefore given by:

$$\Pi(x) = \int_0^{\bar{k}(x)} kdG(k) + (\lambda_S + (\lambda_C - \lambda_S)x)(1 - G(\bar{k}(x))).$$

(5)

The first term captures the expected disruption cost arising from bail-ins. The second term corresponds to the cost of funding a bail-out. Let us highlight that the latter is increasing in the home country contribution due to the higher cost of funds in a country undergoing a sovereign crisis.

The ex-post optimal resolution policy corresponds to the home contribution $x^* \in [0, 1]$ that minimizes $\Pi(x)$. Notice that Assumption 2 implies that any home contribution leading to an unconditional bail-out (i.e., $x \leq \frac{1}{1+C}$) is dominated by the autarky solution that home countries bail out banks only when the disruption bail-in cost is sufficiently large. We can hence focus on the interval $x \in \left(\frac{1}{1+C}, 1\right]$. In order to understand the drivers of the central authority decision, consider the marginal effect of an increase in the home country’s contribution $x$ on the expected aggregate social cost of resolution $\Pi(x)$. Differentiating (5) yields:

$$\Pi'(x) = (\lambda_C - \lambda_S) \left[1 - G(\bar{k}(x))\right] - (1 + \lambda_S)(1 - x)(1 + \lambda_C)g(\bar{k}(x)).$$

(6)

The first term in Expression (6) is a cost effect, which accounts for the increase in the social cost of funding the bail-out by requiring the home country undergoing a crisis to contribute a higher share. This effect is positive due to the wedge in the cost of public funds between the home and the foreign country, thus pushing down the optimal home contribution to bail-outs. The second term is an overstatement effect, accounting for the diminished incentives
of the home country to call for a bail-out when its contribution to bail-outs increases. This effect is negative, pushing up the optimal home contribution to the funding of bail-outs.

The overstatement effect appears because the home authority does not fully internalize the cost of a bail-out. In particular, the aggregate cost of funding a bail-out is given by \( \lambda_S + (\lambda_C - \lambda_S)x \), while the home authority only faces a cost \( F(x) \) (c.f. Expression (4) above). The wedge between the two, which is given by \( (1 + \lambda_S)(1 - x) \), vanishes when the home country is required to fund bail-outs in its entirety (i.e., when \( x = 1 \)). As a consequence, we have that \( \Pi'(1) > 0 \), which in turn implies that \( x^* < 1 \). Hence, the ex-post optimal resolution policy entails risk-sharing across countries, which reduce the aggregate resolution cost in states in which only one country is undergoing a crisis.

The analysis for each of the other bank failure states can be conducted analogously. Observe that in none of the remaining states the cost of public funds in the home country is higher than in the foreign country. As a result, the bail-out cost effect identified in (6) is weakly negative. Hence, both the cost and overstatement effects push up the optimal home contribution to bail-outs, so that it is optimal that the home country of a failing bank funds its bail-out in its entirety. Therefore, there is no risk-sharing between the countries in the remaining states.

The following proposition summarizes our discussion in this section.

**Proposition 1 (Ex-post optimal resolution policies)** The vector \( x^* = (x^*_i, x^*_f) \) of home contributions to bail-outs characterizing the ex-post optimal banking union satisfies:

(i) When the failing bank is located in a country undergoing a crisis, and the neighboring country is stable, that is, in the bank failure states \( (i, \sigma) \in \{(1, CS), (2, SC)\} \), the home contribution is given by \( x^*_i = x^* \in \left( \frac{1}{1 + \lambda_S}, 1 \right) \), which is uniquely characterized by the following condition:

\[
\Pi'(x^*) = 0.
\]

(ii) In any other bank failure state, that is, for \( (i, \sigma) \notin \{(1, CS), (2, SC)\} \), home countries fund bail-outs in their entirety, that is, \( x^*_{i, \sigma} = 1 \).
4.2 Feasibility of ex-post optimal resolution policies

In this section we ask whether the ex-post optimal banking union $x^*$ characterized in Proposition 1 satisfies the participation constraint of the two countries at the initial date, in which case the optimal feasible banking union coincides with the ex-post optimal banking union.

We denote $U_i(x^*)$ the expected utility, as of $t = 0$, of the agents in country $i$ within the banking union $x^*$. Considering that the ex-post optimal resolution policies differ from those in autarky only in the bank failure states $(1, CS)$ and $(2, SC)$, in which the home contribution to bail-outs is $x^* < 1$, we can write the strong country’s (positive or negative) net gain from joining the banking union $x^*$ as:

$$U_1(x^*) - U_1 = (1 - p)q^{CS} \left( \frac{\Pi(x^*)}{\text{Agg. resolution cost}} - \frac{1}{2} q^{SC} q^{CS} (1 - x^*) (1 + \lambda_3) (1 - G(F(x^*))) \right) - \frac{1}{2} q^{SC} (1 - x^*) (1 + \lambda_3) (1 - G(F(x^*))) ,$$

where $U_1$ is as described in Expression (3). The interpretation of this expression goes as follows. The first term captures the expected bank resolution cost for the strong country in the bank failure state $(1, CS)$. Note that it is obtained by subtracting the weak country’s cost from the aggregate resolution cost. The second term accounts for the transfer to the weak country in the bank failure state $(2, SC)$.

We can rewrite Expression (8) more compactly as follows:

$$U_1(x^*) - U_1 = (1 - p)q^{CS} (\Pi^C - \Pi(x^*)) - (1 - p)q^{SC} q^{CS} (1 - x^*) (1 + \lambda_3) (1 - G(F(x^*))) .$$

The first term captures the gains attained by the creation of a banking union, which reduces the aggregate resolution cost when the strong country is undergoing a crisis and the weak country is stable (note that $\Pi(x^*) < \Pi(1) = \Pi^C$). The second term stands for the expected net transfers from the strong to the weak country implied by the foreign contribution to bail-outs $1 - x^*$ in states $(1, CS)$ and $(2, SC)$. Note that the term is proportional to the difference $q^{SC} - q^{CS}$ in the probabilities of these two states. Since $q^{SC} - q^{CS} = q^S_1 - q^S_2 > 0$,
the expected net transfer is positive for country 1. Whenever the second term in Expression (9) dominates the first one, the ex-post optimal banking union is not feasible, as the strong country prefers to remain in autarky.\footnote{The expression for the weak country’s gain from joining the ex-post optimal banking union can be obtained by swapping the direction of the difference $q^{CS} - q^{SC}$ in Expression (9), which shows that the weak country is a net receiver of funds and thus always finds it beneficial.} The next proposition states that the ex-post optimal banking union is feasible if only if the degree of country asymmetry is not too large:

**Proposition 2 (Feasibility of the ex-post optimal banking union)** There exists $a \in (0, 1)$ such that the optimal feasible banking union exhibits ex-post optimal resolution policies if and only if $q^{CS} \geq a q^{SC}$.

### 4.3 Optimal feasible banking union with large country asymmetry

In this section we characterize the optimal feasible banking union when country asymmetry is large enough so that the ex-post optimal banking union does not meet the participation constraint of the strong country. In order to formalize the problem, we generalize the notation and objects introduced in Section 4.1 for the derivation of the ex-post optimal resolution policies at the bank failure states $1, CS$ and $2, SC$. For a given pair of public finances $\sigma$ at $t = 1$, we denote by $\lambda_{1,\sigma}$ and $\lambda_{2,\sigma}$ the social cost of funds in countries 1 and 2, respectively.\footnote{That is, $\lambda_{1,SC} = \lambda_{1,SS} = \lambda_{2,CS} = \lambda_{S}$ and $\lambda_{1,CS} = \lambda_{1,CC} = \lambda_{2,CC} = \lambda_{C}$.}

In addition, for a resolution policy described by a home contribution to bail-outs $x$, we denote by $k(x; i, \sigma)$ the bail-out threshold that it induces, and we let $\Pi(x; i, \sigma)$ denote its expected aggregate resolution cost. Their expressions, which are analogous to Expressions (4) and (5), respectively, are given by:

$$
\mathcal{K}(x; i, \sigma) = \max \left\{ (1 + \lambda_{1,\sigma})x - 1, 0 \right\},
$$

$$
\Pi(x; i, \sigma) = \int_{0}^{x} k dk G(k) + \left( \lambda_{1,\sigma} + \lambda_{2,\sigma} - \lambda_{i,\sigma} \right) x (1 - G(\mathcal{K}(x; i, \sigma))).
$$

It is also convenient to introduce the function $t(x; i, \sigma)$, which accounts for the expected gross social cost incurred by the foreign country (country $i$) in contributing to the funding of bail-outs in the bank failure state $(i, \sigma)$. Its expression is given by:

$$
t(x; i, \sigma) = (1 - x) \left( 1 + \lambda_{i,\sigma} \right) \left( 1 - G(\mathcal{K}(x; i, \sigma)) \right).
$$
Let \( x = (x_i) \) be a banking union. We can write the expected utility at \( t = 0 \) of agents in country \( i \) within the banking union \( x \) as follows:

\[
U_i(x) = \frac{pR - 1 - (1 - p) \sum \Pi(x_{i,j}[i,\sigma])}{\text{NPV}} \left( \frac{\text{Aggregate resolution cost}}{\text{Cost of transfers from foreign country}} \right) - (1 - p) \sum q^\sigma t(x_{i,j}[i,\sigma]) .
\]

Note that the utility decomposition in Expression (13) accounts for any possible home contribution to bail-outs in each bank failure state \((i, \sigma)\). The second term stands for the expected (net) cost incurred by country \( i \) in the resolution of its own bank. This expression is obtained by subtracting the expected cost of transfers incurred by the foreign country from the aggregate resolution cost. The third term in (13) takes into account the expected cost of transfers to fund bail-outs at the foreign country.

Adding up Expression (13) for country 1 and 2 we can write the aggregate welfare in the banking union as follows:

\[
W(x) = U_1(x) + U_2(x) = 2(pR - 1 - (1 - p) \sum \Pi(x_{i,j}[i,\sigma]) .
\]

The expression captures the NPV of the two bank projects, as well as the expected aggregate resolution costs in each bank failure state. Note that the expressions \( t(x_{i,j}[i,\sigma]) \) and \( t(x_{i,j}[i,\sigma]) \) cancel out, as they constitute transfers among countries.

The optimal feasible banking union is the solution to the following maximization problem:

\[
\max_{x} W(x) \quad \text{s.t.} \quad U_1(x) \geq U_1(\text{PC}_1) \quad U_2(x) \geq U_2(\text{PC}_2) .
\]

It follows from (14) that solving the maximization problem in (15) is equivalent to minimizing the aggregate resolution costs, subject to the participation constraints of the two countries.

The following proposition provides necessary and sufficient conditions for a feasible banking union to exist when country asymmetry is large, and provides its characterization whenever it exists.
Proposition 3 (Optimal banking union with large country asymmetry) Let $a$ be as defined in Proposition 2 and assume that $q^{CS} < aq^{SC}$ so that the ex-post optimal banking union is not feasible. Then, there exists a banking union that strictly increases aggregate welfare relative to autarky if and only if $q^{SC} < 1$. In such case, the optimal banking union $\hat{x} \equiv (\hat{x}_{i,a})$ is unique and satisfies:

(i) Only the participation constraint of the strong country binds, that is, $U_1(\hat{x}) = \overline{U}_1$ and $U_2(\hat{x}) > \overline{U}_2$.

(ii) If a bank fails in a country that is undergoing a crisis, and the other country is stable, then the home contribution to a bail-out is smaller (resp. larger) than ex-post optimal if the failing bank is located in the strong (resp. weak) country, that is, $\hat{x}_{1,CS} < x^* < \hat{x}_{2,SC}$.

In addition, there is always foreign aid, even if the failing bank is located in the weak country, that is, $\hat{x}_{2,SC} < 1$.

(iii) If a bank fails, and the state of the public finances of the two countries is the same, then there is foreign aid to fund a bail-out if and only if the failing bank is located in the strong country, that is, $\hat{x}_{1,SS} < \hat{x}_{2,SS} = 1$ and $\hat{x}_{1,CC} < \hat{x}_{2,CC} = 1$.

(iv) If a bank fails in a country that is stable, and the other country is undergoing a crisis, then there is no foreign aid to fund bail-outs, that is, $\hat{x}_{1,SC} = \hat{x}_{2,CS} = 1$.

The optimal distortion of the ex-post optimal policies to ensure the participation of the strong country balances the increase on the aggregate resolution cost and the effect on softening the participation constraint of the strong country. In order to shed light on the departure from the optimal feasible banking union, we analyze below a series of stylized economies in which, besides $\sigma = SC$, only one additional pair of public finances occurs with positive probability. The nature of the risk-sharing agreements in these simple setups extend to the general case characterized in Proposition 3.

Optimal banking union in stylized economies with $q^{SC} \in (0,1)$

(i) $q^{CS} > 0$ and $q^{SC} + q^{CS} = 1$ (Countries’ public finances perfectly negatively correlated).
When the countries public finances are perfectly negatively correlated the strong country can obtain foreign aid in the bank failure state \(1, CS\) and contribute to the funding of bail-outs at the weak country in the bank failure state \(2, SC\). Observe that in any such bank failure states, there exists a threshold \(x < x^*\) such that any amount of foreign aid reduces the aggregate resolution costs as long as the home contribution to bail-outs \(x_i;\) lays in the interval \(x_i; \in (x, 1)\). The proposition states that feasible risk-sharing is optimally achieved by reducing the (more frequent) strong country foreign aid below the ex-post optimal level, and raising the (less frequent) weak country aid above the ex-post optimal level, that is:

\[
0 < 1 - \tilde{x}_{2, SC} < 1 - x^* < 1 - \tilde{x}_{1, CS}.
\]

Notice that both distortions contribute to softening the strong country participation constraint. On the one hand, reducing the strong country’s contribution to fund bail-outs at the weak country leads to a strict increase in the strong’s country expected payoff. This can be seen by differentiating the strong country’s expected utility laid out in Equation (13) with respect to the weak country’s home contribution, that is:

\[
\frac{\partial U_1 (x)}{\partial \tilde{x}_{2, SC}} \bigg|_{\tilde{x}_{2, SC} = x^*} = -(1 - p)q^{SC} \frac{\partial t (x^*|2, SC)}{\partial \tilde{x}_{2, SC}} > 0.
\]

Observe that \(t(x|i, \sigma)\) is strictly decreasing in \(x\) for all bank failure states \((i, \sigma)\), reflecting the fact that higher contributions by the home country reduce both the likelihood of bail-outs and the amount that the foreign country is required to contribute when they are conducted. On the other hand, observe that reducing the strong country’s home contribution to the bail-out of its own bank also increases its payoff strictly, since we have that:

\[
\frac{\partial U_1 (x)}{\partial \tilde{x}_{1, CS}} \bigg|_{\tilde{x}_{1, CS} = x^*} = \frac{\partial H(x^*|1, CS)}{\partial \tilde{x}_{1, CS}} - \frac{\partial t (x^*|1, CS)}{\partial \tilde{x}_{1, CS}} = (1 + \lambda_C) \left[ 1 - G \left( \tilde{x}^* (x^*|1, CS) \right) \right] > 0.
\]

Moreover, it is always convenient to distort both resolution policies. Since the ex-post optimal policy satisfies \(H(x^*) = 0\), distortions around the ex-post optimal arrangement have a negligible impact on the aggregate resolution cost.

\[\text{(ii) } q^{SS} > 0 \text{ and } q^{SC} + q^{SS} = 1 \text{ (Strong country always stable)}.\]
If the strong country is always stable and the two countries are stable contemporaneously with positive probability, the strong country can obtain foreign support to bail out its bank in the latter contingency. In order to see that a banking union is feasible in this economy, consider the derivative of the aggregate resolution costs function in the bank failure state \((1, SS)\), which is given by:

\[
\Pi'(x_{1, SS}|1, SS) = \frac{(\lambda_S - \lambda_S) \left[ 1 - G \left( \mathcal{F}(x_{1, SS}|1, SS) \right) \right]}{(1 + \lambda_S) (1 - x_{1, SS})} \left( 1 + \lambda_S \right) g(\mathcal{F}(x|1, SS)).
\]

Observe that foreign aid in state \((1, SS)\) leads to an overall reduction of welfare, because the overstatement effect is at work and there is no cost gain. However, its impact on the overall welfare is small when the strong country home contribution \(x_{1, SS}\) to a bail-out is large, that is, \(\Pi'(x_{1, SS} = 1|1, SS) = 0\). The reason is that in this case the strong country internalizes to a large extent the cost of bailing out a home bank. Therefore, a home contribution \(\hat{x}_{1, SS} < x^*_1\) softens the strong country participation constraint with a negligible overall welfare effect. The strong country can thus be compensated for its contribution to the weak country bail-outs in state \((2, SC)\) through foreign aid in state \((1, SS)\). In addition, the optimal feasible risk-sharing requires that the support given by the strong country be below the ex-post optimal level, that is, \(1 - \hat{x}_{2, SC} < 1 - x^*\). As explained above, the reason is that the impact of small distortions from \(x^*\) on aggregate welfare is negligible, while substantially softening the participation constraint of the strong country.

(iii) \(q^C > 0 \) and \( q^{SC} + q^{CC} = 1 \) (Weak country always undergoing a crisis).

Even when the weak country is always undergoing a crisis, the banking union can provide risk-sharing across countries, as long as the strong country may experience a crisis with positive probability. A welfare enhancing banking union is feasible because the aggregate resolution cost associated with the provision of foreign aid to the strong country in the bank failure state \((1, CC)\) is initially insignificant relative to the gains from the provision of foreign aid to the weak country in the bank failure state \((2, SC)\). The underlying reason is of the same nature as in the previous economy.
The above examples describe the optimal feasible banking union in the most simple economies with \( q^{SC} < 1 \). Observe that the main features of the resolution policies described in the stylized examples extend to the general case characterized in Proposition 3.

The proposition also states that when \( q^{SC} = 1 \), so that the strong country is always stable and the weak country is always undergoing a crisis, there is no possibility of creating a banking union. The reason is that in such an economy the only way of compensating the strong country for providing foreign aid in state \((2, SC)\) would be through support in the state \((1, SC)\). In this case, in addition to the distortion induced by the overstatement effect, there would be a cost loss in any transfer from the weak to the strong country. This is also the reason why in any optimal banking union countries undergoing crises never provide foreign aid to stable countries.

In order to assess the gains and losses induced by the banking union, we can write:

\[
U_1(\bar{x}) + U_2(\bar{x}) - (U_1 + U_2) = (1 - p) \left[ q^{CS} (\Pi^C - \Pi(\bar{x}_{1CS})) + q^{SC} (\Pi^C - \Pi(\bar{x}_{2SC})) \right] > 0
\]

\[
- (1 - p) \left[ q^{SS} (\Pi(\bar{x}_{1SS}|1, SS) - \Pi^S) + q^{CC} (\Pi(\bar{x}_{1CC}|1, CC) - \Pi^C) \right] > 0
\]

The first term includes the reduction of the aggregate resolution cost in the bank failure states in which there are risk-sharing motives to mutualize the funding of bail-outs. Since \( \bar{x}_{1CS} < x^* < \bar{x}_{2SC} \), this reduction is smaller than in the ex-post optimal banking union. The second term in (19) captures the social cost of the excessive amount of bail-outs of the bank in the strong country in states in which there is no risk-sharing motive to provide foreign aid, but simply the need to compensate the strong country to form the banking union.

Finally, observe that the participation constraint of the strong country binds. Hence, entering the banking union does not affect the expected aggregate welfare of the strong country’s agents. However, forming a banking union leads to a redistribution of welfare in that country from the public to the bank owners, as the bank is bailed out more frequently within the banking union.

We conclude this section by highlighting a policy implication of our results: When country
asymmetry is large, an aggregate welfare maximizing banking union requires the design of risk-sharing rules that are country-specific, explicitly recognizing the ex-ante differences among countries. Aiming at a symmetric banking union may render the project unfeasible. It is worth emphasizing that while the country-specific resolution policies in the optimal banking union may seem to privilege the strong country ex-post, it is indeed the weak country that appropriates the entire welfare gains associated with the creation of a banking union from an ex-ante perspective.

5 Net transfers across countries in an optimal banking union

A general principle governing EU wide risk-sharing policies is that they should not lead to permanent transfers across states.\(^{27}\) This position has been spearheaded by countries with strong economies in an attempt to prevent that the creation of common fiscal backstops becomes a hidden instrument to subsidize countries with weaker fiscal positions.

In this section, we argue that the optimal feasible banking union among countries with a large degree of asymmetry characterized in Proposition 3 may lead to the weak country being a net contributor of funds. In particular, this is the case if the strong country is always stable. More generally, we illustrate how the degree of asymmetry and the correlation of public finances among countries shape transfers in the optimal feasible banking union and determine which country is a net fiscal contributor.

Throughout this section we fix the value of the probability \(q^{SC} > 0\). Moreover, for ease of exposition, and without any significant loss of insight, we let \(q^{CC} = 0\). The distribution of public finances pairs at \(t = 1\) is thus entirely described by the probability \(q^{CS} \in [0, q^{SC}]\), where the upper bound of the latter interval stems from Assumption 1. Notice that \(q^{CS}\) stands for the probability that the strong country experiences a sovereign crisis, given that

\(^{27}\)This principle has been reaffirmed in the recent “Interparliamentary conference on stability, economic coordination and governance in the EU,” celebrated in Bratislava in October, 2016. In particular, in the context of the establishment of a common fiscal capacity at the EU level, the background note for the meeting stated that it “must not represent a disguised form of permanent transfers from one part of the union to another. Without effective safeguards in the design of the mechanism, which will ensure its fiscal neutrality amongst the Member States in the medium term, it could prove difficult to address the concerns of some Member States that they will become overall net contributors.” http://www.ipex.eu/IPEXLEWEB/dossier/files/download/082d1c55797141801579925d315019b.do
$q^{CC} = 0$. The value $q^{CS} = 0$ represents the case of maximal asymmetry, whereby the strong country is always stable. On the other extreme, when $q^{CS}$ gets arbitrarily close to $q^{SC}$, we have that countries are (almost) symmetric. For each $q^{CS} \in [0, q^{SC})$, we denote by $T(q^{CS})$ the expected net contribution of funds from the strong country to the weak country in the optimal feasible banking union.

Consider first the case in which the asymmetry among countries is not too large, that is, $q^{CS} > a q^{SC}$, where $a \in (0, 1)$ corresponds to the value described in Proposition 2, so that the ex-post optimal banking union is feasible. The expected net contribution of funds by the strong country is given by:

$$T(q^{CS}) = (1-p)(q^{SC} - q^{CS})(1-x^*)(1-G(x^*)) > 0.$$  (20)

When countries are not too asymmetric, the strong country is a net contributor of funds. Notwithstanding, the participation constraint of the strong country is slack. In addition, as $q^{CS}$ decreases (so that the strong country becomes less likely to experience a fiscal crisis), we have that $T(q^{CS})$ increases, so that the strong country becomes a larger contributor of funds. As long as the ex-post optimal banking union is feasible, transfers among countries only take place when the home country undergoes a crisis, and the foreign country is fiscally stable. Hence, the less likely the strong country is to experience a crisis, the higher its expected net fiscal contribution to the banking union.

This is no longer the case when the ex-post optimal banking union is not feasible, that is, when $q^{CS} < a q^{SC}$. Ensuring the participation of the strong country requires distortions of the ex-post optimal resolution policies to reduce the net transfers from the strong country to the weak one in each state. These distortions may be large enough to revert the position of the strong country as a net contributor within the banking union. In fact, the weak country is a net contributor when the strong country is always stable, that is, for $q^{CS} = 0$, as we show in turn. When the strong country is always stable, there are only two states in which there are transfers of funds across countries: The strong country contributes in state $(2, SC)$ and receives a compensation in state $(1, SS)$. Hence, using the characterization of the optimal banking union $\tilde{\pi} \equiv (\tilde{\pi}_{i,s})$ in Proposition 3, we have that:

$$T(q^{CS} = 0) = (1-p)q^{SC}(1 - \tilde{x}_{2,SC})(1 - G(\tilde{x}_{2,SC}|SC)) - (1-p)q^{SS}(1 - \tilde{x}_{1,SS})(1 - G(\tilde{x}_{1,SS}|SS)),$$  (21)
Moreover, we can use the fact that the strong country’s participation constraint is binding and write Expression (13) as:

\[
(1 - p)q^{SS}(\Pi(\tilde{x}_{1,SS}|1, SS) - \Pi^F) = (1 - p)(1 + \lambda_S) \left[ q^{SS}(1 - \tilde{x}_{1,SS}) \left( 1 - G(\tilde{x}_{1,SS}|SS) \right) - q^{SC}(1 - \tilde{x}_{2,SC}) \left( 1 - G(\tilde{x}_{2,SC}|SC) \right) \right],
\]

or:

\[-(1 + \lambda_S)T(q^{CS} = 0) = (1 - p)q^{SS}(\Pi(\tilde{x}_{1,SS}|1, SS) - \Pi^F) > 0,
\]

where we have used that \( \tilde{x}_{1,SS} < 1 \), and the fact that \( \Pi(x|1, SS) \) is strictly decreasing in \( x \). The latter inequality implies that the strong country is a net receiver of funds. More precisely, the gross (social) value of the expected net transfer that the strong country receives (left hand side) amounts to the increase in aggregate resolution cost that the foreign aid in state \( (1, SS) \) generates (right hand side), which stems from the overstatement effect that the foreign contribution to bail-outs \( 1 - \tilde{x}_{1,SS} \) induces without any associated cost gain.

Our analysis emphasizes the need to distinguishing between the objectives of the different parties involved (namely, aggregate welfare, and domestic welfare maximization, for central and domestic authorities, respectively) and the instruments employed to achieve them (such as risk-sharing through cross-country contributions to the funding of bail-outs). The main policy message from this analysis is that imposing zero net expected cross-country transfers is not the optimal way of ensuring each country’s legitimate demand that the creation of a banking union is not detrimental to its constituency.

### 6 Early intervention

So far we have circumscribed the authorities’ decisions to the resolution of a bank when the realized value of its assets is not enough to repay debt investors. Authorities can in practice *early intervene* a bank to protect depositors and other senior debtholders when they believe that the bank is likely to fail at a future date. Early intervention ahead of a potential further deterioration of the bank assets allows authorities to ensure the repayment of all senior debtholders without the need to recourse to public funds.

In this section we extend the model to allow for both late resolution and early intervention actions. We analyze how these two are optimally set in a banking union. We introduce an
interim date, which we refer to as $t = 1/2$, in which the relevant resolution authority observes a signal $p_i$, standing for the probability that bank $i$’s project succeeds at $t = 1$. At $t = 0$, $p_i$ is regarded as a random variable with support $[0, 1]$, cumulative distribution function $F(\cdot)$ and strictly positive probability density function $f(\cdot)$, with $\int_0^1 dF(p_i) = p$. We also assume that at $t = 1/2$ the pair of public finances $\sigma \in \{SS, SC, CS, CC\}$ at $t = 1$ is common knowledge. An early intervention of the bank at $t = 1/2$ leads to the liquidation of the bank assets, which yields a payoff of $L \geq 1$. The authority uses these proceeds to repay the unit principal owed to the bank debtholders and distributes the residual payoff among the bank’s owners. Note that we assume that early intervention actions are based on a signal that is observed by both the central and domestic authorities. The rationale for this is that early intervention prevents the bail-in of debtholders, which makes the intervention process smoother and less prone to creating spillovers on the rest of the financial sector.

In this set-up a banking union consists of a pair of vectors $(z, \mathcal{p})$. The vector $z = (x_{i,\sigma})$ has the same meaning as in the baseline model. The vector $\mathcal{p} = (p_{i,\sigma})$ consists of the early intervention thresholds for the interim signal, below which the relevant authority intervenes the bank.

We first analyze the ex-post optimal intervention thresholds $\mathcal{p}(x_{i,\sigma}|i, \sigma)$, which depend on the resolution vector $x_{i,\sigma}$ through its effect on the aggregate resolution cost. They are characterized by the following indifference condition:

$$\mathcal{p}(x_{i,\sigma}|i, \sigma) R - (1 - \mathcal{p}(x_{i,\sigma}|i, \sigma)) \Pi(x_{i,\sigma}|i, \sigma) = L. \quad (24)$$

The left hand side of this expression accounts for the aggregate expected utility associated with letting the bank continue until $t = 1$. In particular, the first term captures the expected return of the bank assets. The second term includes the expected aggregate resolution costs, which takes into account both the probability that the bank fails and the resolution policies that will be adopted in that contingency. The right hand side of (24) accounts for the liquidation proceeds from the early intervention of the bank.

We now turn to the analysis of the feasibility of the ex-post optimal banking union $(z^*, \mathcal{p}^*)$, which maximizes the aggregate utility in the economy. The ex-post optimal banking union can be easily derived by backwards induction: It consists of the ex-post optimal late resolution policies $z^*$ characterized in Proposition 1, and the associated ex-post optimal early intervention thresholds $\mathcal{p}(x_{i,\sigma}^*|i, \sigma)$ given by Equation (24) evaluated in each ex-post
optimal value $x^*_r$ of home contributions.

The ex-post optimal banking union is feasible if and only if:

$$U_i(x^*, p^*) - U_i \geq 0.$$  

(25)

In a similar vein to the derivation of Expression (9), we can write this expression as:

$$R_i^1 \sum_{j=1}^n \left( \int_{p^{(j)}}^{1} (1-p) dF(p) \right) q^{CS} \left( \Pi_i^C - \Pi_i(x^*) \right) - \left( \int_{p^{(j)}}^{1} (1-p) dF(p) \right) \left( q^{CS} - q^{CS} \right) (1-x^*)(1 + \lambda_S) \left( 1 - G_k(x^*) \right) \geq 0,$$  

(26)

or:

$$q^{CS} \left( \Pi_i^C - \Pi_i(x^*) \right) - \left( q^{CS} - q^{CS} \right) (1-x^*)(1 + \lambda_S) \left( 1 - G_k(x^*) \right) \geq 0.$$  

(27)

Hence, the ex-post optimal banking union in the extended model is feasible if and only the the ex-post optimal banking union in the baseline model is feasible, that is, when country asymmetry is not too large, in the sense of Proposition 2.

Let us move to the analysis of the optimal feasible banking union when country asymmetry is large. In this case, not only late resolution decisions, but also early intervention actions, might not be ex-post optimal. We say that a banking union $(x, p)$ is forbearing in the early intervention of bank $i$ in state $\sigma$ if the intervention threshold $\Pi_i$ lays below the optimal intervention threshold characterized by Equation (24), that is, if $\Pi_i < \Pi_i(x^*|i, \sigma)$.

Similarly, we say that the banking union is tight in the early intervention of bank $i$ in state $\sigma$ if $\Pi_i > \Pi_i(x^*|i, \sigma)$.

In order to assess whether forbearance or tightness in early intervention are optimal we derive an analogous expression to (13) for the extended model:

$$U_i(x, p) = \sum_{\sigma} q^{CS} \left( \int_{p_\sigma}^{1} p dF(p) \right) R + F(p_\sigma)L - 1$$  

$$- \sum_{\sigma} \left( \int_{p_\sigma}^{1} (1-p) dF(p) \right) \left( \Pi_{i,\sigma} - \frac{t(x^*|i, \sigma)}{\text{Cost of transfers from foreign country}} \right) - \frac{t(x^*|\sigma)}{\text{Cost of transfers to foreign country}} - 1.$$  

(28)
Observe that, unlike Expression (13), the expression above takes into account that the bank is liquidated whenever the (state-contingent) signal lays below the (state-contingent) early intervention threshold, yielding a value of \( L \).

Suppose that for some bank failure state \((i, \sigma)\) the banking union prescribes a resolution policy given by \( x_{i,\sigma} < 1 \), so that country \( i \) receives foreign aid in that contingency. Using (24), we have that the derivative of the expected utility in country \( i \) with respect to the interim threshold \( p_{i,\sigma} \), evaluated at the ex-post optimal threshold \( p_{i,\sigma} = p(x_{i,\sigma}|i, \sigma) \), is given by:

\[
\frac{\partial U_i(x, p)}{\partial p_{i,\sigma}} \bigg|_{p_{i,\sigma} = p(x_{i,\sigma}|i, \sigma)} = q^p \left[ L - p(x_{i,\sigma}|i, \sigma) R - (1 - p(x_{i,\sigma}|i, \sigma)) \left( t(x_{i,\sigma}|i, \sigma) f(p(x_{i,\sigma}|i, \sigma)) \right) \right],
\]

or:

\[
= -q^p t(x_{i,\sigma}|i, \sigma) f(p(x_{i,\sigma}|i, \sigma)) < 0.
\] (29)

Hence, whenever a country receives foreign aid, early intervention forbearance increases the country’s expected utility. The intuition is that foreign transfers make the bank continuation more profitable to the recipient country than to the entire economy. Moreover, the definition of the ex-post optimal intervention threshold implies that:

\[
\frac{\partial W(x, p)}{\partial p_{i,\sigma}} \bigg|_{p_{i,\sigma} = p(x_{i,\sigma}|i, \sigma)} = 0.
\]

Hence, the aggregate welfare effect of marginal changes of the early intervention threshold around its ex-post optimal value is negligible. As a result, the participation constraint of the strong country, which is binding when country asymmetry is large, can be relaxed with a negligible aggregate welfare impact by allowing some early intervention forbearance in the states in which the strong country receives foreign aid. Analogously, if the banking union prescribes some amount of foreign aid to the weak country in some state, then it is ex-ante optimal that the central authority be tight in the intervention of the weak country’s bank to soften the participation constraint of the strong country. The next proposition characterizes the properties of early intervention policies in the optimal feasible banking union:

**Proposition 4 (Optimal early intervention with large country asymmetry)** Let \( a \) be as defined in Proposition 2 and assume that \( q^{CS} < a q^{SC} \) so that the ex-post optimal banking union is not feasible. Moreover, assume that \( q^{SC} < 1 \). In an optimal feasible banking union

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The proposition states that when country asymmetry is large the bank in the strong country will benefit from some early intervention forbearance. In contrast, the bank in the weak country will experience some tight early interventions. These two early intervention distortions complement the late resolution distortions in softening the strong country participation constraint. The preferential treatment of the bank in the strong country suggests that there could be a tension when countries differ substantially between the objective of minimizing resolution costs within a banking union and that of ensuring a level-playing field across banks.

7 Other extensions

In this section we extend the model in two directions. In Section 7.1 we analyze the optimal resolution mechanism when foreign support to the funding of bail-outs can be a mix of transfers to the failing bank's debtholders (direct recapitalization, as in the baseline model), and one-period loans to the fragile sovereign (indirect recapitalization). In Section 7.2 we introduce cross-border debt holdings and analyze the extent of risk-sharing in the presence of foreign ownership of debt claims.

7.1 Direct versus indirect aid

In the baseline model we have considered financial aid from the foreign country in the form of a transfer to the debtholders of the failing bank. This form of external support, which resembles the Direct Recapitalization Instrument (DRI) established by the European Stability Mechanism (ESM) in 2014, differs from the indirect recapitalization instrument (IRI), in which the ESM grants a loan to a fragile sovereign to support the resolution of domestic failing banks. The current agreements among EU members establish a pecking order in the use of these instruments: Access to the DRI can only be granted "if indirect recapitalization is not possible," and its use requires some contribution by the sovereign in distress.
In this section we extend the model to allow for both direct transfers and loans issued by the foreign country, and inquire about the optimality of prioritizing indirect foreign aid. We introduce an additional date, $t = 2$, at which the sovereigns may also raise public funds. Countries may be stable or undergo a crisis at $t = 2$. To keep the setup as simple as possible, we assume that a country undergoing a crisis becomes stable at $t = 2$ with probability $\rho_C \in (0, 1)$ and that a stable country continues to be stable with probability $\rho_S \in (0, 1]$. We denote the expected cost of public funds at $t = 2$ given the sovereign state $h \in \{S, C\}$ at $t = 1$ as $\tilde{\lambda}_h = \rho_h \lambda_S + (1 - \rho_h) \lambda_C$. We restrict loans to be fiscally neutral (in expectation) for the lending country, so that any subsidy among countries is channeled through direct transfers. Hence, the gross interest rate on a loan granted by a country in state $h \in \{S, C\}$ is given by $R_h = \frac{1 + \lambda_h}{1 + \lambda_S}$.

Formally, and similarly to the baseline model, we can describe a banking union by a pair $(x_{i,\sigma}, y_{i,\sigma})$, establishing for each of the bank failure states $(i, \sigma)$ the home country contribution $x_{i,\sigma}$ and the amount $y_{i,\sigma}$ borrowed as indirect aid to fund bail-outs. The remaining amount $1 - x_{i,\sigma} - y_{i,\sigma}$ is provided by the foreign country in the form of direct foreign aid. As in the baseline model, a standard mechanism-design approach allows us to focus on resolution mechanisms in which the decision of whether to bail out or to bail in the failing bank is delegated to the home resolution authority.

In order to illustrate the role for loans in this setup, consider a bank failure while the home country is undergoing a crisis and the foreign country is stable. Partially funding a bail-out through a foreign country loan plays the same role as home contributions in curbing the incentives of the domestic authority to overstate the need of foreign support, as the loan has to be paid back at $t = 2$. Moreover, since the country undergoing the crisis pays back the loan at a time at which it is stable with positive probability, substituting home contributions with a loan from the foreign country reduces the expected cost of funds for the borrowing country, since $\left(1 + \frac{1}{\tilde{\lambda}_C}\right) R_S < 1 + \lambda_C$. Moreover, by construction of $R_S$, the loan is fiscally neutral for the lending country. Therefore, indirect foreign aid dominates home contributions. Nonetheless, since $\tilde{\lambda}_C > \lambda_S$, direct foreign aid constitutes a less costly source of public funds from an aggregate perspective. Hence, as in the baseline model, there is an aggregate cost advantage of using direct foreign aid to fund banks’ bail-outs. Consequently, the optimal funding mix consists of a combination of a direct transfer and a loan from the
foreign country, thereby eliminating home contributions in the resolution of banks.

When both countries are either concurrently stable, or simultaneously undergoing a crisis, there is no gain from using loans. Hence, any loan could be substituted out by a contemporaneous home contribution with no payoff consequence for any of the countries. In order to simplify the exposition, in the following proposition we focus on the set of mechanisms in which the use of loans strictly improves welfare. A similar analysis to that conducted in Section 4 leads to the following result:

**Proposition 5 (Banking union with direct and indirect foreign aid)** Assume that $q^{SC} < 1$.

The optimal banking union $(\hat{x}, \hat{y})$ is such that:

(i) In states $(i, \sigma) \in \{(1, CS), (2, SC)\}$ bail-outs are funded with a combination of strictly positive amounts of direct (transfer) and indirect (loan) foreign aid— and no contemporaneous home contributions—, that is, $\hat{x}_{i,\sigma} = 0$ and $\hat{y}_{i,\sigma} \in (0, 1)$ (and thus $1 - \hat{x}_{i,\sigma} - \hat{y}_{i,\sigma} \in (0, 1)$).

(ii) In states $(i, \sigma) \notin \{(1, CS), (2, SC)\}$, indirect (loan) foreign aid is not issued, that is, $\hat{y}_{i,\sigma} = 0$, so that any source of foreign aid is channeled through direct transfers.

The proposition states that in every contingency in which it is optimal to issue loans, these must be combined with a direct transfer from the issuing country to reduce the aggregate cost of bail-outs. In addition, in those states, it is also optimal not to require home contributions, as the loans provide incentives for local authorities not to overstate the need of bail-outs.

Unlike the ESM design of the DRI as a last resort instrument, this result suggests that the DRI should be considered as an integral part of optimal resolution schemes, which should be systematically used in combination with the IRI. Moreover, the access to the DRI should be granted without requiring the home sovereign undergoing a crisis to contribute to the funding of bail-outs contemporaneously.

### 7.2 Cross-border debt holdings

In this section we show that the presence of cross-border liabilities provides an additional motive for countries to enter a banking union and increases the welfare gains generated by such an agreement. Formally, we assume that a fraction $\beta$ of each bank’s debt is held by agents of the foreign country. The case $\beta = 0$ thus corresponds to the baseline model. We keep the assumption that bail-ins originate a disruption cost of $k$ at the home country.
that is privately observed by the home authority. This assumption is particularly plausible for banks whose core lending activities are domestic, in which case the potential contagion to other banks will be presumably confined to domestic institutions whose assets are more correlated with those of the failing bank.\footnote{The analysis under the assumption that a bank bail-in generates contagion costs also in the foreign country whose magnitude is privately observed by the foreign authority would lead to a mechanism design problem in which the principal (the central authority) has to provide incentives to reveal their private information to two agents (both the home and the foreign authority). The formal analysis in such a case becomes much more complex and is beyond the scope of this paper.}

Consider the resolution of a bank at a country undergoing a crisis when the foreign country is stable. Let $x$ denote the home contribution to a bail-out in the baseline economy and $x'$ a home contribution in the economy with cross-border liabilities. The following comparison ensues:

<table>
<thead>
<tr>
<th>Utility following bail-out</th>
<th>Cross-border</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home</td>
<td>$1 - \beta - (1 + \lambda_C)x'$</td>
<td>$1 - (1 + \lambda_C)x$</td>
</tr>
<tr>
<td>Foreign</td>
<td>$\beta - (1 + \lambda_B)(1 - x')$</td>
<td>$-(1 + \lambda_B)(1 - x)$</td>
</tr>
</tbody>
</table>

Setting $x' \equiv x - \frac{\beta}{1 + \lambda_C}$, it follows that the utility-upon-bail-out difference between the cross-border and the baseline economy is given by:

- Home: $1 - \beta - (1 + \lambda_C)x' - (1 - (1 + \lambda_C)x) = 0$
- Foreign: $\beta - (1 + \lambda_B)(1 - x') - (-(1 + \lambda_B)(1 - x)) = \frac{\lambda_C - \lambda_B}{1 + \lambda_C} > 0$.

These expressions show that starting with any home contribution $x$ to bail-outs in the baseline model, we can reduce the home contribution to $x'$ in the case in which there are cross-border debt holdings, maintaining the home country’s utility unaltered and strictly increasing the foreign country’s utility. The reason is that when a fraction of the bank’s debt is held abroad, the foreign country’s contribution to a bail-out constitutes a partial transfer to its own agents.

These arguments can be generalized to the remaining bank-failure states, as we state in the following proposition.

**Proposition 6 (Banking union with cross-border liabilities)** Suppose that a fraction $\beta > 0$ of each bank’s debt is held by agents from the foreign country. Then the welfare gains relative to autarky induced by the optimal feasible banking union are strictly larger than in the baseline model.
The presence of cross-border liabilities provides an additional motive for countries to be willing to share the costs of bailing out banks. The proposition shows that such motive adds to the risk-sharing motive highlighted in the baseline model and increases the welfare gains that a banking union can attain.

8 Conclusion

In this paper, we analyze the optimal bank resolution decisions and the design of a fiscal backstop in a banking union. The rationale for a banking union is to provide a risk-sharing agreement across countries when one of the sovereigns may be in financial distress at the time of having to resolve a failing domestic bank. We address the potential for risk-sharing in a banking union in a setup that takes into consideration two features that are prevalent in the euro area banking union project: Supranational resolution authorities must rely on the information provided by domestic resolution authorities and countries differ on the strength of their public finances.

The informational friction limits the extent of risk-sharing because countries face an incentive to overstate the bail-in cost to obtain funding from the neighboring country to bail out home troubled banks. We show that the ex-post aggregate welfare resolution policies in a banking union entail co-funding of bail-outs if and only if banks are located in countries undergoing a crisis and the foreign country is stable. In addition, ex-post optimal decisions have to be distorted when country asymmetry is large to ensure the participation of the strong country in the banking union. In particular, we find that the required distortions reduce the amount of foreign aid to the weak country and increase the transfers to the strong country, thereby increasing the probability of bail-outs in the strong country and decreasing it in the weak country.

Our findings highlight the importance of establishing country-specific resolution policies for heterogenous countries to benefit from a banking union. Even though asymmetric resolution policies privilege the strong country’s bank, it is the weak country’s agents that appropriate the surplus created by the banking union. Indeed, the optimal feasible banking union when country asymmetry is large leads to a net welfare gain of the weak country’s agents, and to a redistribution of welfare from the taxpayer to the bank owners within the strong country.
Our model also highlights that requiring expected transfers to net out across countries is suboptimal. In particular, we find that strong countries should be the net receiver of funds when their public finances are always stable.

The need for country-specific policies to ensure the participation of the strong countries in the union extends to the case in which authorities can early intervene a bank at a time in which the liquidation of the bank assets yields sufficient proceeds to repay debtholders in full. We find that the optimal feasible banking union exhibits forbearance in the early intervention decisions of the bank in the strong country and is tight in intervening the weak country’s bank.

Finally, we find that bridge loans from a stable country to a country undergoing a crisis improve upon resolutions in which the home country is required to make contemporaneous contributions. However, loans should be used in combination with direct transfers from the stable country to reduce the overall cost of bank resolutions. This result suggests that direct foreign support should be an integral part of public backstop instruments.

Our paper abstracts from considering cross-country debt-holdings or international spillovers. While these are important aspects to take into account in the design of a banking union, we want to highlight that there is a risk-sharing rationale to construct a banking union even when local banking systems are not much interrelated. Higher degrees of interdependence may lead to more risk-sharing across countries and may render the banking union easier to implement. The analysis of the creation of banking unions among countries with more imbricated banking systems might be a matter for future research.
Appendix

A Truth-telling bank resolution mechanisms

In this Appendix we first characterize the different types of truth-telling resolution mechanisms and then select the only type that is relevant for the analysis of an optimal feasible banking union. We show, as stated in Section 2, that the relevant truth-telling mechanisms can be described by the required home contribution to bail-outs $x$, and can be implemented by delegating the resolution decision (bail-out or bail-in) to the home authority.

We first adopt without loss of generality the following convention: for the knife-edge cases in which under a particular resolution mechanism the home authority is indifferent between reporting a value of $k$ that leads to a bail-out and another one leading to a bail-in, we assume that it chooses to report the former.\footnote{Without this convention, there could be other truth-telling schemes on top of those described in Lemma 1 in this Appendix in which the close interval is open on its left boundary. Nonetheless, these other mechanisms are ex-ante payoff equivalent to those described in Lemma 1.}

Let $\{b_{i,\tau}(k), x_{i,\tau}(k)\}$ be a resolution mechanism in bank failure state $(i, \sigma)$. Suppose that the bail-in cost observed by the home authority is $k$. The expected utility of agents in country $i$ if the home authority reports $\hat{k}$ is given by:

$$V(\hat{k}, k) = \underbrace{b_{i,\tau}(\hat{k})}_{\text{Deptholders' payoff}} - \underbrace{(1 - b_{i,\tau}(\hat{k})) k}_{\text{Bail-in Cost}} - \underbrace{(1 + \lambda_{i,\sigma}) x_{i,\tau}(\hat{k})}_{\text{Cost Public Funds}}, \quad (A1)$$

where $\lambda_{i,\sigma}$ denotes the deadweight cost associated with raising public funds in country $i$ in the state of public finances $\sigma$. The first term in this expression corresponds to the transfer that the bank’s debtholders receive through a bail-out. The second term stands for the bail-in costs. The last term includes the expected gross social cost of the funds country $i$ has to provide for the resolution of its bank. The truth-telling property of the resolution mechanism can then be written as:

$$V(k, k) \geq V(\hat{k}, k) \text{ for all } k \text{ and } \hat{k} \in [0, k_{\text{max}}]. \quad (A2)$$

The next lemma characterizes incentive compatible resolution mechanisms as we argue below.

\textbf{Lemma 1} A resolution mechanism for a failing bank in a fragile country is truth-telling if and only if it belongs to one of the three following classes:

1. (Unconditional bail-in) $b_{i,\tau}(k) = 0$ for all $k \in [0, k_{\text{max}}]$.

2. (Unconditional bail-out) There exists $\hat{x} \in [0, 1]$ such that $b_{i,\tau}(k) = 1$ and $x_{i,\tau}(k) = \hat{x}$ for all $k \in [0, k_{\text{max}}]$. 
3. (Threshold) There exists \( x \in (\underline{x}, 1) \) and \( \overline{b}_{i, \sigma}(x) \in (0, k_{\text{max}}) \) such that \( b_{i, \sigma}(k) = 1 \) if and only if \( k \geq \overline{b}_{i, \sigma}(x) \), in which case \( x_{i, \sigma}(k) = x \). Moreover, we have that
\[
\overline{b}_{i, \sigma}(x) = (1 + \lambda_{i, \sigma}) x - 1,
\]
where \( \underline{x} \in (0, 1) \) is uniquely determined by the condition \( \overline{b}_{i, \sigma}(\underline{x}) = 0 \).

**Proof** Necessity Suppose that \( \{b_{i, \sigma}(\hat{x}), x_{i, \sigma}(\hat{x})\} \) is an incentive compatible (truth-revealing) mechanism. Let us define the set \( \Gamma^1 = \{k \in [0, k_{\text{max}}] \text{ s.t. } b_{i, \sigma}(k) = 1\} \). If \( \Gamma^1 \) is empty then case 1 in the lemma is satisfied. Let us suppose that \( \Gamma^1 \) is non empty. Taking into account that \((1 + \lambda_{i, \sigma}) x \) is strictly increasing in \( x \), it is immediate to show that truth-telling implies that there exists a constant \( \overline{x} \) such that \( x_{i, \sigma}(k) = \overline{x} \) for all \( k \in \Gamma^1 \).

Let us now prove that if \( k' > k \) for some \( k \in \Gamma^1 \) then \( k' \in \Gamma^1 \). In fact, incentive compatibility implies
\[
V(k', k') \geq V(k, k') \Rightarrow b_{i, \sigma}(k') - (1 - b_{i, \sigma}(k')) k' - (1 + \lambda_{i, \sigma}) x_{i, \sigma}(k') \geq \overline{x} \geq 1 - (1 + \lambda_{i, \sigma}) \overline{x},
\]
\[
V(k, k) \geq V(k', k) \Rightarrow 1 - (1 + \lambda_{i, \sigma}) \overline{x} \geq b_{i, \sigma}(k') - (1 - b_{i, \sigma}(k')) k - (1 + \lambda_{i, \sigma}) x_{i, \sigma}(k'),
\]
and combining the two inequalities we have
\[
-(1 - b_{i, \sigma}(k')) k' \geq -(1 - b_{i, \sigma}(k')) k,
\]
which can only be satisfied if \( b_{i, \sigma}(k') = 1 \).

Let \( \overline{\kappa} = \inf(\Gamma^1) \). We must have \( \overline{\kappa} \in \Gamma^1 \). In fact, if that is not the case then \( b_{i, \sigma}(\overline{\kappa}) = 0 \).

By definition of \( \overline{\kappa} \), for any \( \varepsilon > 0 \) there exists \( k' \in \Gamma^1 \) and such that \( 0 < k' - \overline{\kappa} < \varepsilon \). Incentive compatibility implies:
\[
V(k', k') \geq V(k', \overline{\kappa}) \Rightarrow \overline{\kappa} \geq 1 - (1 + \lambda_{i, \sigma}) \overline{x},
\]
\[
V(k', k') \geq V(k', k') \Rightarrow 1 - (1 + \lambda_{i, \sigma}) \overline{x} \geq k'.
\]
Combining the two inequalities and taking limits \( \varepsilon \to 0 \) we conclude that
\[
\overline{\kappa} = 1 - (1 + \lambda_{i, \sigma}) \overline{x}.
\]
But then when the local authority observes \( \overline{\kappa} \) it is indifferent between reporting \( \overline{\kappa} \), which leads to a bail-in, and reporting any \( k \in \Gamma^1 \), which leads to a bail-out. Our convention established at the beginning of this Appendix then states that the local authority would report \( k \in \Gamma^1 \) and the mechanism would not be truth-revealing. Hence we must have \( \overline{\kappa} \in \Gamma^1 \) and the fact that for all \( k \geq \overline{\kappa} \) we have \( k \in \Gamma^1 \) then implies that \( \Gamma^1 = [\overline{\kappa}, 1] \). If \( \overline{\kappa} = 0 \) then
case 2 in the lemma is satisfied. If \( k > \overline{k} \) then for any \( k < \overline{k} \) it has to be the case that \( b_{i,o}(k) = 0 \) and incentive compatibility implies
\[
V(\overline{k}, \overline{k}) \geq V(k, \overline{k}) \iff 1 - (1 + \lambda_{i,o}) \pi \geq -\overline{k} \\
V(k, k) \geq V(\overline{k}, k) \iff -k \geq 1 - (1 + \lambda_{i,o}) \pi.
\]
Taking limits \( k \to \overline{k} \) and combining the two inequalities we have that
\[
1 - (1 + \lambda_{i,o}) \pi = -\overline{k},
\]
and then case 3 in the lemma is satisfied.

**Sufficiency** It is straightforward to check that the mechanisms in the 3 cases in the lemma satisfy incentive compatibility.

Some of the truth telling mechanisms characterized in the previous lemma cannot be part of an optimal feasible banking union because they are Pareto dominated. Note that the notion of Pareto domination is the right one to discard resolution mechanisms and not aggregate welfare domination. This is because the optimal feasible banking union problem is a constrained problem in which aggregate welfare maximization is subject to the endogenous participation of the two countries. We have:

**Lemma 2** The following truth-telling mechanisms are Pareto dominated and can be discarded for the analysis of the ex-post optimal resolution policies and of the optimal feasible banking union:

1. The unconditional bail-in mechanism is Pareto dominated by a threshold mechanism with home contribution to bail-outs \( x = 1 \).

2. The unconditional bail-out mechanism characterized by a home contribution \( x > \bar{x} \) to bail-outs is Pareto dominated by a threshold mechanism with home contribution to bail-outs \( x = \bar{x} \).

**Proof** The first statement is an immediate consequence of the assumption \( c_{\text{max}} > \lambda_{C} \) and the fact that the foreign country is indifferent between an unconditional bail-in policy and a threshold mechanism in which bail-outs are entirely locally funded (i.e., such that \( x = 1 \)).

Second, by construction the home country prefers a threshold mechanism with home contribution to bail-outs \( x = \bar{x} \) to an unconditional bail-out mechanism with the same home contribution. The foreign country also prefers the former because it strictly reduces the probability that it has to contribute to the funding of bail-outs.

We conclude that it suffices to restrict our attention to the threshold mechanisms, which are described by a home contribution to bail-outs \( x > \bar{x} \), and the unconditional bail-out
mechanisms with home contribution to bail-outs $x \leq \kappa$, which can be also thought as a family of threshold mechanisms where the induced bail-out threshold is $\kappa = 0$. This justifies our restriction in the entire paper to resolution mechanisms that are described by the required home contribution to bail-outs $x$ in the entire interval $x \in [0, 1]$, and in which the decision on whether to conduct a bail-out or a bail-in is delegated to the home authority.

B Proofs

Proof of Proposition 1 We have proved all the statements of the proposition in the text preceding the proposition, except for:

i) No $x \leq \frac{1}{1+\lambda_C}$ is optimal.

In fact, we have that:

$$\Pi(0) = \int_0^{\lambda_C} k dG(k) + (1 - G(\lambda_C)) < \int_0^{\lambda_{\text{max}}} k dG(k) = \Pi(0),$$

where the inequality follows from Assumption 2. Since $\Pi(x) > 0$ for all $x \leq \frac{1}{1+\lambda_C}$, the inequality above implies that $\Pi(x) \geq \Pi(0) > \Pi(1)$ for all $x \leq \frac{1}{1+\lambda_C}$.

ii) The function $\Pi(x)$ is convex in the interval $x \in \left[\frac{1}{1+\lambda_C}, 1\right]$ and thus has a unique minimum in this interval.

Plugging the expression for the p.d.f. and the c.d.f. of a uniform distribution in the expression for $\Pi(x)$ in (6) we have that:

$$\Pi(x) = \frac{(\lambda_C - \lambda_S) k_{\text{max}} - \mathcal{T}(x)}{k_{\text{max}}} - (1 + \lambda_S) (1 - x) (1 + \lambda_C) \frac{1}{k_{\text{max}}} - \frac{1}{k_{\text{max}}} (1 + 2 \lambda_S - \lambda_C) > 0,$$

Differentiating the previous expression with respect to $x$ we obtain:

$$\Pi''(x) = \frac{1 + \lambda_C}{k_{\text{max}}} (1 - (\lambda_C - \lambda_S) + (1 + \lambda_S)) = \frac{1 + \lambda_C}{k_{\text{max}}} (1 + 2 \lambda_S - \lambda_C) > 0,$$

where in the last inequality we have used that Assumption 2 implies that $2\lambda_S > k_{\text{max}} > \lambda_C$.

Proof of Proposition 2 From (9) we have that $U_1(x^*) - U_1 \geq 0$ if and only if:

$$q^{CS} (\Pi - \Pi(x^*)) \geq (q^{SC} - q^{CS}) (1 - x^*) \left(1 + \lambda_S (1 - G(\mathcal{T}(x^*)))\right),$$

where:

$$a \equiv \frac{(1 - x^*) (1 + \lambda_S) (1 - G(\mathcal{T}(x^*)))}{\Pi - \Pi(x^*) + (1 - x^*) (1 + \lambda_S (1 - G(\mathcal{T}(x^*)))} \in (0, 1).$$

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Finally, from Assumption 1, and using that \( q_C^2 > q_C^1 \), we have that if the participation constraint of country 1 is satisfied then the participation constraint of country 2 is also satisfied. We conclude that a banking union with ex-post optimal resolution policies satisfies the participation constraint of the two countries if and only if Expression (B1) is satisfied.

**Proof of Proposition 3** We refer to a general banking union as \( \bar{x} \). Let us first state some properties of the partial derivatives of the aggregate welfare function and the expected utility in each country that have either been discussed in Section 4.3 or that can be easily derived from the expressions introduced in that section.

- For any \((i, \sigma)\):
  \[
  \frac{\partial W(x)}{\partial x_{i,\sigma}} = -(1 - p)q^\sigma \frac{\partial \Pi(x_{i,\sigma}|i, \sigma)}{\partial x_{i,\sigma}}.
  \]
- For any \(i\) and \(\sigma \in \{SS, CC\}\), if \(q^\sigma > 0\):
  \[
  \frac{\partial W(x)}{\partial x_{i,\sigma}} > 0 \text{ if } x_{i,\sigma} < 1, \text{ and } \frac{\partial W(x)}{\partial x_{i,\sigma}} = 0 \text{ if } x_{i,\sigma} = 1.
  \]
- For \((i, \sigma) \in \{(1, SC), (2, CS)\}\), if \(q^\sigma > 0\):
  \[
  \frac{\partial W(x)}{\partial x_{i,\sigma}} > 0.
  \]
- For \((i, \sigma) \in \{(1, CS, (2, SC)\}\), if \(q^\sigma > 0\):
  \[
  \frac{\partial W(x)}{\partial x_{i,\sigma}} > 0 \text{ if } \frac{1}{1 + \lambda_{i,\sigma}} < x_{i,\sigma} < x^*, \text{ and } \frac{\partial W(x)}{\partial x_{i,\sigma}} = 0 \text{ if } x_{i,\sigma} = x^*; \text{ and } \frac{\partial W(x)}{\partial x_{i,\sigma}} < 0 \text{ otherwise.}
  \]
- For any \((i, \sigma)\), if \(q^\sigma > 0\):
  \[
  \frac{\partial U_i(x)}{\partial x_{i,\sigma}} < 0, \text{ and } \frac{\partial U_i(x)}{\partial x_{j,\sigma}} > 0.
  \]

Let \( \bar{x} \equiv (\bar{x}_{i,\sigma}) \) be an optimal banking union. Let us denote \( PC_i \) the participation constraint of country \(i\). We first prove the existence of a feasible banking union when \( q^{SC} < 1 \) and the properties of the optimal one. We then prove that there is no proper feasible banking union when \( q^{SC} = 1 \).

**Optimal banking union when \( q^{SC} < 1 \)**

We conduct the proof of the statements in the proposition in a sequence of steps.

i) **Any optimal banking union strictly increases aggregate welfare relative to autarky.**
It suffices to find a banking union that strictly increases aggregate welfare relative to autarky. Since \( q^{SC} < 1 \), at least one of the three following inequalities is satisfied:

\( a) \ q^{CS} > 0 \).

From (13) we have that

\[
\frac{1}{q^{CS}} \frac{\partial U_1(1)}{\partial x_{1,CS}} = \frac{1}{q^{SC}} \frac{\partial U_2(1)}{\partial x_{2,SC}} = -(1 - p)(1 + \lambda_c) \left(1 - G(\lambda_c)\right),
\]

(B2)

where 1 denotes the autarky banking union. Let us consider the family of banking unions \( x(y) \), indexed by the parameter \( y \geq 0 \), which is characterized by: \( x_{1,CS}(y) = 1 - y \), \( x_{2,SC}(y) = 1 - \gamma y \) with \( \gamma \equiv \frac{q^{CS}(1+\lambda_c)}{q^{SC}(1+\lambda_S)} - \varepsilon \) and \( \varepsilon > 0 \), and \( x_{i,s}(y) = 1 \) for any other \( (i, \sigma) \). Using (B2), we have that

\[
\left. \frac{dU_1(x(y))}{dy} \right|_{y=0} = \frac{\partial U_1(1)}{\partial x_{1,CS}} - \gamma \frac{\partial U_1(1)}{\partial x_{2,SC}} = \varepsilon (1 - p) q^{CS} \left(1 + \lambda_S \right) \left(1 - G(\lambda_c)\right), \quad \text{and}
\]

\[
\left. \frac{dU_2(x(y))}{dy} \right|_{y=0} = \frac{\partial U_2(1)}{\partial x_{1,CS}} - \gamma \frac{\partial U_2(1)}{\partial x_{2,SC}} = \left(1 - p\right) \left(1 - G(\lambda_c)\right) \left(1 + \lambda_c\right) \frac{(1 + \lambda_c)^2 - (1 + \lambda_S)^2}{1 + \lambda_S} q^{CS} - \varepsilon q^{SC} \left(1 + \lambda_c\right).
\]

We have that since \( q^{CS} > 0 \), then for \( \varepsilon \) sufficiently small \( \left. \frac{dU_1(x(y))}{dy} \right|_{y=0} > 0 \) and \( \left. \frac{dU_2(x(y))}{dy} \right|_{y=0} > 0 \). Then, there exists \( y > 0 \) such that in the banking union \( x(y) \) both countries are strictly better off than in autarky.

\( b) \ q^{SS} > 0 \).

From (13) we have that

\[
\frac{\partial U_1(1)}{\partial x_{1,SS}} = \frac{\partial U_2(1)}{\partial x_{2,SS}} = -(1 - p) q^{SS} \left(1 + \lambda_S \right) \left(1 - G(\lambda_S)\right).
\]

(B3)

Let us consider the family of banking unions \( x(y) \) indexed by the parameter \( y \geq 0 \) characterized by: \( x_{1,SS}(y) = 1 - y \), \( x_{2,SC}(y) = 1 - \gamma y \) with \( \gamma \equiv \frac{q^{SS}(1+G(\lambda_S))}{q^{SC}(1+\lambda_S)} - \varepsilon \) and \( \varepsilon > 0 \), and \( x_{i,s}(y) = 1 \) for any other \( (i, \sigma) \). Using (B2) and (B3), the arguments of the previous case can be reproduced in an analogous manner and show that there exists \( y > 0 \) such that in the banking union \( x(y) \) both countries are strictly better off than in autarky.

\( c) \ q^{CC} > 0 \).

Analogous to case b.
Notice that if \( q'' = 0 \) for some \( \sigma \), the specific resolution policies in the bank failure states \((i, \sigma)\) are irrelevant. For the sake of shortening our arguments we will assume throughout the rest of the proof of the case \( q'' SC < 1 \) that \( q'' > 0 \) for all \( \sigma \in \{SS, CC, CS\} \). The reader can easily check that our arguments are valid if \( q'' > 0 \) for some \( \sigma \in \{SS, CC, CS\} \), which has to be the case since \( q'' SC < 1 \).

ii) Exactly one of the participation constraints is binding.

Suppose none of the participation constraints is binding. Then \( \hat{x} = x^* \). But we have \( q'' CS < a q'' SC \), and Proposition 2 implies \( x^* \) is not feasible.

Suppose \( PC_i \) is binding. Then \( PC_j \) is not binding because otherwise \( i \) would imply that \( \hat{x} \) is not optimal.

iii) If \( PC_1 \) is binding then \( \hat{x} \) satisfies \( \hat{x}_{1,SS} < \hat{x}_{2,SS} = 1, \hat{x}_{1,CC} < \hat{x}_{2,CC} = 1, \hat{x}_{1,CS} = 1, \hat{x}_{1,CS} < x^* \).

If \( PC_i \) is binding then \( ii \) implies that \( PC_{i} \) is not binding.

For \( \sigma \neq SC \), the properties of the derivatives of \( W(\mathbf{x}) \) and \( U_1(\mathbf{x}) \) with respect to \( x_{2,\sigma} \) when \( x_{2,\sigma} < 1 \) and the optimality of \( \hat{x} \), immediately imply that \( \hat{x}_{2,SS} = \hat{x}_{2,CC} = \hat{x}_{2,CS} = 1 \).

For \( \sigma \in \{SS, CC\} \), the properties of the derivatives of \( W(\mathbf{x}) \) and \( U_1(\mathbf{x}) \) with respect to \( x_{1,\sigma} \) when \( x_{1,\sigma} = 1 \) and the optimality of \( \hat{x} \), immediately imply that \( \hat{x}_{1,SS} < 1 \) and \( \hat{x}_{1,CC} < 1 \).

The properties of the derivatives of \( W(\mathbf{x}) \) and \( U_1(\mathbf{x}) \) with respect to \( x_{1,CS} \) when \( x_{1,CS} \geq x^* \) and the optimality of \( \hat{x} \), immediately imply that \( \hat{x}_{1,CS} < x^* \).

iv) If \( PC_1 \) is binding then \( \hat{x} \) satisfies \( x^* < \hat{x}_{2,SC} < 1 \).

If \( PC_1 \) is binding then \( ii \) implies that \( PC_2 \) is not binding and the properties of \( \hat{x} \) in \( iii \) are satisfied.

The properties of the derivatives of \( W(\mathbf{x}) \) and \( U_1(\mathbf{x}) \) with respect to \( x_{2,SC} \) when \( x_{2,SC} \in \left[ \frac{1}{1 + SC}, x^* \right] \) and the optimality of \( \hat{x} \), imply that \( \hat{x}_{2,SC} \notin \left[ \frac{1}{1 + SC}, x^* \right] \).

Suppose that \( \hat{x}_{2,SC} < \frac{1}{1 + SC} \) and thus \( \Pi(\hat{x}_{2,SC}|2, SC) = 0 \). We thus have that:

\[
\Pi(\hat{x}_{2,SC}|2, SC) \geq \Pi(0|2, SC) > \Pi(1|2, SC),
\]

(B4)

where the last inequality follows from Assumption 2.

Let us now prove that the following inequality holds:

\[
\Pi(\hat{x}_{1,CS}|1, CS) > \Pi(1|1, CS).
\]

(B5)

We have from \( iii \) that \( \hat{x}_{1,CS} < x^* \). If \( \hat{x}_{1,CS} \leq \frac{1}{1 + SC} \) the same arguments as for the bank failure state \((2, SC)\) imply the inequality. Suppose that \( \hat{x}_{1,CS} > \frac{1}{1 + SC} \) so that \( \Pi(\hat{x}_{1,CS}|1, CS) > 0 \). Let us denote \( \mu > 0 \) the Lagrange multiplier of the participation constraint of country 1 at the banking union \( \hat{x} \). Optimality of \( \hat{x} \) implies that

\[
\frac{\partial W}{\partial x_{2,SC}} \bigg|_{x_{2,SC}=\hat{x}_{2,SC}} \geq -\mu \frac{\partial U_1}{\partial x_{2,SC}} \bigg|_{x_{2,SC}=\hat{x}_{2,SC}}, \text{ with equality iff } \hat{x}_{2,SC} > 0.
\]

(B6)
Using that \( F(x_2, SC) = 0 \) for all \( x_{2, SC} < \frac{1}{1 + \lambda_S} \), we can compute the marginal derivatives in the inequality above and obtain after some algebra

\[
\mu \geq \frac{\lambda C - \lambda_S}{1 + \lambda_S}.
\]  

(B7)

In addition, using that \( \bar{F}_{1, CS} \in \left( \frac{1}{x_{1, CS}}, x^* \right) \) the optimality of \( \hat{x} \) implies that:

\[
\frac{\partial W}{\partial x_{1, CS}|_{x_{1, CS} = \bar{F}_{1, CS}}} = -\mu \frac{\partial U_i}{\partial x_{1, CS}|_{x_{1, CS} = \bar{F}_{1, CS}}},
\]

(B8)

which after some algebra can be written as:

\[
[\mu (1 + \lambda_C) + \lambda_C - \lambda_S] = \frac{(1 + \lambda_S) (1 - \bar{F}_{1, CS}) (1 + \lambda_C)}{k_{\text{max}} - ((1 + \lambda_C)x_{1, CS} - 1)}.
\]

(B9)

Notice that \( x^* \) is defined as the solution of (B8) for \( \mu = 0 \), so that from the equality above the following equality follows:

\[
\lambda_C - \lambda_S = \frac{(1 + \lambda_S) (1 - x^*) (1 + \lambda_C)}{(k_{\text{max}} - ((1 + \lambda_C)x^* - 1))}.
\]

(B10)

Dividing Expressions (B9) and (B10) we obtain:

\[
\frac{1 + \lambda_C}{\lambda_C - \lambda_S} = \frac{1 - \bar{F}_{1, CS}}{1 - x^*} \frac{k_{\text{max}} - ((1 + \lambda_C)x^* - 1)}{k_{\text{max}} - ((1 + \lambda_C)x_{1, CS} - 1)},
\]

Taking into account that \( \bar{F}_{1, CS} < x^* \) and using (B7) we obtain from the expression above:

\[
2 < \frac{1 + \lambda_C}{1 + \lambda_S} + 1 = \frac{1 + \lambda_C}{\lambda_C - \lambda_S} + 1 = \frac{1 - \bar{F}_{1, CS}}{1 - x^*} \frac{k_{\text{max}} - ((1 + \lambda_C)x^* - 1)}{k_{\text{max}} - ((1 + \lambda_C)x_{1, CS} - 1)} \leq \frac{1 - \bar{F}_{1, CS}}{1 - x^*},
\]

and thus:

\[
1 - x^* < x^* - \bar{F}_{1, CS}.
\]

Finally, since the function \( \Pi(x_{1, CS}|1, CS) \) achieves a minimum at \( x_{1, CS} = x^* \), the inequality above implies (B5).

For any other \( (i, \sigma) \notin \{(1, CS), (2, SC)\} \), it follows from \( iii) \) and the properties of the derivatives of \( W(x) \) with respect to \( x_{i, \sigma} \) that:

\[
\Pi(\hat{x}_{i, \sigma}|i, \sigma) \geq \Pi(1|i, \sigma).
\]

(B11)

Taking (B4), (B5) and (B11) together we have that \( W(\hat{x}) < W(1) \), which contradicts the optimality of \( \hat{x} \). We conclude that it cannot be the case that \( \bar{F}_{2, SC} < \frac{1}{1 + \lambda_C} \), and thus we must have \( \bar{F}_{2, SC} > x^* \).
Finally, taking (iii) into consideration, if $\tilde{x}_{2, SC} = 1$ it would follow that all the home contributions of country 2 are equal to 1. Moreover, since $q^{SC} < 1$, there would be at least one home contribution of country 1 that occurs with positive probability and is different from 1. As a result, $PC_2$ would not be satisfied.

v) If $PC_1$ is binding then $\bar{x}$ satisfies $\bar{x}_{1, SC} = 1$.

Suppose that $PC_1$. Let $\mu > 0$ denote again the Lagrange multiplier of the participation constraint of country 1 at $\bar{x}$. Using (iv), we have that (B6) is satisfied with equality. Some algebra rearrangements lead to:

$$\frac{\lambda_C - \lambda_S - \mu (1 + \lambda_S)}{1 + \mu} = \frac{(1 + \lambda_S) (1 - \tilde{x}_{2, SC}) (1 + \lambda_C)}{k_{max} - k(\tilde{x}_{2, SC}, 1)} > 0,$$

(B12)

and thus the Lagrangian multiplier $\mu$ satisfies:

$$\mu \leq \frac{\lambda_C - \lambda_S}{1 + \lambda_S}. \quad \text{(B13)}$$

Suppose that $\tilde{x}_{1, SC} < 1$. The optimality of $\bar{x}$ implies that:

$$\left. \frac{\partial W}{\partial x_{1, SC}} \right|_{x_{1, SC} = \tilde{x}_{1, SC}} \geq -\mu \left. \frac{\partial U_1}{\partial x_{1, SC}} \right|_{x_{1, SC} = \tilde{x}_{1, SC}} \quad \text{with equality if } \tilde{x}_{1, SC} > 0. \quad \text{(B14)}$$

Let us first consider the case that $\tilde{x}_{1, SC} > \frac{1}{1 + \lambda_S}$, so that the expression above is satisfied with equality and $k(\tilde{x}_{1, SC}, 1, SC) > 0$. Some algebra rearrangements lead to:

$$\mu (1 + \lambda_S) - (\lambda_C - \lambda_S) = \frac{(1 + \lambda_C) (1 - \tilde{x}_{1, SC}) (1 + \lambda_S)}{k_{max} - k(\tilde{x}_{1, SC}, 1)} > 0,$$

which implies that (B13) cannot be satisfied. Suppose, on the contrary, that $\tilde{x}_{1, SC} \leq \frac{1}{1 + \lambda_S}$, so that $k(\tilde{x}_{1, SC}, 1, SC) = 0$. We have that Expression (B14) can be written as:

$$\mu \leq \frac{\lambda_C - \lambda_S}{1 + \lambda_S} \quad \text{with equality if } \tilde{x}_{1, SC} > 0,$$

which, taking into consideration (B13), implies that $\tilde{x}_{1, SC} = 0$. Let us now show that we would have $U_1(\bar{x}) > U_1(1)$, thereby contradicting the fact that $PC_1$ is binding. In fact, taking into account (iii) we have that an optimal banking union with $PC_1$ binding strictly increases the utility in country 1 conditional on the pair of public finances at $t = 1$ being $\sigma \neq SC$. Moreover, taking into account that $\tilde{x}_{1, SC} = 0$, the utility in country 1 conditional on the pair of public finances at $t = 1$ being $\sigma = SC$ is given by:

$$U_1(\bar{x}, \sigma = SC) = pR + (1 - p) \cdot 1 - (1 - p) (1 - \tilde{x}_{2, SC}) (1 + \lambda_S) (1 - G(k(\tilde{x}_{2, SC}, 2, SC))), \quad \text{(B15)}$$

where the first term captures the expected payoff of the bank project, the second term includes the transfers to country 1 induced by the bail-outs entirely funded by country 2.
whenever bank 1 fails, and the last term captures the transfers country 1 does to country 2 to contribute to the funding of bail-outs in the latter country. In contrast, under autarky the utility in country 1 conditional on the pair of public finances at \( t = 1 \) being \( \sigma = SC \) is given by:

\[
U_1(1|\sigma = SC) = pR - (1 - p) \left[ \int_0^{\lambda_1} kdG(k) + \lambda_0(1 - G(\lambda_0)) \right].
\]  

(B16)

Using that \( \tilde{x}_{2,SC} > x^* \) and the fact that \( \tilde{y}(x^*|2,SC) > \lambda_0 \), which follows from Proposition 1, it follows that \( \tilde{y}(\tilde{x}_{2,SC}|2,SC) > \lambda_0 \). Using this inequality, (B15) and (B16), the inequality \( U_1(\tilde{x}|\sigma = SC) > U_1(1|\sigma = SC) \) can be written after some algebraic manipulation as:

\[
\int_0^{\lambda_1} (1 + k) dG(k) + \int_{\lambda_0}^{\tilde{y}(\tilde{x}_{2,SC}|2,SC)} (1 + \lambda_0)dG(k) + \int_{\tilde{y}(\tilde{x}_{2,SC}|2,SC)}^{\lambda_2} \tilde{x}_{2,SC}(1 + \lambda_0)dG(k) > 0,
\]

and is thus satisfied. This concludes the proof that when \( \tilde{x}_{1,SC} = 0 \) we have that \( U_1(\tilde{x}) > U_1(1) \), contradicting the assumption that \( PC_1 \) is binding.

\( vi) \) If \( PC_2 \) is binding then \( \tilde{x} \) satisfies:

\[ \tilde{x}_{2,SC} < x^* < \tilde{x}_{1,CS} < 1, \tilde{x}_{2,SS} < \tilde{x}_{1,SS} = 1, \tilde{x}_{2,CC} < \tilde{x}_{1,CC} = 1, \tilde{x}_{1,SC} = \tilde{x}_{2,CS} = 1. \]

The statements in this item extend those in \( iii)-v) \) and its proof is identical.

\( vii) \) \( PC_2 \) is not binding.

Suppose that \( PC_2 \) is binding. Taking \( vi) \) into consideration, considering the characterization in Proposition 1 of the ex-post optimal banking union \( x^* \), and given the inequalities \( \frac{\partial U_1(\tilde{x})}{\partial x_{1,SC}} < 0 \) and \( \frac{\partial U_1(\tilde{x})}{\partial x_{2,SS}} > 0 \), it follows that:

\[
U_1(\tilde{x}) < U_1(x^*).
\]

From this inequality, and using the fact that \( q^{CS} < q^{SC} \), Proposition 2 implies \( U_1(\tilde{x}) < U_1(\tilde{x}) \), which means that \( PC_1 \) is not binding.

To conclude the proof it suffices to notice that \( ii) \) and \( vi) \) imply that \( PC_1 \) is binding. Then, the statements in the proposition result from \( iii)-v) \).

\( viii) \) There exists a unique optimal banking union

Let \( \tilde{x} \) and \( \tilde{x}' \) denote two optimal banking unions. Then, they satisfy Properties \( iii)-v) \). Let \( \mu \) and \( \mu' \) be the Lagrange multiplier of the participation constraint of country 1 at \( \tilde{x} \) and \( \tilde{x}' \), respectively. Using (B12), we have that:

\[
\tilde{x}_{2,SC} \leq \tilde{x}'_{2,SC} \text{ iff } \mu \leq \mu' \text{ with equality iff } \mu = \mu'.
\]

(B17)

We can conduct similar arguments in the other bank failure states in which there is positive foreign aid, namely in the states \( (1, \sigma) \), with \( \sigma \in \{SS,CC,SC\} \), and find that:

\[
\tilde{x}_{1,\sigma} \leq \tilde{x}'_{1,\sigma} \text{ iff } \mu \leq \mu' \text{ with equality iff } \mu = \mu'.
\]

(B18)
We have that \( U_1(\tilde{x}) = U_1(x) = U \). Now, (B17) and (B18) imply that \( U_1(\tilde{x}) \geq U_1(x) \)
iff \( \mu \leq \mu' \), with equality iff \( \mu = \mu' \). It follows that \( \mu = \mu' \) which, in turn, from (B17) and (B18) implies that \( \tilde{x} = x' \).

**Infeasibility of a banking union \( x \) with \( W(x) > W(1) \) when \( q^{SC} = 1 \)**

Suppose that \( q^{SC} = 1 \) and that there exists a banking union \( x \) with \( W(x) > W(1) \). Let \( \tilde{x} \) denote the optimal banking union which, a fortiori, satisfies \( W(\tilde{x}) > W(1) \). Since country asymmetry is maximal, we have from Proposition 2 that \( \tilde{x} \neq x^* \), and at least one of the participation constraints is binding. If \( PC_2 \) is binding, then the properties of the derivatives of \( W(x) \) and \( U_2(x) \) with respect to \( x_{i,SC} \), and the optimality of \( \tilde{x} \), immediately imply that \( \tilde{x}_{1,SC} = 1 \). Moreover, \( W(\tilde{x}) > W(1) \) implies that \( \tilde{x}_{2,SC} < 1 \). But then, it immediately follows that \( U_1(\tilde{x}) < U_1(1) \).

We thus have that \( PC_2 \) is binding. The fact that \( W(\tilde{x}) > W(1) \) implies that \( \tilde{x}_{2,SC} < 1 \). From here the arguments in step v) of the proof of the case \( q^{SC} < 1 \) conducted above could be reproduced, which implies that \( \tilde{x}_{1,SC} = 1 \). But this, again, would imply that \( U_1(\tilde{x}) < U_1(1) \). □

**Proof of Proposition 4** Let \((x, p)\) be a feasible banking union and let \( W(x, p) \) and \( U_1(x, p) \) denote the aggregate welfare function and the expected utility in country \( i \), respectively. It is easy to check that the partial derivatives of these functions with respect to the \( x_{i,p} \) have the same properties as those of their baseline model analogous described at the beginning of the proof of Proposition 3. In addition, we have that, if \( \frac{\partial \bar{W}(x, p)}{\partial x_{i,p}} \neq 0 \), then the quotient \( \frac{\partial \bar{W}(x, p)}{\partial x_{i,p}} \) is independent of \( p \). Taking this into account, the statements i)-vi) in the proof of Proposition 3 can be proved with no changes in this extended set-up.

The following properties will be used frequently in the rest of the proof and can be easily checked:

\[
\frac{\partial W(x, p)}{\partial \bar{x}_{i,p}} = 0, \quad \text{and} \quad \frac{\partial W(x, p)}{\partial \bar{p}_{i,p}} < 0 \text{ iff } \bar{p}_{i,p} > \bar{p}(x_{i,p}|i, \sigma) \tag{B19}
\]

\[
\frac{\partial U_1(x, p)}{\partial \bar{p}_{i,p}} < 0 \text{ if } \bar{p}_{i,p} > \bar{p}(x_{i,p}|i, \sigma) \text{ or if } \bar{p}_{i,p} = \bar{p}(x_{i,p}|i, \sigma) \text{ and } x_{i,p} < 1 \tag{33}
\]

\[
\frac{\partial U_1(x, p)}{\partial \bar{x}_{i,p}} > 0 \text{ if } x_{i,p} < 1, \quad \text{and } \quad \frac{\partial U_1(x, p)}{\partial \bar{p}_{i,p}} = 0 \text{ if } x_{i,p} = 1.
\]

Let us now prove that statement vi) in the proof of Proposition 3 remains valid, namely, \( PC_2 \) cannot be binding in the optimal banking union. Suppose, on the contrary, that \( PC_2 \) is binding. We argue that this leads to distortions relative to the policies in the ex-post optimal banking union \((x', p')\) that imply \( U_1(\tilde{x}, \tilde{p}) < U_1(x', p') \) and, since by assumption
the country asymmetry is large enough so that the banking union \((x^*, p^*)\) does not satisfy \(PC_1\), we would have a contradiction.

Since \(PC_2\) is binding, we must have that \(\tilde{x}_{1,CS} > x^*\) and the ex-post optimal intervention threshold \(p(\tilde{x}_{1,CS}|1, CS)\), conditional on \(\tilde{x}_{1,CS}\), satisfies \(p(\tilde{x}_{1,CS}|1, CS) > p^*\). In addition, since \(PC_2\) is binding, the properties in (B19) and the optimality of \((\tilde{x}, \tilde{p})\) imply that:

\[
\tilde{p}_{1,CS} > p(\tilde{x}_{1,CS}|1, CS) > p^*.
\]  

Similarly, for \(\sigma = SS, CC, SC\), we have that \(\tilde{x}_{1,\sigma} = x^*_{1,\sigma} = 1\) and:

\[
\tilde{p}_{1,\sigma} > p(\tilde{x}_{1,\sigma}|1, \sigma) = p(1|1, \sigma).
\]

It follows that the distortions in the resolution and early intervention policies of \((\tilde{x}, \tilde{p})\), relative to \((x^*, p^*)\), in the bank failure states that involve the bank in country 1 strictly decrease the utility in that country.

Let us now consider the resolution and early intervention decisions of bank 2 in state \(\sigma = SC\), that is \(\tilde{x}_{2,SC}\) and \(\tilde{p}_{2,SC}\). In order to prove that the distortions induced by the fact that \(PC_2\) is binding reduce the utility in country 1 it suffices to show that:

\[
\left(\int_{\tilde{x}_{2,SC}}^{x^*} (1 - p_2)dF(p_2)\right) q^t(\tilde{x}_{2,SC}|2, SC) \geq \left(\int_{p^*}^{1} (1 - p_2)dF(p_2)\right) q^t(x^*|2, SC),
\]

where the left hand side and the right hand side account for the social gross cost of the transfers from country 1 to country 2 under \((\tilde{x}_{2,SC}, \tilde{p}_{2,SC})\) and \((x^*, p^*)\), respectively. Suppose, on the contrary, that (B22) is not satisfied. Since we have that \(\tilde{x}_{2,SC} < x^*\), it has to be the case that \(\tilde{p}_{2,SC} > p^*\). Let \((\tilde{x}', \tilde{p}')\) be the banking union that differs from \((\tilde{x}, \tilde{p})\) only in the policies for bank 2 in the state \(\sigma = SC\), which are defined to be \(\tilde{x}_{2,SC}' = x^*\) and \(\tilde{p}_{2,SC}' = p^*\). The increase in \(x_{2,SC}\) from \(\tilde{x}_{2,SC}\) to \(\tilde{x}_{2,SC}' = x^*\) is aggregate welfare increasing and, conditional on \(\tilde{x}_{2,SC}' = x^*\), the reduction in \(\tilde{p}_{2,SC}\) from \(\tilde{p}_{2,SC}\) to \(\tilde{p}_{2,SC}' = p^*\) is also aggregate welfare increasing, which implies that \(W(\tilde{x}', \tilde{p}') > W(\tilde{x}, \tilde{p})\). Moreover, the fact that (B22) is not satisfied means, by construction, that \(U_2(\tilde{x}', \tilde{p}') > U_2(\tilde{x}, \tilde{p})\). The optimality of \((\tilde{x}, \tilde{p})\) then implies that:

\[
U_1(\tilde{x}', \tilde{p}') < U_1(\tilde{x}, \tilde{p}).
\]

Notice that the increase in \(x_{2,SC}\) from \(\tilde{x}_{2,SC}\) to \(\tilde{x}_{2,SC}' = x^*\) increases the utility in country 1, while the reduction in \(\tilde{p}_{2,SC}\) from \(\tilde{p}_{2,SC}\) to \(\tilde{p}_{2,SC}' = p^*\) decreases it. As a result, the inequality above implies that there exists a banking union \((\tilde{x}', \tilde{p}')\) that differs from \((\tilde{x}, \tilde{p})\) only in the policies for bank 2 in state \(\sigma = SC\), which are given by \(\tilde{x}_{2,SC}' = x^*\), \(\tilde{p}_{2,SC}' \in (p^*, \tilde{p}_{2,SC})\), and satisfy:

\[
U_1(\tilde{x}', \tilde{p}') = U_1(\tilde{x}, \tilde{p}) \text{ and } W(\tilde{x}', \tilde{p}') > W(\tilde{x}, \tilde{p}).
\]

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The equality and inequality above in turn imply that $U_2(\tilde{x}_s', \tilde{p}_s') > U_2(\bar{x}, \bar{p}) = \overline{U}_2$. But then the banking union $(\tilde{x}_s', \tilde{p}_s')$ would be feasible and thus optimal.

Analogous arguments show that the distortions in the policies for bank 2 in states $\sigma = SS, CC, CS$ induced by the fact that $PC_2$ is binding reduce the utility in country 1 relative to that under the ex-post optimal banking union. This concludes the proof that $PC_2$ cannot be binding.

We must thus have that $PC_1$ is binding. The statements in the proposition regarding the forbearance in early interventions of bank 1 and tightness in those of bank 2 are then an immediate consequence of the properties in (B19) and the optimality of $(\tilde{x}, \tilde{p})$. $\blacksquare$

**Proof of Proposition 5** An arbitrary funding mix $(\tilde{x}_{i, \sigma}, \tilde{y}_{i, \sigma})$ for bail-outs at country $i$ in state $\sigma$ consists of: the home contribution $x_{i, \sigma}$ at $t = 1$, the foreign loan $y_{i, \sigma}$ repaid by the home country at $t = 2$, and the foreign contribution $1 - x_{i, \sigma} - y_{i, \sigma}$ at $t = 1$. Notice that both home contributions and foreign loans provide incentives to the home authority not to overstate the need for bail-outs. In addition, by construction of the interest rate charged by the foreign country on loans, the expected utility of this country only depends on is direct foreign contribution.

Let $(\hat{x}, \hat{y})$ be an optimal banking union. In order to prove the proposition we will argue that for any bank failure state $(i, \sigma)$ we have that either $\tilde{x}_{i, \sigma} = 0$ or $\tilde{y}_{i, \sigma} = 0$, so that the optimal funding mix is characterized by a single variable $(\hat{y}_{i, \sigma})$ in states for which $\tilde{x}_{i, \sigma} = 0$, and $\tilde{x}_{i, \sigma}$ in states for which $\tilde{y}_{i, \sigma} = 0$. The results in the proposition then can be obtained by reproducing the arguments in the proofs of Propositions 1, 2 and 3.

i) If $(i, \sigma) \in \{(1, SC), (2, SC)\}$, then $\tilde{x}_{i, \sigma} = 0$.

Suppose that $\tilde{x}_{i, \sigma} > 0$. Since $\lambda_C > \lambda_C$ and $R_S \leq 1$ (because $\lambda_S \geq \lambda_S$), we have that

\[ 1 + \lambda_C > \left(\frac{1 + \lambda_C}{1 + \lambda_C}\right) R_S. \]

This implies that the cost for the home country of home contributions is strictly higher than repaying foreign loans. Hence, the funding mix $(0, y'_{i, \sigma})$ such that $y'_{i, \sigma} = \tilde{x}_{i, \sigma} + \hat{y}_{i, \sigma}$ strictly increases the home country’s welfare relative to $(\tilde{x}_{i, \sigma}, \hat{y}_{i, \sigma})$, without changing the foreign country’s welfare. A banking union that only differs from $(\hat{x}, \hat{y})$ in the funding mix in the state $(i, \sigma)$, which is defined to be $(0, y'_{i, \sigma})$, would thus be feasible and increase aggregate welfare.

ii) If $(i, \sigma) \in \{(1, SC), (2, CS)\}$, then $\tilde{y}_{i, \sigma} = 0$.

Suppose that $\tilde{y}_{i, \sigma} > 0$. Since $\lambda_S \geq \lambda_S$ and $R_C > 1$ (because $\lambda_C > \lambda_C$), we have that

\[ 1 + \lambda_S < \left(\frac{1 + \lambda_S}{1 + \lambda_S}\right) R_C. \]

This implies that the cost for the home country of repaying foreign loans is strictly higher than that of home contributions. Hence, the funding mix $(x'_{i, \sigma}, 0)$ such that $x'_{i, \sigma} = \tilde{x}_{i, \sigma} + \hat{y}_{i, \sigma}$ strictly increases the home country’s welfare relative to $(\tilde{x}_{i, \sigma}, \hat{y}_{i, \sigma})$, without changing the foreign country’s welfare. A banking union that only differs from $(\hat{x}, \hat{y})$ in the funding mix in the state $(i, \sigma)$, which is defined to be $(x'_{i, \sigma}, 0)$, would thus be feasible.
and increase aggregate welfare.

iii) If \((i, \sigma) \in \{(1, SS), (1, CC), (2, SS), (2, CC)\}\), then \(\hat{y}_{i, \sigma} = 0\).

Suppose that \(\hat{y}_{i, \sigma} > 0\). We have that \(1 + \lambda_S = \left(1 + \lambda_S \right) R_S\) and that \(1 + \lambda_C = \left(1 + \lambda_C \right) R_C\). This implies that the cost for the home country of repaying foreign loans equals that of home contributions. Hence, the funding mix \((x'_i, 0)\) such that \(x'_i = \hat{x}_{i, \sigma} + \hat{y}_{i, \sigma}\) provides the same welfare as \((\hat{x}_{i, \sigma}, \hat{y}_{i, \sigma})\) for the two countries. A banking union that only differs from \((\hat{x}, \hat{y})\) in the funding mix in the state \((i, \sigma)\), which is defined to be \((x'_i, 0)\), is thus pay-off equivalent to \((\hat{x}, \hat{y})\).

**Proof of Proposition 6** We introduce some notation. For a given bank failure state \((i, \sigma)\) and home contribution \(x\) we denote \(k^0(x|j i; \sigma)\) the bail-out threshold in the economy with cross-border liabilities. We have:

\[
\hat{k}^0(x|j i; \sigma) = \max \{(1 + \lambda_{i, \sigma}) x - (1 - \beta), 0\}.
\]

From the expression for the analogous threshold \(k^0(x|j i; \sigma)\) in the baseline model given in (10) we have for \(x \geq \frac{\hat{q}}{1 + \lambda_{i, \sigma}}\) that:

\[
\hat{k}^0(x - \frac{\beta}{1 + \lambda_{i, \sigma}}|i, \sigma) = k^0(x|j i; \sigma) = \hat{k}(x|j i; \sigma).
\]

Notice that for \(x < \frac{q}{1 + \lambda_{i, \sigma}}\) the equality also holds if we replace \(x - \frac{\beta}{1 + \lambda_{i, \sigma}}\) by \(\max \left\{ x - \frac{q}{1 + \lambda_{i, \sigma}}, 0 \right\}\).

For any country \(j \in \{1, 2\}\), bank-failure tuple \((i, \sigma)\), and home contribution \(x\), we denote the overall utility at \(t = 1\) of agents in country \(i\) when there are cross-border liabilities \(\beta\) as \(U^0_i(x|j i; \sigma)\). In addition, we denote \(U_j(x|j i; \sigma) = U^0_i(x|j i; \sigma)\).

Finally, we have that the overall utility as of \(t = 0\) in country \(j\) associated with a banking union \(x\) when there are cross-border liabilities \(\beta\), which we denote by \(U^\beta_j(x)\), satisfies:

\[
U^\beta_j(x) = \sum_{i, \sigma} q^\beta U_j(x|j i; \sigma). \tag{34}
\]

We can now proceed with the proof. Consider the optimal feasible banking union \(x^*\) of the baseline economy. We are going to construct a new banking union \(x' = (x'_i)\) satisfying for every \(j\) and \((i, \sigma)\) the following inequality:

\[
U^\beta_j(x'_i|j i; \sigma) - U^\beta_j(1|j i; \sigma) \geq U_j(x'_i|j i; \sigma) - U_j(1|j i; \sigma), \tag{35}
\]

with strict inequality for some \(i\) and \((j, \sigma)\). This will in particular imply that the banking union \(x'\) is feasible in the economy with cross-border liabilities and that the welfare gains relative to autarky induced by \(x'\) are strictly larger than those induced by \(x\) in the baseline economy.
To streamline the expressions and exposition to follow, we make two simplifying assumptions. First, for all \((i, \sigma)\) we have \(\overline{G}^{i}(1|i, \sigma) > k_{\text{max}}\), so that there are no bail-outs in autarky in the cross-border economy. A necessary and sufficient condition for the assumption to be satisfied is that \(\lambda_{i, \sigma} + \beta > k_{\text{max}}\). Second for all \(\sigma\), we have \(q'' > 0\).

Consider the bank failure state \((i, \sigma)\). Suppose \(\lambda_{i, \sigma} \geq \lambda_{i, \sigma}'\). Let:
\[
x_{i, \sigma}' = \max \left\{ x_{i, \sigma} - \frac{\beta}{1 + \lambda_{i, \sigma}}, 0 \right\}.
\]  
(36)

We have by construction that:
\[
\overline{G}^{i}(x_{i, \sigma}'|i, \sigma) = \overline{G}(x_{i, \sigma}'|i, \sigma),
\]  
(37)

and that:
\[
x_{i, \sigma}' - x_{i, \sigma} \leq \frac{\beta}{1 + \lambda_{i, \sigma}}.
\]  
(38)

We have the following utility gains for the home country in the baseline and cross-border economies:
\[
U_{i}(x_{i, \sigma}'|i, \sigma) - U_{i}(1|i, \sigma) = \int_{\overline{G}^{i}(x_{i, \sigma}'|i, \sigma)}^{\lambda_{i, \sigma}} \left( k - \overline{G}(x_{i, \sigma}'|i, \sigma) \right) dG(k) + \left( 1 - G(\lambda_{i, \sigma}) \right) \left( \lambda_{i, \sigma} - \overline{G}(x_{i, \sigma}'|i, \sigma) \right),
\]
\[
U_{i}^{\beta}(x_{i, \sigma}'|i, \sigma) - U_{i}^{\beta}(1|i, \sigma) = \int_{\overline{G}(x_{i, \sigma}'|i, \sigma)}^{\lambda_{i, \sigma}} \left( k - \overline{G}(x_{i, \sigma}'|i, \sigma) \right) dG(k),
\]

where we have used that \(\overline{G}^{i}(1|i, \sigma) > k_{\text{max}}\) and that \(\overline{G}(1|i, \sigma) = \lambda_{i, \sigma}\). Using (37) we have from the expressions above that:
\[
U_{i}^{\beta}(x_{i, \sigma}'|i, \sigma) - U_{i}^{\beta}(1|i, \sigma) > U_{i}(x_{i, \sigma}'|i, \sigma) - U_{i}(1|i, \sigma),
\]
and (35) is strictly satisfied for \(j = i\).

We have the following utility gains for the foreign country:
\[
U_{i}^{\beta}(x_{i, \sigma}'|i, \sigma) - U_{i}^{\beta}(1|i, \sigma) = - \left( 1 - \overline{G}(x_{i, \sigma}'|i, \sigma) \right) \left( 1 + \lambda_{i, \sigma} \right) \left( 1 - x_{i, \sigma}' \right),
\]
\[
U_{i}^{\beta}(x_{i, \sigma}'|i, \sigma) - U_{i}^{\beta}(1|i, \sigma) = - \left( 1 - \overline{G}(x_{i, \sigma}'|i, \sigma) \right) \left( 1 + \lambda_{i, \sigma} \right) \left( 1 - x_{i, \sigma}' - \beta \right),
\]

where in the second expression we have used that when in any bail-out of bank \(i\) the measure \(\beta\) of debtholders in country \(\overline{7}\) are repaid and that in autarky there are no bail-outs. Taking into account (36), (37), (38), and the assumption that \(\lambda_{i, \sigma} \geq \lambda_{i, \sigma}'\), we have from the expressions above that:
\[
U_{i}^{\beta}(x_{i, \sigma}'|i, \sigma) - U_{i}^{\beta}(1|i, \sigma) \geq U_{i}^{\beta}(x_{i, \sigma}'|i, \sigma) - U_{i}^{\beta}(1|i, \sigma) + \left( 1 - \overline{G}(x_{i, \sigma}'|i, \sigma) \right) \beta \lambda_{i, \sigma} - \lambda_{i, \sigma}' \lambda_{i, \sigma} \geq U_{i}^{\beta}(x_{i, \sigma}'|i, \sigma) - U_{i}^{\beta}(1|i, \sigma),
\]

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and (35) is satisfied for $j = 1$.

Suppose $\lambda_{i,\sigma} < \lambda_{2,\sigma}$, then Proposition 1 and 3 imply that the optimal feasible banking union satisfies $x^*_{i,\sigma} = 1$. Let $x^*_{i,\sigma} = 1$. Then we trivially have for any $j = 1, 2$ that:

$$U^G_j(x^*_{i,\sigma} | i, \sigma) - U^G_j(1 | i, \sigma) = U^G_j(x^*_{i,\sigma} | i, \sigma) - U^G_j(1 | i, \sigma) = 0,$$

and (35) is satisfied for any $j$. ■
References


The views expressed in this paper are our own and do not necessarily coincide with those of Banca d’Italia. We would like to thank Thorsten Beck, Elena Carletti, Guillermo Caruana, Jean-Eduard Colliard, Giovanni Dell’Ariccia, Xavier Freixas (discussant), Florian Heider, Agnese Leonello (discussant), Gerard Llobet, Gyöngyi Lóránth, Robert Marquez, David Martinez-Miera, Martin Oehmke, Rafael Repullo, Javier Suarez (discussant), Xavier Vives, Lucy White and seminar audiences at the 28th European Summer Symposium in Financial Markets (Gerzensee, evening sessions), the II MadBar Workshop at Universitat Pompeu Fabra, the 1st ESCB Financial Stability Workshop, the 25th Meeting of the Spanish Finance Association, the 42nd Simposio of the Spanish Economic Association, and the Universitat de les Illes Balears for helpful comments. Sergio is grateful to Ministerio de Economia, Industria y Competitividad for financial support under Grant ECO2017-82385-P.

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