Working Paper Series

No 70 / March 2018

The variance risk premium and capital structure

by
Babak Lotfaliei

ESRB
European Systemic Risk Board
European System of Financial Supervision

ABSTRACT

This paper investigates how the asset-return variance risk premium changes leverage. I find that the premium lowers leverage by increasing risk-neutral bankruptcy probability and costs in a model where asset returns have stochastic variance with risk premium. Empirically, the model calibrations verify significant reduction in optimal leverage, closer to observed leverage than the model without the premium. In model-free regressions, I also document negative correlation between leverage and the variance premium. The most negative correlation is among investment-grade firms with low asset beta and historical variance but high variance premium because their assets have high exposure to market variance premium.

Keywords: Variance Risk Premium, Capital Structure, Optimal Leverage.

JEL classification: G32, G33, G12.

I. Introduction

It is not yet clear how risky time-variations in asset variance that create the variance risk premium(s) (VRP) impact leverage. This question is important because asset variance has a first-order effect on leverage: first, high asset variance that measures business uncertainty reduces leverage because variance increases debt costs, e.g. overhang and bankruptcy costs. Second, return variance of an underlying security directly impacts option prices. Debt and equity are options on the firm assets. The firm assets' variance directly impacts debt and equity because their underlying security is the firm assets (Merton, 1974). VRP is the spread between risk-neutral (RN) and historical variances created by risky time-variations in variance. Since variance directly impacts option prices, VRP also has a first-order impact on them.[1] If VRP of the underlying security has a critical role in pricing options, then, analogically, asset VRP has a critical role in pricing debt and equity. This critical role also implies that asset VRP impacts leverage. But, the dynamic and direction of any potential impact is not documented as asset VRP is absent in most capital structure studies (see Strebulaev and Whited (2011) for a review[2]).

To answer the question, this paper investigates the effect of asset VRP on leverage by addressing two challenges: first, it is complicated to theoretically consider risky variance in the already-complex capital structure models (Huang and Huang, 2012). Hence, the relationship between VRP and leverage remains unclear which leads to the second challenge: it is not obvious empirically which firms' leverage is affected by asset VRP. This paper proposes that asset returns have VRP and, therefore, RN asset variance is greater than

[1]See Christoffersen, Heston, and Jacobs (2013), van der Ploeg (2006), and Christoffersen and Diebold (2000) for derivative studies. Historical variance is also called "physical", "realized", or "$\mathbb{P}$-measure" variance. I pick this term because historical variance usually measures physical variance. RN variance is also called "risk-adjusted", "$\mathbb{Q}$-measure" or "pricing measure" variance. Asset variance is short for asset-return variance.

[2]See also Sundaresan (2013), Bhamra, Kuehn, and Strebulaev (2010), Broadie, Chernov, and Sundaresan (2007) and Anderson and Sundaresan (1996) where historical and RN asset variances are identical.

historical variance. I find that asset VRP negatively impacts leverage and, in the cross-section of the firms, the strongest negative relationship is for investment-grade firms. VRP reduces the optimal leverage significantly, on top of the variance level itself, because the firms borrow conservatively to hedge against the risk of future increases in variance.

Theoretically, this paper introduces the concept of hedging asset VRP to determine leverage in corporate finance. Asset VRP recently has been used to only explain some asset-pricing puzzles. Choi and Richardson (2016) report time-varying asset variance based on market data such as equity variance and Barras and Malkhozov (2016) show that VRP exists and measure it both in options and equity markets. I relax the deterministic asset variance assumption in the earlier studies by considering asset VRP. I only focus on trade-off theory in Brennan and Schwartz (1978)'s extended settings, while the results are complementary to other capital structure theories. Without VRP, the model nests Leland (1998)'s dynamic capital structure that has optimal default, leverage, and debt rollover but there is no upward restructuring and transaction costs. The model with VRP results in tractable formulas for debt, equity, and optimal default policy, which makes the optimal leverage calculations simpler. The model shows that high VRP increases RN default probability. High RN default probability reduces tax benefits and increases debt costs, which lowers optimal leverage.

By introducing the missing factor of the risk premium for asset variance (proxied by VRP), this paper helps to reducing an empirical paradox: contrary to earlier theoretical predictions, empirical studies observe that "large, liquid, profitable firms with low expected distress costs use debt conservatively" (Graham, 2000). Historical asset variance has a key role in the observation because underleveraged firms usually have low historical asset variance. This implies low expected distress costs. Low distress costs provide an opportunity to have more debt, which these firms seemingly do not utilize. However, most of these capital structure studies consider asset variance to be deterministic[3] and ignore variance risk. Since

[3]See also Denis (2012), Welch (2004) or Frank and Goyal (2007) for a broader analysis. Without VRP, this paper's calibrations are limited to historical variance and collapse to earlier models that propose higher

rating agencies consider profitability and historical asset risk, I use ratings to highlight such underleveraged firms. Investment-Grade (IG) firms are large and profitable and they have the lowest leverage and historical asset variance. Therefore, the largest difference between actual and proposed leverage is for IG firms. Between 2002 and 2015, for example, an average IG firm has leverage close to 37% but the model without VRP predicts leverage close to 53%. However, with VRP, the model yields lower optimal leverage that is closer to observed leverage: in the case of IG firms, the model implies 38% leverage on average because of their high asset VRP. Hence, VRP is an important missing piece in the capital structure analysis.

This paper also analyzes why RN variance remains relatively high while historical variance is low for IG firms. The spread in variance, VRP, widens for IG firms because they hold asset portfolios with low idiosyncratic variance and high exposure to market variance.[4] With deterministic market variance and no VRP, only the level of the market variance and fear of direct shocks to market reduces leverage via asset beta. Chen, Xu, and Yang (2013) and Schwert and Strebulaev (2014) document several effects of systematic variance level on the capital structure. This paper complements these studies by proposing high VRP as a new effect of exposure to market variance on leverage; with VRP, the shocks to asset variance are additional to the direct shocks to asset return. Idiosyncratic variance of assets does not change asset beta. But, idiosyncratic variance cushions the market variance shocks and VRP to be transferred to the firm. While a firm may have low asset beta and total variance, it chooses conservative leverage to hedge exposure to market VRP due to low idiosyncratic asset variance and high asset VRP. In the case of IG firms, relatively high RN variance from the exposure to market VRP lowers IG firms' leverage more than other firms' leverage.

optimal leverage for the firms compared to actual leverage.

[4]Possible explanation that requires further research is holding a large and diverse portfolio of the assets. Idiosyncratic variance is also called diversifiable variance. Market variance is also called systematic or non-diversifiable variance.

Empirically, I test the theoretical implications and verify that the assets' VRP reduces optimal leverage, particularly for IG firms. Between 1997 and 2015, I calibrate the model to data using a method similar to Jones, Mason, and Rosenfeld (1984) and Huang and Huang (2012). The calibrations without VRP do not agree with observed leverage because the model is misspecified from ignoring VRP. The model that only considers historical asset variance concludes that IG firms have the highest underleverage. I also find that IG firm's underleverage directly correlates with their unexplained credit premium, or credit premium puzzle. This finding complements the studies showing that IG firms contribute the most to the credit premium puzzle, e.g. Eom, Helwege, and Huang (2004). However, the model with VRP yields leverage which is much closer to observed leverage. The results are robust to calibrations on individual firms in each rating class following a method similar to Schaefer and Strebulaev (2008) for a shorter period between 2002 and 2015.[5] Model-free regressions with controls for other factors, such as cash holdings and profitability, also produce the two results: a) VRP proxy has negative correlation with target leverage of the firms, which relatively has stronger negative effect than profitability. This result supports the importance of VRP-leverage relationship because profitability has first-order effect on leverage (Titman and Wessels, 1988), b) VRP has stronger effect on IG firms than an average firm.

Finally, in a supporting analysis, while IG firms have the lowest asset beta and historical variance, this paper verifies that IG firms' asset variance has the largest correlation and exposure to market variance among all rated firms. Since market variance is priced, high correlation explains their high asset VRP. Robust calibrations with this exposure also support the earlier results. In sum, the empirical results confirm the propositions: asset VRP reduces leverage and, consequently, underleverage, where the highest VRP belongs to IG firms.

This paper contributes to the earlier empirical work on firm leverage in three ways. First,

[5]Bond-yield data in the calibrations on the firm-years is limited to the 2002-2015 period, while data for average representative firms in ratings is available for longer time from FRED, between 1997 and 2015.

it adds asset VRP as a factor to determine optimal leverage.[6] The calibrations with VRP produce results closer to observed leverage than calibrations excluding VRP. The regressions also show that asset VRP is as important as profitability in determining the target leverage for all the firms. Second, it shows how ratings highlight firms with more severe underleverage and connect underleverage to the credit-spread puzzle. Chen (2010) also links underleverage and the credit premium puzzle by analyzing their relationship to common risk factors in time-series at the macro level. However, this paper extends the analysis to the cross-section of the firms and determines the categories of the firms to which the puzzles are relevant. Without VRP, this paper finds almost one-to-one correlation between underleverage and credit-spread puzzles across IG firms. Third, I propose that asset VRP and hedging the exposure to market variance risk complement credit-score targeting as identified by Kisgen (2009) and Kisgen (2006) in determining the capital structure of the rated firms.

This paper also makes three contributions to the literature on structural models and optimal capital structure. This paper is the first, to my knowledge, to extend the work on hedging variance risk from asset pricing[7] to capital structure both theoretically and empirically. Some structural models suggest that a dynamic capital structure can explain the leverage choice of the firms. However, they have difficulty explaining the observed abnormal low leverage because they are limited to the constant-variance assumption. This paper relaxes the constant-variance assumption and finds that VRP reduces optimal leverage through increasing RN default probability and hedging variance risk. Second, using a framework similar to Tahani (2005), I derive approximate closed-form solutions for asset prices and the default policy with stochastic variance. The tractable formulas are simpler to derive compared to the perturbation method suggested by Fouque, Sircar, and Solna (2006).[8] Hence, this

[6]List of the other factors are available in many studies, such as Öztekin (2015), Fan, Titman, and Twite (2012), Graham and Leary (2011), Frank and Goyal (2009), and Rajan and Zingales (1995).

[7]For example, see Campbell, Giglio, Polk, and Turley (2015), Garlappi and Yan (2011) and Campbell, Hilscher, and Szilagyi (2008).

[8]McQuade (2013) and Barsotti (2012) also recently use the perturbation method. Ericsson, Elkamhi,

paper's method technically makes it easier and possible to calculate optimal leverage because the closed-form is used for the optimal default. Third, this paper adds a missing factor and model misspecification to the earlier literature concerning the sources of underleverage, such as macro conditions (Korajczyk and Levy, 2003) or errors in tests (Strebulaev, 2007). I confirm that ignoring VRP in a constant-variance framework implies puzzling higher optimal leverage than observed leverage and, thus, the underleverage.

II. The Model

Let's consider a firm with unlevered-asset[9] return process that follows Geometric Brownian with stochastic variance:

$$(1) \qquad \mathbb{P}: \begin{cases} \frac{d\nu}{\nu} = (\mu-\delta)dt + \sqrt{V}dW_1^p \\ dV = \kappa(\theta - V)dt + \sigma\sqrt{V}dW_2^p \end{cases} \qquad \mathbb{Q}: \begin{cases} \frac{d\nu}{\nu} = (r-\delta)dt + \sqrt{V}dW_1 \\ dV = \lambda(\theta^* - V)dt + \sigma\sqrt{V}dW_2, \end{cases}$$

where $\mathbb{P}$ is physical measure, $\mathbb{Q}$ is risk-neutral measure, μ is the return drift, δ is the payout rate of cash or leak in the cashflows, V is variance, $\sqrt{V}$ is volatility, κ is the mean-reversion speed, σ is variance's volatility, θ is the mean variance (long-run or long-term variance), r is the risk-free rate, θ^* is the RN mean variance, and λ is the RN speed of mean-reversion. W_1 and W_2 are independent Brownian motions under $\mathbb{Q}$ and W_1^p and W_2^p are independent Brownian motions under $\mathbb{P}$ (see Appendix A for a description of all variables). W_1^p and W_1 are direct shocks to asset return. W_2^p and W_2 are shocks to variance.

With stochastic variance, $\mathbb{Q}$-measure is not unique and this paper assumes that the firm's asset-return process under $\mathbb{Q}$ is a process similar to Heston (1993) where $\theta^* = \frac{\kappa\theta}{\lambda}$. Cox, Ingersoll, and Ross (1985) present the general equilibrium model behind the processes. The

and Du (2011) use numerical integration.

[9] Unlevered asset value, ν, is present value of all future asset cash flows (see Online Appendix 1 for details).

difference between the risk-neutral and physical mean-reversion speeds $(\lambda - \kappa)$ represents VRP.[10] If hypothetically one could have invested on variance as a security, VRP is intuitively the return premium of this security, the spread between physical and RN returns. VRP is unitless and negative $(\lambda < \kappa)$ which implies variance's mean under the pricing measure is higher than the historical mean. Carr and Wu (2009) and Bakshi and Kapadia (2003) report empirical evidence on negative VRP. Negative VRP holds because intuitively the risk-averse agents dislike risky variance and consider a higher RN mean for variance when variance is stochastic with priced risk. For example, no VRP sets RN variance equal to historical variance. This paper reports the absolute value of VRP in the calibrations. Similar to McQuade (2013), Stein and Stein (1991) and Hull and White (1987), variance is not asymmetric for the asset returns, $Corr_{asset}(\frac{d\nu}{\nu}, dV) = 0$, which helps to keep the model tractable. Despite this assumption, the model replicates asymmetric stock variance for the equity process $(Corr_{equity} < 0)$ as a stylized fact in the markets (see Appendix B).

A. VRP and Market Variance Risk

Let's assume that market variance has priced risk with the following process:

$$
\begin{cases} P: \quad dV_s = \kappa_s(\theta_s - V_s)dt + \sigma_s\sqrt{V_s}dW^p_{sys} & \theta^*_s = \kappa_s\theta_s/\lambda_s, \\ Q: \quad dV_s = \lambda_s(\theta^*_s - V_s)dt + \sigma_s\sqrt{V_s}dW_{sys} & VRP_s = \lambda_s - \kappa_s, \end{cases} \tag{2}
$$

where V_s is market variance, κ_s is market variance's mean-reversion speed and θ_s is mean market variance in physical measure. σ_s is volatility of market variance, λ_s is market variance's mean-reversion speed under pricing measure, θ^*_s is mean market variance in pricing measure, and VRP_s is market VRP. W^p_{sys} and W_{sys} respectively are variance shocks under physical and RN measures. For the firm, total variance in Equation 1 breaks down into

[10] Online Appendix 2 describes more details about the underlying economic assumptions such as risk premiums ($\mu - r$ and $\lambda - \kappa$), and why $\lambda - \kappa$ represents VRP.

systematic priced variance and idiosyncratic variance:

$$
\mathbb{P}:\begin{cases} \frac{d\nu}{\nu}=(\mu-\delta)dt+\sqrt{bV_s}dW_s^p+\sqrt{V_i}dW_i^p & \beta=\sqrt{b}, \quad \alpha=1-\frac{V_I}{V}, \quad \kappa=\alpha\kappa_s, \\ V=V_i+bV_s, \quad dV=bdV_s, & \sigma=\sigma_s\sqrt{\alpha b}, \quad \theta=b\theta_s/\alpha, \end{cases}
$$

(3)

$$
\mathbb{Q}:\begin{cases} \frac{d\nu}{\nu}=(r-\delta)dt+\sqrt{bV_s}dW_s+\sqrt{V_i}dW_i & \lambda=\alpha\lambda_s, \quad \theta^*=b\theta_s^*/\alpha, \\ V=V_i+bV_s, \quad dV=bdV_s, & \theta^*=\kappa\theta/\lambda, \quad VRP=\alpha VRP_s, \end{cases}
$$

where V_i is constant idiosyncratic variance, β is asset beta, and b is a positive constant. α isexposure and mean correlation of asset variance with market variance which matches proportional systematic variance. W_i^p and W_i respectively are idiosyncratic shocks under physical and RN measures independent from W_s^p and W_s which are shocks to market. While it has no effect on asset beta which transfers market-return shocks, idiosyncratic variance reduces the impact of market variance shocks and VRP to be transferred to the firm.[11]

If I shut down the stochastic variance and its priced risk, the formulas collapse into a model similar to Strebulaev (2007). When market variance is deterministic and has no priced risk, only the market variance level would affect the firms' leverage decision via asset beta and total variance. But, if market variance has priced risk, having proportionally high asset-return exposure to market variance makes a firm more likely to have high asset VRP: while market variance may remain low, market VRP can be high due to uncertain future of market variance. The VRP channel complements the direct effect of exposure to market-return shocks suggested by the earlier studies through beta. With asset VRP, the assets are additionally exposed to the variance shocks. Hence, RN asset variance and asset VRP also depend on the exposure to market variance.

There is no qualitative difference between decomposing variance or only using total variance. Therefore, I abstract from the decomposition for brevity in the derivations and only present the model and calibrations with total variance using Equation 1.

[11]Table 15 in Online Appendix 3 has a numerical example based on empirical values.

B. Capital Structure

The firm with unlevered assets is all equity and optimally recapitalizes some equity with debt. For simplicity, the firm faces no information asymmetry and agency costs as the manager has aligned objectives with the shareholders. Unlevered asset value is not affected by financial decisions and financial manager also cannot control the business risk of the firm, V, which is exogenously imposed by the business environment. Based on the classical framework of Modigliani and Miller (1958), tax savings from interest payments, default costs, and debt rollover are the only frictions. The only effect of VRP is on optimal default policy and RN default probability that affects the value for tax savings and default costs. The levered value of the firm is the unlevered value plus the expected net debt benefits (tax savings of interests less the costs of bankruptcy). The levered firm continuously issues and rollovers debt with infinite maturity. But, average actual debt maturity is $M = 1/m$ because the firm replaces old perpetual debt at a continuous rate m by issuing new perpetual debt with coupon c and face value p. The firm effectively has outstanding debt with coupon payments C and face value P at any point of time after issuance where $p = mP, c = mC$. The coupon payments symbolically represent interest expenses of the firm. Due to the rollover, the continuous debt service is the total coupon rate, C, plus the net partial principal repayment, mP. The debt rollover has no transaction costs but creates extra continuous debt service equal to mP.

After recapitalization, shareholders decide when to file for bankruptcy. They pick a boundary, L. For the asset value below L, they file for bankruptcy because there is no value in serving debt anymore. Shareholders fix the boundary for a period, T. At the beginning of each period, they set the boundary and they commit to declare bankruptcy when unlevered value falls below the boundary. Most of earlier research assumes a constant boundary due to constant variance. A very large T results in a constant boundary. Changing the boundary at certain points of time creates a degree of freedom over fixing the boundary for an infinite horizon. At default, shareholders recover zero value by losing the firm to creditors. Creditors liquidate the firm and receive the value of the firm less the bankruptcy costs, $(1-\rho)L$. ρ is the proportional bankruptcy cost (PBC) rate.

When equity holders decide about the boundary, they maximize the equity's value. The shareholders' optimization problem requires optimal boundary, L^*, to satisfy the "smooth pasting" condition ($L^* : \frac{\partial Eq}{\partial \nu}|_{(\nu=L^*)} = 0$, where Eq is equity value function). L^* maximizes equity value and $\frac{\partial Eq}{\partial L^*} = 0$. Smooth-pasting condition assures that equity derivative remains continuous at L^* and equal to 0.

Without VRP, the model reduces to the dynamic model in Leland (1998) without optimal risk shifting, cash-flow-triggered default, transaction costs, and rebalancing. These assumptions are not in the model because qualitatively they do not change the results but they add extra layers of complexity to the model. Even with the current simplifying assumptions, the model equations are hard to setup and solve because of stochastic variance and require applying approximations to derive the optimal policies.

The assumption about outstanding debt being perpetual allows for time homogeneity and formulating equity and debt functions from a recursive relation: given states $\nu_{nT} \in [L^*(V_{nT}), \infty]$ and $V_{nT} \in (0, \infty]$ at time nT where $n = \{0, 1, 2...\}$, shareholders choose optimal L^*_{nT} as only a function of the current variance state, V_{nT}. Then, at time kT, L^*_{kT} is a function of V_{kT} while ν is only checked for crossing the boundary L^*_{kT}. The time homogeneity at decision points means that, if by coincidence $V_{kT} = V_{nT}$, then optimal policies are the same, $L^*_{kT} = L^*_{nT}$, because variance is the only state variable to determine the optimal boundary. If the firm is not in default, the optimal default boundary is dependent only on variance and independent from the current value of the firm; shareholders do not change their mind about the boundary, if the firm value changes. This result has already been used in previous works, such as Leland (1994) and Leland and Toft (1996), for constant variance cases.[12] Figure 1 shows the structure of the model.

Insert Figure 1 about here.

[12]Proposition 1 in Online Appendix 4 shows that the same result is valid for a more general case.

Table 1 compares the model with some of the studies and highlights model features. For example, Hsu, Sa-Requejo, and Santa-Clara (2010) use stochastic boundary, but it has just random variations without an underlying factor. This paper considers stochastic asset variance as the underlying cause for time-varying optimal default boundary. Ericsson et al. (2011) show the effect of asset VRP on credit risk and McQuade (2013) analyzes asset VRP and Fama-French factors. This paper extends their work from asset pricing into the capital structure using a novel approximation technique which allows me to find optimal leverage and determine the effect of asset VRP on leverage.

Compared to numerical integration in Ericsson et al. (2011), the approximation method provides direct formulas for debt and equity. Without the formulas, optimal leverage calculations require 3 levels of numerical calculations: calculating optimal leverage requires the calculation of optimal default which, itself, depends on calculation of debt and equity values. Basically, the calculations of debt and equity values add one layer of numerical complexity. However, this paper's formulas allow to even derive semi-closed form for optimal default and, only optimal leverage requires numerical calculation which substantially reduces the numerical complexity. Perturbation method based on Fouque et al. (2006) requires fast mean-reversion speed to deliver accurate estimation. However, the calibrations in this paper show that volatility has very slow mean-reversion speed, specially under RN measure. This slow mean-reversion speed may reduce the accuracy of the perturbation method. In this paper, I extend the technique in Romano and Touzi (1997) to calculate debt and equity values, which is less sensitive to mean-reversion speed. The formulas perform reasonably and converge[13], when compared to simulation results.

Insert Table 1 about here.

[13] See Online Appendix 5

C. Securities valuation

C.1. Debt valuation

Perpetuity of debt allows writing the formula in recursive form, given time homogeneity.At the beginning of each period, the debt value, d, is :

$$d(\nu_0, V_0) = E^Q\Big(\tfrac{c+mp}{m+r} + (1 - I_{\tau<T})\Big[e^{-rT}(d(\nu_T, V_T) - \tfrac{c+mp}{m+r})\Big] + I_{\tau<T}\Big[e^{-r\tau}((1-\rho)mL - \tfrac{c+mp}{m+r})\Big]\Big) \tag{4}$$

where τ is default time and $I_{\tau<T}$ is 1, if default happens prior to time T. The value of debt is simply the cash flow from coupons and debt rollover until the holder re-evaluates it at time T or receives the recovery value at default. The first term is the present value of risk-free perpetuity with debt retirement. The second term is the present value of debt at time T given no default. The final term is the present value of debt in default. The coupon and face value of outstanding debt has linear relation with coupon and face value of d ($c = mC, p = mP$), which implies $d = mD$. Using the approach in Romano and Touzi (1997), I define ζ as $ln(\nu/L)$ and future variance, $\hat{V}$, as average variance between time 0 and T. They follow (see Appendix C):

$$\begin{cases} d\zeta = (r - \delta - \frac{1}{2}\hat{V})dt + \sqrt{\hat{V}}dW \\ \hat{V} = (\int_0^T V_s ds)/T \end{cases} \tag{5}$$

The transformed state variables facilitate the valuation. However, both new variables are still random. I condition the term inside the expectation operator in Equation 4 on future variance. Using the conditioning, the debt formula is (see Appendix D.1 for details):

$$\begin{aligned} d &= \tfrac{c+mp}{m+r} + \big((1-\rho)\, mL - \tfrac{c+mp}{m+r}\big)\, E(e^{-H_b\zeta_0}) \\ h &= \frac{r-\delta-\frac{1}{2}\hat{V}}{\sqrt{\hat{V}}} \qquad H_b = \frac{\sqrt{h^2+2(r+m)}+h}{\sqrt{\hat{V}}} \end{aligned} \tag{6}$$

The only random variable is future variance within the expectation operator (the last term in the equation). Based on the solution for the expectation, debt value is:

$$
\begin{aligned}
&d = \tfrac{c+m\,p}{m+r} + \left((1-\rho)\; mL - \tfrac{c+m\,p}{m+r}\right)\, \left(1 - \tfrac{1}{2}(A_b\,\zeta_0 - B_b\,{\zeta_0}^2)\right) e^{-\zeta_0 \hat{H}_b} \\
(7) \qquad &A_b = \hat{H}_b''.E[(\hat{V}_0 - E[\hat{V}_0])^2], \qquad B_b = {\hat{H}_b}'^2.E[(\hat{V}_0 - E[\hat{V}_0])^2] \\
&\hat{H}_b = H_b|_{\hat{V}_0=E[\hat{V}_0]}, \quad \hat{H}_b' = \tfrac{\partial H_b}{\partial \hat{V}_0}|_{\hat{V}_0=E[\hat{V}_0]}, \quad \hat{H}_b'' = \tfrac{\partial^2 H_b}{\partial \hat{V}_0^2}|_{\hat{V}_0=E[\hat{V}_0]}
\end{aligned}
$$

There is no available closed-form solution for the future variance moments. Appendix E derives the formulas for the moments. Taylor expansion up to second order approximates the expectation term around the expected variance.[14] Hull and White (1987) and Sabanis (2003) have also used a similar approximation technique.

C.2. Equity and Optimal Default Boundary

Similar steps produce the values for the tax benefits and bankruptcy costs of debt (see Appendix D.2), where *tax* is the tax rate:

$$
(8) \qquad TB(\zeta_0, \hat{V}_0) = tax \times \frac{C}{r} - tax \times \frac{C}{r} E\Big(e^{-H.\zeta_0}\Big), \quad H = \frac{\sqrt{h^2+2r}+h}{\sqrt{\hat{V}}},
$$

$$
(9) \qquad BC(\zeta_0, \hat{V}_0) = \rho L E\Big(e^{-H.\zeta_0}\Big)
$$

In the contingent-claim framework discussed in Ericsson and Reneby (1998), the expectation term is the value of a claim that pays $1 at default, or the discounted RN default probability:

$$
(10) \qquad E\Big(e^{-H.\zeta_0}\Big) \simeq e^{-\zeta_0.\hat{H}}\Big[1 - \frac{1}{2}(A\zeta_0 - B\zeta_0^2)\Big]
$$

[14]Online Appendix 5 compares the values from the simulation and approximation results to also show that the second order is acceptable.

where all the parameter definitions, such as A and B, and derivations are in Appendix E. Equity is simply the unlevered value plus the tax benefits less debt and the bankruptcy costs:

$$Eq(\zeta_0, \hat{V}_0) = \nu_0 + TB(\zeta_0, \hat{V}_0) - D(\zeta_0, \hat{V}_0) - BC(\zeta_0, \hat{V}_0) \tag{11}$$

All the valuations depend on the optimal default level. The optimal boundary is:

$$\frac{\partial Eq}{\partial \nu}|_{(\nu=L^*)} = 0 \Rightarrow L^* = \frac{\frac{C+mP}{r+m}(\hat{H}_b + \frac{1}{2}A_b) - \frac{tax \times C}{r}(\hat{H} + \frac{1}{2}A)}{1 + (1-\rho)(\hat{H}_b + \frac{1}{2}A_b) + \rho(\hat{H} + \frac{1}{2}A)} \tag{12}$$

Here, the boundary adjusts for asset VRP and nests earlier models: if m is zero and without VRP, the optimal boundary is very similar to the formula in Leland (1994).

D. Comparative Statics: Optimal Leverage and Default Boundary

The comparative statics show that asset VRP reduces both the optimal default boundary and leverage. In the statics, the model parameters match the empirical values. Risk-free and asset payout rates are respectively equal to 5% and 3% to match the historical rates between 1997 and 2015. Variance volatility is 30% to match the standard deviation of variance in equity data. Historical variance mean-reversion speed, κ, is set to 4 to make sure that the Feller condition is met similar to Ericsson et al. (2011). The average asset variance is 0.04, 20% squared, close to the asset variance in low-risk firms reported by Elkamhi, Ericsson, and Parsons (2012). The managers update their decision for boundary every year, $T = 1$. Effective tax rate is 25% and debt rollover rate is 10%, which are the tax and minimum rollover rate used in Leland (1998). The tax rate is lower than conventional 35% to include personal taxes. PBC rate is in the 30% to 60% range with an average of 45% as calculated by Glover (2016). Earlier studies may use lower costs, e.g. Bris, Welch, and Zhu (2006) and Davydenko, Strebulaev, and Zhao (2012), but Glover (2016) argues that earlier low estimates have selection bias and fixing for the bias increases the bankruptcy costs. Elkamhi et al. (2012) also estimate 50% cost that includes other early distress costs, such as lost customers. For optimal leverage, the decision variable is the amount of debt to borrow, P, and maximizes the total levered value of the firm, $P : \partial(Eq + D)/\partial P = 0$. The coupon rate is set to make the face value of debt, p, match with the market value, d. If both face value and coupons are optimized, the managers choose 0 face value to avoid debt rollover and the answer is degenerate; the model collapses into a static model. Finally, the default boundary is at the optimal level.

VRP decreases the optimal default boundary (see Figure 2). Intuitively, with high VRP, equity holders are more patient to file for bankruptcy in hopes for getting out of trouble due to the uncertainty in variance. Equity is analogous to a call option on the assets and call's value increases with RN variance of the underlying asset. This relation makes equity holders more likely to wait because high RN variance implies high value for their call option that only benefits from the upside risk. Thus, shareholders are more patient when VRP is high, and they lower the optimal boundary.

Another explanation for the negative effect of VRP on the boundary is through the real-option theory. In Dixit and Pindyck (2012)'s real-option framework, filing for bankruptcy is a real option held by shareholders and they compare it with the costs of serving debt (Geske, 1977). *Ceteris paribus*, an option experiences appreciation when its underlying asset's VRP goes up. This principle applies to the real option to default: high VRP increases the value of holding the real option to default, which reduces optimal default threshold.

The optimal market leverage ratio decreases with high asset VRP in Figure 3. High VRP implies high RN variance because it represents the relative difference between RN and historical variances. Even if default boundary is lower for high RN variance, overall, high RN variance increases RN default probability. Hence, VRP increases the present value of the bankruptcy costs and lowers the tax benefits, which both reduce optimal market leverage. These results lead to the following hypothesis:

Hypothesis 1. *The asset variance risk premium decreases target leverage, ceteris paribus (H1).*

Insert Figure 2 about here.

Insert Figure 3 about here.

III. Empirical Analysis

A. Data, puzzles and variance risk

Model calibrations with data show that asset VRP decreases optimal leverage to a level closer to actual leverage, especially for IG firms. I also compare the contribution of the model with VRP versus the model without VRP in explaining the empirical trends.

The sample includes matched fiscal year-end financial data of the US firms from Compustat, the ratings from S&P, and equity historical volatilities for 365 days from Optionmetrics. Data is between 1997 and 2015 due to the availability range for Optionmetrics data. I drop financial firms (SIC codes 6000-6999), utility firms (SIC codes 4900-4999), non-public firms, small firm-years with book asset value (AT) below \$10 millions and subsidiaries (STKO=1 and 2) since they have different bankruptcy policies (for example, see Luciano and Nicodano (2014)). Equity market cap is shares times share price. Equity volatility is 365-day standard deviation of equity returns. The linear interpolation fills equity-volatility time series for missing data dates, where volatility represents variance with the square-root transformation. The time series of the US treasury notes as risk-free rate and yield spreads are from Federal Reserve. The yield data is available for an average rated firm from the Bank-of-America Merrill-Lynch (BOA-ML) US Corporate Option-Adjusted Spreads (OAS). In each rating, the spread is between OAS index of all the bonds and the spot risk-free rate. BOA-ML spreads are available only for the average rated firms (e.g. see Hong and Sraer (2013) that also use same data). Therefore, this paper calibrates the model to the representative firms in each category instead of each firm-year data point. Later, in robustness check, I use firm-year data using a different source and results do not change. The simple averages of the firm-years' characteristics in each rating represent the firms in the rating. The first four columns of Table 2 show the statistics for all the firm-years in the sample. In data, an average IG firm's equity is larger and financial risks are smaller with lower equity volatility, yield spread, and financial leverage. The trends in data across ratings are similar to earlier studies such as Huang and Huang (2012). Further discussion about the table follows.

Place Table 2 about here

A.1. Leverage and credit premium without VRP

The calibration results for all the representative firms are also in Table 2. The calibration without VRP in this paper is standard procedure in the literature (Jones et al., 1984; Schaefer and Strebulaev, 2008) (calibration details such as the procedure and formulas are in Appendix F). In brief, the calibrations adjust asset value and volatility to match model-implied equity value and volatility with empirical values (2-by-2 calibration: 2 equations, 2 unknowns). When rating improves, calibrated asset size has increasing trend and historical asset risk has declining trend in the last columns. These trends together support the stylized observation that the historical probability of default for better ratings are smaller as reported by the rating agencies. The trends also are similar to reports by the earlier studies (Elkamhi et al., 2012; Schaefer and Strebulaev, 2008; Eom et al., 2004). Figure 4 shows the counter-intuitive positive correlation between asset volatility and leverage without VRP from Table 3. Since IG firms' historical asset variance is lower and size is larger than the other firms, they have lower historical default risk. Without asset VRP, it seemingly contradicts intuition when they choose low leverage despite their low historical default risk.

Insert Figure 4 about here.

Insert Figure 5 about here.

I use the calibrated parameters for representative firms without VRP to calculate model-implied leverage and yield spread. Middle columns in Table 2 compare these model-implied values with their empirical twins. In Figure 5, I calculate the percentage difference between model-implied and empirical values.

For SG firms, the model without VRP reports lower optimal leverage than observed leverage. A similar trend appears later in the calibrations with VRP for SG firms. This "overleverage" is normal and matches the intuition: low grade firms should have lower leverage, but they do not afford buying back debt due to facing financial distress and debt buyback costs. In other words, they naturally are overlevered because their financial distress makes debt almost irreversible for them. As long as the costs to buyback debt are higher than default or issuing new debt, they remain overlevered. For example, one source of high refinancing cost for them is the stylized hold-out problem. Empirical evidence also supports this established observation. Simple transaction cost is one of the empirically identified reasons for overleverage (Gilson, 1997). Another empirical reason is financial distress for these firms. Korteweg (2010) reports that very low "refinancing probability is consistent with market frictions preventing these highly overlevered firms from immediately unlevering, causing significant friction costs due to financial distress." Although optimal leverage for these firms are lower than their leverage and they benefit the most from unlevering, the overlevered firms are very unlikely to unlever. So from now on, I mostly focus on IG firms.

Modeling a firm's decisions without asset VRP leads to serious model misspecification, which is also not limited to leverage. For IG firms, the misspecification creates some gap between observed leverage and optimal leverage which has a one-on-one relationship with the mismatch in the credit spread. Of course, there is no expectation for the model to explain all the credit spread due to other priced factors such as liquidity (Bongaerts, De Jong, and Driessen, 2011). However, the gap is the largest for IG firms which have liquid bonds due to their ratings. The calibrations ignore asset VRP and, consequently, are misspecified. In the finance literature, there are separate reports of the mismatch in credit premium and leverage without noticing any connection across ratings. For example, the gap in the credit spreads is reported by Huang and Huang (2012). The gap in leverage is implicit in capital structure studies without a thorough analysis, such as in Elkamhi et al. (2012). The big picture that this paper portrays combines these familiar pieces in the literature together with a thorough analysis on leverage. Chen (2010) also finds that there is connection between the credit spread and leverage puzzles, but only in time series without priced variance risk as variance depends only on macroeconnomic conditions. However, I show that the puzzles are cross-sectionally connected: the unexplained leverage and credit spreads are larger only in top ratings, which supports the idea that ignoring VRP creates misspecification not limited to credit premium. Moreover, I relax deterministic variance with priced variance risk. Next, I show that asset VRP alleviates the mismatch in explaining the empirical trends.

A.2. Calibration with VRP

Table 3 shows the calibration results. Compared to the model without VRP, model calibration with VRP has one extra equation and parameter: asset VRP adjusts to match model-implied yield spread to empirical values (3-by-3 calibration: 3 equations, 3 unknowns).[15] Similar to the calibrations without VRP, IG firms have relatively larger size and lower mean asset volatility in the model with VRP. Thus, the model with VRP also replicates low historical risk and default probability of the top ratings.

Place Table 3 about here

Regarding VRP, Ait-Sahalia and Kimmel (2007) report RN mean-reversion speed of 5 on VIX as equity-market variance index. In Table 3, the mean-reversion speed of asset variance is smaller than 5 in RN measure for two reasons, idiosyncratic variance and no leverage. First, leverage inflates equity variance compared to asset variance (Merton, 1974). *Ceteris*

[15]As a stylized finding, the structural models with constant variance do not produce large enough yield spreads, which leads to the credit premium puzzle (Jones et al., 1984; Elton, Gruber, Agrawal, and Mann, 2001; Amato and Remolona, 2003; Eom et al., 2004; Huang, 2010). There is no yield spread in the calibration for the model without VRP because this model cannot match the observed yield spreads in data. But, the model with VRP perfectly matches the yield spread. Later in Section C.1, when I drop yield spread from calibrations and apply 2-by-2 calibration to both models with and without VRP, the results do not change.

paribus, high leverage increases the financial risk of a firm, which increases equity's variance almost linearly with leverage. Thus, leverage of S&P-500 firms also inflates mean-reversion speed of equity-return variance in VIX compared to the speed of asset variance. Second, individual firm-level speed is also deflated compared to market's speed due to idiosyncratic variance, assuming that idiosyncratic variance is not time-varying as much as market variance (see Equation 3). While the physical mean-reversion speed is constant for all the firms, RN speed is smaller in IG firms because of high VRP. Ericsson et al. (2011) also report similar ranges for VRP and mean-reversion speed.

First and foremost, Table 3 shows that optimal leverage is lower with asset VRP (H1), especially for IG firms that tend to have the highest asset VRP. Contrary to the earlier low levels of historical variance and default probability for IG firms, the trend of the assets' VRP is reverse. High VRP implies that IG firms have relatively high RN variance and RN default probability, even with low historical variance and historical default probability. For IG firms, the comparison confirms that the model with VRP significantly alleviates their underleverage compared to the model without VRP. Hence, the following hypothesis is implied by the calibrations, which I verify statistically in addition to H1 in the next section:

Hypothesis 2. *Asset VRP reduces the investment-grade firms' leverage more than the other firms, ceteris paribus (H2).*

It is interesting to notice how ignoring VRP may mislead a researcher about the leverage behavior: historical asset variance and default risk for IG firms are lower than the average. Many theories of capital structure and intuition expect their low historical risk to create potential for increasing debt in their capital structure. But, this expected high optimal leverage conflicts with low leverage choice of IG firms. However, taking VRP into account reconciles them: some firms just try to hedge their variance risk. Thus, model misspecification, i.e. not considering VRP, contributes to seemingly paradoxical leverage choice of the firms, which adds to the other explanations.

B. Regression analysis

The regressions have the advantage of being model-free compared to the calibrations. In the linear regressions, the assets' VRP proxy has negative effect on leverage of the firms, while there are controls for other factors such as cash holdings and internal funds. On the same sample used for calibrations, the ratio of equity's RN and historical volatility is the assets' VRP proxy. Equity RN volatility is volatility implied by option prices. Since both RN and historical equity volatilities are inflated by the leverage, the ratio cancels out the leverage effect. The regressions also have the classical factors used by the earlier studies (For example, see Bae, Kang, and Wang (2011)): a) asset tangibility to control for bankruptcy costs, b) profitability to control for pecking order, c) cash holdings to control for financial flexibility d) natural logarithm of revenues to control for size e) Tobin's Q to control for the growth of the business, f) unlevered historical equity volatility to represent historical asset volatility. Appendix A has more details about the calculation of the variables, such as Compustat codes. Table 4 shows the descriptive statistics of these variables.

Place Table 4 about here

It is interesting to notice some anecdotal factors that imply IG firms should have more debt (see Figure 6): IG firms have more tangible assets relative to their debt, which implies lower bankruptcy costs because tangible assets can be used as collateral for debt (Jensen and Meckling, 1976). They have less risk, low historical asset variance[16] and asset beta, and they are larger in size, where both reduce the chances of information asymmetry (Hennessy and Whited, 2007) or default (asset beta is unlevered equity beta). These size and historical risk trends also appear in the calibrations. IG firms have high payout rate, mostly to the shareholders, signaling less concerns for financial flexibility as suggested by DeAngelo, DeAngelo,

[16]Only CCC and lower ratings have low unlevered volatility due to their extreme leverage: their unlevered volatility is low because unlevered volatility is equity volatility$\times(1 - leverage)$ and does not closely reflect their true asset volatility. Only calibrations address this issue (see their asset volatility in Table 2 and Figure 4).

and Whited (2011) and hold less cash for internal funding of their projects. Regardless of all these positive factors pushing them towards having higher leverage, they have the lowest leverage. In Figure 7, the only factor that sends a negative signal for low leverage is asset VRP; the proxy for VRP and firm exposure to market variance risk for IG firms are the highest on average, which also anecdotally imply H2. Exposure to market variance risk is defined in Section C.1 with more detailed analysis.

Place Figure 6 about here

Place Figure 7 about here

To verify these anecdotal findings, this paper runs the regression that considers leverage adjustments by the firms following DeAngelo and Roll (2015), Banerjee, Dasgupta, and Kim (2008), and Fama and French (2002):

$$(13) \qquad Lev_{i,t} - Lev_{i,t-j} = \Psi[Target_{i,t} - Lev_{i,t-1}], \qquad Target_{i,t} = \sum_k a_k X_{k,i,t}$$

where Ψ is adjustment speed, $Target$ is target leverage, Lev is leverage, i is the firm index, t is time index, X has the independent variables, and k is the index for independent variables. With the assumption that j is for one period and there is an intercept in the regression, the relation turns into:

$$(14) \qquad Lev_{i,t} = (1 - \Psi)Lev_{i,t-1} + a_0 + \sum_k \Psi a_k X_{k,i,t} + \epsilon_i + \epsilon_t + \epsilon$$

where clustered-error controls are ϵ_i, the firm, and ϵ_t, time. There are several independent variables. Average leverage of the industry for each firm controls the industry effect and there is no difference between having fixed industry dummies or the leverage average. Dummies for years and firms control the fixed effects. The other independent variables are the same as listed in Table 4 without the asset payout rate. The regressions are on the whole sample and the subsamples of IG and SG firm-years. Factors are standardized to analyze the relative importance.

Place Table 5 about here.

Table 5 shows the results.[17] Similar to Bae et al. (2011), R-squared is above 50% due to including fixed effect dummies and lagged leverage. Asset historical volatility has the largest effect on leverage. The overall strong effect of volatility is similar to the results in Chen, Wang, and Zhou (2015). One standard deviation of a firm-year from the average drops target leverage by 6%. But, historical volatility has less strong effect in the IG subsample.

On the other hand, one standard deviation of the assets' VRP from the average among IG firms has stronger negative effect on leverage than SG firms. The VRP coefficient in the IG sample is almost relaively twice as the SG sample: the asset VRP coefficient is 50% of the historical volatility coefficient for the IG subsample, while it is 30% in the SG sample (3.1%/6.4% and 2.9%/8.6%). Similar results also appear on the subsample in robustness check. Another noteworthy result is the relatively higher impact of asset VRP than the profitability in data. Titman and Wessels (1988) and Myers (1993) argue that profitability is a widely considered factor to have first-order effect on leverage. If all the factors are ranked based on their standardized impact, asset VRP stands second in line next to asset volatility and above the other factors. In sum and similar to the calibrations, asset VRP has negative effect on leverage in the regressions (H1) and the negative effect is relatively stronger in IG firm-years (H2) than the average.

C. Robustness check

C.1. Calibration with exposure to market variance risk

As an alternate asset VRP measure, I check exposure of the firms' asset variances to market variance and, consequently, market VRP. I show three outcomes. First, I confirm IG firms' high exposure to market VRP, which explains their high asset VRP and its negative effect

[17] None of the subsamples have zero-leverage observations, so the issues raised by Elsas and Florysiak (2015) or Lotfaliei (2015) does not apply to this paper's analyses.

on their leverage. Second, I replicate results in Ericsson et al. (2011) and McQuade (2013) about improved credit spreads with VRP to show that using this new method is reasonable. Finally, the calibration results are robust to VRP estimation method which parsimoniously uses exposure to market variance risk instead of credit spread. The model with VRP still shows significantly closer leverage ratio to observed leverage ratio.

For each firm-year between 1997 and 2015, I calculate 90-day correlation between squared CBOE's VIX index and squared 30-day option-implied equity volatility. Since VIX is based on 30-day options, I use 30-day volatility for each firm rather than 365-day volatility. The volatility is squared to represent variance. Assuming that leverage is constant during the 90-day period before the data date, the correlation is not inflated by the firm's leverage. Leverage inflates both the covariance of equity variance with squared VIX and the standard deviation of equity variance. Leverage cancels out when the covariance is divided to volatility of variance in correlation calculation. The average correlation across all the firm-years in each rating represents the exposure of an average firm in the rating to market variance, and, subsequently, to market VRP. Therefore, I use the correlation as a measure of exposure

Insert Figure 8 about here.

Figure 8 shows support for H2 where an average IG firm has higher exposure to market variance than an average firm while they have lower asset beta. Compared to Figure 6 and Figure 7, Figure 8 report the averages across major categories rather than subcategories. Unlevered asset beta is unlevered historical equity beta from the market returns in past year. Asset beta shows exposure to market return shocks, while correlation shows exposure to market variance shocks. For example, across the extreme ratings, the IG subsample in the figure has almost twice as much exposure than the SG subsample. But, SG firms have higher asset beta than IG firms. A similar trend in asset beta across ratings is also reported by Huang and Huang (2012) which has larger gap between IG and SG firms. Hence, the asset betas are more parsimonious with lower gap in this paper. If IG firms have higher exposure to market variance, then they are more likely to face high asset VRP. It is different from hedging asset beta which is about shocks to market return. IG firms have indeed low asset beta while their asset VRP is high. These firms choose low leverage in this case to hedge VRP and the risk of their asset variance to be affected by a market-wide variance shock. Next, I verify this behavior through leverage calibration via the exposure.

Insert Table 6 about here.

Table 6 shows the robust calibrations where model with VRP generate more realistic leverage ratios (Appendix F has the calibration details). The earlier calibrations for the model with VRP include yield spread. This extra parameter may imply an unfair calibration advantage towards the model with VRP. In addition, any error in measuring yield spread only would affect the calibrations in the model with VRP. Hence, I change the calibration method to check the robustness of results as follows: for representative firms, both models with or without VRP are only calibrated with 2-by-2 equations. In 2-by-2 equations, asset volatility and value are calibrated to equity volatility value. Model with VRP has 2 extra parameters, instant asset volatility and asset VRP. Instant volatility is set to mean asset volatility. Asset VRP is asset variance exposure, α, times market VRP as in Equation 3. I multiply the exposure times 4 which is estimated by Ait-Sahalia and Kimmel (2007) as the market VRP. The asset VRP estimation is more parsimonious in this method and lower than asset VRP in Table 3. Nevertheless, the results are similar to earlier calibrations in Table 3. Not only the results support the negative relationship between asset VRP and leverage (H1), but also the model with asset VRP still produces leverage closer to observed leverage for IG firms relative to other firms (H2).

Insert Table 7 about here.

In order to validate this calibration method, I report the model-implied yield spreads in Table 7. If considering asset VRP alleviates the underleverage puzzle (H1) and there is connection between credit and underleverage puzzles as in Figure 5, then, one expects to also see improvement in credit puzzle with using asset VRP. There is no expectation to explain *all* the credit spreads due to other priced factors such as liquidity. However, if the VRP intuition is valid, there is expectation to improve model-implied spreads, especially for IG firms. I use the parameters from Tables 6 and 2, such as asset VRP, asset volatility and value, to calculate model-implied yield spreads with and without asset VRP. The calibration without VRP shows the classical credit premium puzzle. But, the model with VRP indeed produces yield spreads which are closer to the observed spreads. Moreover, the highest improvement is in IG category due to their high asset VRP.

These results are similar to Ericsson et al. (2011) and McQuade (2013). They report improved credit spreads with asset VRP, but this paper also connects the credit spreads to observed leverage. Regarding the exposure of firm asset to VRP, Dorion (2010) also reports that the ratio of systematic volatility for IG firms' assets is higher than an average firm across ratings. Using a different method, he shows that the diversification gain in volatility is smaller by putting IG firms in a portfolio than the others.

As explained in Section A, the differences in asset beta and asset VRP are due to idiosyncratic volatility's impact on VRP. The magnitude of idiosyncratic volatility does not change asset beta but it reduces asset VRP. Idiosyncratic variance cushions the impact of the shocks to market variance being transferred to the firm. When idiosyncratic variance is relatively low, the systematic portion of total variance is larger and the effect of market variance change is larger in IG firms. Since market variance has premium, the premium results in higher VRP on IG firm's assets. Hence, IG firms have higher VRP than SG firms, while IG firms have lower asset beta. In sum, the firms, particularly IG firms, reduce leverage because, in addition to other factors, their assets has VRP through market VRP.

C.2. Calibration using firm-year data

In order to make sure that the results of calibrations are not driven by using representative firms and Jensen's inequality, I run calibrations on firm-year data. It turns out that earlier representative-firm calibrations are more parsimonious because underleverage in model without VRP is larger in firm-year calibrations. Calibrating to firm-year data has the advantages that statistical test on the calibrations is possible and it also delivers the time-series of estimated asset VRP. The disadvantage is to have a much smaller sample leaning towards IG firms. Nevertheless, the inferences for H1 and H2 are similar.

Data collection process is similar to Section A, but I replace representative-firm yield spread data with firm-years. I collect corporate bond characteristics, prices, and yield from Mergent and TRACE. The sample shrinks to the 2002-15 period because of bond data availability from TRACE. Bond data are for senior unsecured corporate bonds without features such as being callable. Yield spread is bond yield less maturity-matching risk-free rate. Missing yield data for data dates are filled by linear interpolation of daily data. The first four columns of Table 8 show the statistics for all the firm-years in the sample. The sample leans more towards IG firms because bond data's availability is better for these firms. Similar to Table 3, an average IG firm's equity is larger, and equity volatility, yield spread, and financial leverage are lower.

Place Table 8 about here

The calibration results for all the firm-years are in Table 8 where the results are similar to Table 3. Calibration details are in Appendix F. For each firm-year, in the model without VRP, asset value and volatility calibrate so that equity value and volatility match with empirical data (2-by-2 calibration). Calibrations with VRP is 4-by-4; it involves two extra equations (to match empirical yield spread and empirical leverage) and two extra parameters (instant asset volatility and asset VRP).[18] Specifically, the model with asset VRP implies yield spreads that fit their empirical twins as part of the calibration similar to Table 3.

I report calibrated parameters only for the model with VRP for brevity. Asset VRP

[18]Calibration to the leverage does not create an endogeneity problem and, in both models, it is a comparison between "as is" versus "as expected" for the leverage. Nevertheless, this paper also reports similar results when leverage is dropped from calibrations in Table 8 for robustness check in Online Appendix 6.

is close to the numbers from representative-firm calibration. The model with stochastic variance implies a flatter difference between historical instant and mean asset variance for IG firms. Therefore, if there exists a historical asset-volatility term structure, it is flatter for IG firms than the others. This means that the VRP proxy used in Figure 7a for the regressions actually overestimates SG firms' asset VRP proxy and is more parsimonious. Since an SG firm's asset VRP is low and long-run historical volatility is more likely to be higher due to steeper term structure, the ratio of long-run RN to historical volatility is more likely to be smaller than the same ratio based on 1-year values.

The model-implied leverage with and without VRP are based on calibrated parameters. These results show that the calibrations to representative firms are more parsimonious because underleverage in the model without VRP is more severe in firm-year data. In other words, Jensen's inequality lowers model-implied leverage in the representative-firm calibration compared to calibrating to firm-years and, then, averaging. Since IG firms' historical asset variance is lower and size is larger than the other firms, they have lower historical default risk. Without asset VRP, it is counter-intuitive when IG firms have relatively low historical default risk but also choose low leverage. However, calibrating with VRP shows that these firms have high VRP and implies lower leverage.

Table 9 statistically measures the sample differences which controls for firm and year clustered errors using the method recommended by Petersen (2009). I estimate $Leverage_{data} - Leverage_{model} = \alpha + \epsilon_i + \epsilon_t + \epsilon$ where α is the difference, and ϵ_i and ϵ_t control for clustered firm and time errors. Negative α implies underleverage where observed leverage is smaller than model-implied leverage and vice versa. For SG firms, both models with and without VRP overestimate optimal leverage, which highlight these naturally overlevered firms as discussed earlier. For IG firms, the model without VRP significantly underestimates leverage while the model with VRP generates more reasonable results. Hence, ignoring VRP exacerbates the seemingly paradoxical choice of these firms to have conservative leverage.

Place Table 9 about here

In addition to the cross-sectional analysis, this paper reports the time-series behavior of an average firm's estimated asset VRP and compares it with leverage and the market's proxy for VRP. Figure 9 shows the time-series. Asset VRP index is the size-weighted asset VRP from calibrations. Leverage index is size-weighted observed leverage of the firm-years in each year. The market VRP proxy is the ratio of the difference between markets' RN and historical volatility to historical volatility. Market's RN volatility is CBOE's VIX and historical volatility is for 30-day returns of S&P-500. Both VIX and S&P-500 historical volatility are based on equity which inflates them by the leverage of the S&P-500 firms. When the difference is divided to historical volatility, leverage cancels out and market VRP proxy is unlevered. Market VRP is based on the information from the options market. Market VRP looks more volatile than the asset VRP because asset VRP's calculation is based on a longer horizon than market VRP proxy.

Insert Figure 9 about here.

Even without including any information from the options market in this paper's asset VRP index, the average asset VRP and market VRP proxy seem to correlate. They also have negative relationship with leverage. The negative relationship seems stronger during non-crisis times when the VRP indexes are high and imply high variance fears. The fear seems legitimate: during the crisis, the negative economy-wide shock drives historical variance up close to RN variance which is evident in market VRP reduction. This increase validates the market-wide fear of increases in variance, which leads to conservative low leverage during non-crisis times.

C.3. Regression analysis in subsample

For the 2002-15 period subsample, the empirical inferences from running model-free regressions also produce similar results in support of the model implications: VRP has negative effect on the firms' leverage, especially for IG firms. Between 2002 and 2015, this paper collects all the data following the same procedure as in the earlier section from merging Compustat and Optionmetrics with similar filters. The descriptive statistics are in Appendix G and the classical factors, such as asset volatility and Tobin's Q, that determine leverage follow the same trends as in Figures 6 and 7. Among these factors, only asset VRP proxies and exposure to market variance risk support IG firms' relatively low leverage.

Place Table 10 about here

Table 10 shows the results for the regressions. Asset VRP has negative effect on leverage as implied by the theory (H1)and it is the second factor in rank. Asset VRP is also relatively more important for IG firms (H2): the VRP coefficient in the IG sample is almost twice as the SG sample. Another noteworthy result is the relatively higher impact of asset VRP than the profitability in the IG subsample. The profitability seems more important for SG firms which are more likely to experience financial distress. Although the sample seems small, Appendix H reports similar results by estimation of bias-corrected least-square dummy variable (LSDV) with bootstraping to control for small sample. The appendix also addresses the classical Nickell (1981)'s critique due to including lagged leverage and fixed-effect dummies because it reports bias-corrected results using the method from Arellano and Bond (1991). Therefore, both hypotheses are still confirmed in model-free regressions on the subsample.

IV. Conclusion

This paper shows that variance risk premium at the asset level reduces leverage, especially for IG firms which seem to have greater exposure to market variance risk. I present a theoretical model to support these hypotheses and verify them in the empirical tests. Risky time-variations in asset variance produces the variance risk premium that increases the wedge between historical and RN variance. RN asset variance raises the RN probability of default for the firm which increases the costs of debt and reduces the tax savings. Hence, a firm with low historical variance and high variance premium chooses conservative leverage. This behavior hedges future increase in asset variance induced by the exposure to market variance shocks. Empirically, the model calibrations with VRP produce lower leverage closer to actual leverage of the firms than the model excluding VRP. In the time-series, asset VRP is high post crisis, which reflects the market's apprehension of a possible increase in variance during the crisis. This apprehension leads to lower leverage. In the cross-section of the firms ranked by their historical risk, the investment-grade firms hold assets with high exposure to market variance risk, which also seems to explain their conservative leverage. The regressions also validate negative VRP-leverage correlation with more negative correlation in the IG subsample.

The VRP effect cannot be detected in the earlier models with constant variance because they assume that historical and RN variances are equal. It is contrary to the stylized fact in other finance areas that variance is stochastic with priced risk. Focusing only on historical variance of the firm's assets creates model misspecification: without asset VRP, IG firms seem to have potential to increase their leverage considering their low historical risk of default, which remains underutilized. The misspecification not only is tractable in underleverage, but also links underleverage to other seemingly odd observations such as unexpectedly high credit premium identified by Eom et al. (2004) and Huang and Huang (2012).

A potential extension of this paper for future research is to explain the low speed of leverage adjustments reported by Frank and Goyal (2007). Asset variance is an important factor in the speed of leverage adjustments modeled by Goldstein, Ju, and Leland (2001); high variance implies fewer re-adjustments. All these models assume constant variance. But, VRP may be a factor in the puzzling infrequent leverage adjustments and alleviate the debate on the structural models (Welch, 2013; Strebulaev and Whited, 2013). Also, a potential subject for future research is to examine the underlying causes for high exposure to market variance risk in IG firms' assets; IG firms are large firms and may have high operating leverage or well-diversified portfolio of assets with low idiosyncratic variance.

References

Ait-Sahalia, Yacine, Jianqing Fan, and Yingying Li, 2013, The leverage effect puzzle: Disentangling sources of bias at high frequency, *Journal of Financial Economics* 109, 224–249.

Ait-Sahalia, Yacine, and Robert Kimmel, 2007, Maximum likelihood estimation of stochastic volatility models, *Journal of Financial Economics* 83, 413 – 452.

Amato, Jeffery D, and Eli M Remolona, 2003, The credit spread puzzle1, *BIS Quarterly Review* 51.

Anderson, Ronald W, and Suresh Sundaresan, 1996, Design and valuation of debt contracts, *Review of Financial Studies* 9, 37–68.

Arellano, Manuel, and Stephen Bond, 1991, Some tests of specification for panel data: Monte carlo evidence and an application to employment equations, *Review of Economic Studies* 58, 277–297.

Bae, Kee-Hong, Jun-Koo Kang, and Jin Wang, 2011, Employee treatment and firm leverage: A test of the stakeholder theory of capital structure, *Journal of Financial Economics* 100, 130–153.

Bakshi, Gurdip, and Nikunj Kapadia, 2003, Delta-hedged gains and the negative market volatility risk premium, *Review of Financial Studies* 16, 527–566.

Banerjee, Shantanu, Sudipto Dasgupta, and Yungsan Kim, 2008, Buyer−supplier relationships and the stakeholder theory of capital structure, *Journal of Finance* 63, 2507–2552.

Barras, Laurent, and Aytek Malkhozov, 2016, Does variance risk have two prices? evidence from the equity and option markets, *Journal of Financial Economics* 121, 79 – 92.

Barsotti, Flavia, 2012, Optimal capital structure with endogenous default and volatility risk, Dimad working papers, Dipartimento di Matematica per le Decisioni, Universita' degli Studi di Firenze.

Bekaert, G., and G. Wu, 2000, Asymmetric volatility and risk in equity markets, *Review of Financial Studies* 13, 1–42.

Bhamra, Harjoat S., Lars-Alexander Kuehn, and Ilya A. Strebulaev, 2010, The aggregate dynamics of capital structure and macroeconomic risk, *Review of Financial Studies* 23, 4187–4241.

Bongaerts, Dion, Frank De Jong, and Joost Driessen, 2011, Derivative pricing with liquidity risk: Theory and evidence from the credit default swap market, *Journal of Finance* 66, 203–240.

Brennan, Michael J, and Eduardo S Schwartz, 1978, Corporate income taxes, valuation, and the problem of optimal capital structure, *Journal of Business* 103–114.

Bris, Arturo, Ivo Welch, and Ning Zhu, 2006, The costs of bankruptcy: Chapter 7 liquidation versus chapter 11 reorganization, *Journal of Finance* 61, 1253–1303.

Broadie, Mark, Mikhail Chernov, and Suresh Sundaresan, 2007, Optimal debt and equity values in the presence of chapter 7 and chapter 11, *Journal of Finance* 62, 1341–1377.

Bruno, G. S. F., 2005, Estimation and inference in dynamic unbalanced panel-data models with a small number of individuals, *Stata Journal* 5, 473–500(28).

Campbell, John Y, Stefano Giglio, Christopher Polk, and Robert Turley, 2015, An intertemporal CAPM with stochastic volatility, Working paper no 18411, Harvard University.

Campbell, John Y, Jens Hilscher, and Jan Szilagyi, 2008, In search of distress risk, *Journal of Finance* 63, 2899–2939.

Carr, Peter, and Liuren Wu, 2009, Variance risk premiums, *Review of Financial Studies* 22, 1311–1341.

Chen, Hui, 2010, Macroeconomic conditions and the puzzles of credit spreads and capital structure, *Journal of Finance* 65, 2171–2212.

Chen, Hui, Hao Wang, and Hao Zhou, 2015, Stock return volatility and capital structure decisions, *American Finance Association meeting, Boston* .

Chen, Hui, Yu Xu, and Jun Yang, 2013, Systematic risk, debt maturity, and the term structure of credit spreads, *Working Paper Available at SSRN 2024329* .

Choi, Jaewon, and Matthew Richardson, 2016, The volatility of a firm's assets and the leverage effect, *Journal of Financial Economics* 121, 254 – 277.

Christoffersen, Peter, Steven Heston, and Kris Jacobs, 2013, Capturing option anomalies with a variance-dependent pricing kernel, *Review of Financial Studies* 26, 1963–2006.

Christoffersen, Peter F, and Francis X Diebold, 2000, How relevant is volatility forecasting for financial risk management?, *Review of Economics and Statistics* 82, 12–22.

Cox, John C, Jonathan E Ingersoll, and Stephen A Ross, 1985, A theory of the term structure of interest rates, *Econometrica* 53, 385–407.

Davydenko, Sergei A., Ilya A. Strebulaev, and Xiaofei Zhao, 2012, A market-based study of the cost of default, *Review of Financial Studies* 25, 2959–2999.

DeAngelo, Harry, Linda DeAngelo, and Toni M. Whited, 2011, Capital structure dynamics and transitory debt, *Journal of Financial Economics* 99, 235 – 261.

DeAngelo, Harry, and Richard Roll, 2015, How stable are corporate capital structures?, *Journal of Finance* 70, 373–418.

Denis, David J, 2012, The persistent puzzle of corporate capital structure: Current challenges and new directions, *Financial Review* 47, 631–643.

Dixit, Avinash K., and Robert S. Pindyck, 2012, *Investment under Uncertainty*, [E-reader version] (Princeton University Press).

Dorion, Christian, 2010, *Pricing financial derivatives: The impact of business conditions and systematic risk*, PhD thesis, McGill University, Montreal.

Elkamhi, Redouane, Jan Ericsson, and Christopher A Parsons, 2012, The cost and timing of financial distress, *Journal of Financial Economics* 105, 62–81.

Elsas, Ralf, and David Florysiak, 2015, Dynamic capital structure adjustment and the impact of fractional dependent variables, *Journal of Financial and Quantitative Analysis* 50, 1105–1133.

Elton, Edwin J, Martin J Gruber, Deepak Agrawal, and Christopher Mann, 2001, Explaining the rate spread on corporate bonds, *Journal of Finance* 56, 247–277.

Eom, Y. H, J. Helwege, and J. Huang, 2004, Structural models of corporate bond pricing: An empirical analysis, *Review of Financial Studies* 17, 499.

Ericsson, Jan, Redouane Elkamhi, and Du Du, 2011, Time-varying asset volatility and the credit spread puzzle, in *Paris December 2011 Finance Meeting EUROFIDAI-AFFI*.

Ericsson, Jan, and Joel Reneby, 1998, A framework for valuing corporate securities, *Applied Mathematical Finance* 5, 143–163.

Fama, Eugene F., and Kenneth R. French, 2002, Testing trade-off and pecking order predictions about dividends and debt, *Review of Financial Studies* 15, 1–33.

Fan, Joseph P. H., Sheridan Titman, and Garry Twite, 2012, An international comparison of capital structure and debt maturity choices, *Journal of Financial and Quantitative Analysis* 47, 23–56.

Figlewski, S., and X. Wang, 2001, Is the 'Leverage Effect' a leverage effect?, *Working Paper Available at SSRN 256109* .

Fouque, JeanPierre, Ronnie Sircar, and Knut Solna, 2006, Stochastic volatility effects on defaultable bonds, *Applied Mathematical Finance* 13, 215–244.

Frank, Murray Z., and Vidhan K. Goyal, 2007, Chapter 12 - trade-off and pecking order theories of debt, in B. Espen Eckbo, ed., *Handbook of Empirical Corporate Finance*, Handbooks in Finance, 135 – 202 (Elsevier, San Diego).

Frank, Murray Z., and Vidhan K. Goyal, 2009, Capital structure decisions: which factors are reliably important?, *Financial Management* 38, 1–37.

Garlappi, Lorenzo, and Hong Yan, 2011, Financial distress and the cross-section of equity returns, *Journal of Finance* 66, 789–822.

Geske, Robert, 1977, The valuation of corporate liabilities as compound options, *Journal of Financial and quantitative Analysis* 12, 541–552.

Gilson, Stuart C, 1997, Transactions costs and capital structure choice: Evidence from financially distressed firms, *Journal of Finance* 52, 161–196.

Glover, Brent, 2016, The expected cost of default, *Journal of Financial Economics* 119, 284–299.

Goldstein, Robert, Nengjiu Ju, and Hayne Leland, 2001, An EBIT-based model of dynamic capital structure, *Journal of Business* 74, 483–512.

Graham, John R., 2000, How big are the tax benefits of debt?, *Journal of Finance* 55, 1901–1941.

Graham, John R, and Mark T Leary, 2011, A review of empirical capital structure research and directions for the future, *Annual Review of Financial Economics* 3.

Hennessy, Christopher A, and Toni M Whited, 2007, How costly is external financing? evidence from a structural estimation, *Journal of Finance* 62, 1705–1745.

Heston, S. L, 1993, A closed-form solution for options with stochastic volatility with applications to bond and currency options, *Review of Financial Studies* 6, 327–343.

Hong, Harrison, and David Sraer, 2013, Quiet bubbles, *Journal of Financial Economics* 110, 596–606.

Hsu, J., J. Sa-Requejo, and P. Santa-Clara, 2010, A structural model of default risk, *Journal of Fixed Income* 19, 77–94.

Huang, Jing-zhi, 2010, The structural approach to modeling credit risk, in *Handbook of Quantitative Finance and Risk Management*, 665–673 (Springer).

Huang, Jing-Zhi, and Ming Huang, 2012, How much of the corporate-treasury yield spread is due to credit risk?, *Review of Asset Pricing Studies* 2, 153–202.

Hull, J., and A. White, 1987, The pricing of options on assets with stochastic volatilities, *Journal of Finance* 281–300.

Jensen, Michael C, and William H Meckling, 1976, Theory of the firm: Managerial behavior, agency costs and ownership structure, *Journal of Financial Economics* 3, 305–360.

Jones, E Philip, Scott P Mason, and Eric Rosenfeld, 1984, Contingent claims analysis of corporate capital structures: An empirical investigation, *Journal of Finance* 39, 611–625.

Kisgen, Darren J, 2006, Credit ratings and capital structure, *Journal of Finance* 61, 1035–1072.

Kisgen, Darren J., 2009, Do firms target credit ratings or leverage levels?, *Journal of Financial and Quantitative Analysis* 44, 1323–1344.

Korajczyk, Robert A, and Amnon Levy, 2003, Capital structure choice: macroeconomic conditions and financial constraints, *Journal of Financial Economics* 68, 75 – 109.

Korteweg, Arthur, 2010, The net benefits to leverage, *Journal of Finance* 65, 2137–2170.

Leland, Hayne E, 1994, Corporate debt value, bond covenants, and optimal capital structure, *Journal of Finance* 49, 1213–1252.

Leland, Hayne E, 1998, Agency costs, risk management, and capital structure, *Journal of Finance* 53, 1213–1243.

Leland, Hayne E, and Klaus Bjerre Toft, 1996, Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads, *Journal of Finance* 51, 987–1019.

Lotfaliei, Babak, 2015, Zero leverage and the value in waiting to issue debt, Working paper, Finance Department, San Diego State University.

Luciano, Elisa, and Giovanna Nicodano, 2014, Guarantees, leverage, and taxes, *Review of Financial Studies* 27, 2736–2772.

McQuade, Timothy J., 2013, *Essays in Financial and Housing Economics*, PhD thesis, Harvard University.

Merton, Robert C., 1974, On the pricing of corporate debt: The risk structure of interest rates, *Journal of Finance* 29, 449–470.

Modigliani, Franco, and Merton H Miller, 1958, The cost of capital, corporation finance and the theory of investment, *American Economic Review* 48, 261–297.

Murfin, Justin, and Ken Njoroge, 2015, The implicit costs of trade credit borrowing by large firms, *Review of Financial Studies* 28, 112–145.

Myers, Stewart C., 1993, Still searching for optimal capital structure, *Journal of Applied Corporate Finance* 6, 4–14.

Nickell, Stephen, 1981, Biases in dynamic models with fixed effects, *Econometrica* 1417–1426.

Öztekin, Özde, 2015, Capital structure decisions around the world: which factors are reliably important?, *Journal of Financial and Quantitative Analysis* 50, 301–323.

Petersen, Mitchell A., 2009, Estimating standard errors in finance panel data sets: Comparing approaches, *Review of Financial Studies* 22, 435–480.

Petersen, Mitchell A., and Raghuram G. Rajan, 1997, Trade credit: Theories and evidence, *Review of Financial Studies* 10, 661–691.

Rajan, Raghuram G, and Luigi Zingales, 1995, What do we know about capital structure? some evidence from international data, *Journal of Finance* 50, 1421–1460.

Romano, M., and N. Touzi, 1997, Contingent claims and market completeness in a stochastic volatility model, *Mathematical Finance* 7, 399–412.

Sabanis, S., 2003, Stochastic volatility and the mean reverting process, *Journal of Futures Markets* 23, 33–47.

Schaefer, Stephen M., and Ilya A. Strebulaev, 2008, Structural models of credit risk are useful: Evidence from hedge ratios on corporate bonds, *Journal of Financial Economics* 90, 1 – 19.

Schwert, Michael, and Ilya A Strebulaev, 2014, Capital structure and systematic risk, *Stanford University Working Paper 178* .

Stein, Elias M., and Jeremy C. Stein, 1991, Stock price distributions with stochastic volatility: An analytic approach, *Review of Financial Studies* 4, 727–752.

Strebulaev, Ilya A, 2007, Do tests of capital structure theory mean what they say?, *Journal of Finance* 62, 1747–1787.

Strebulaev, Ilya A, and Toni M Whited, 2011, Dynamic models and structural estimation in corporate finance, *Foundations and Trends in Finance* 6.

Strebulaev, Ilya A., and Toni M. Whited, 2013, Dynamic corporate finance is useful: A comment on welch (2013), *Critical Finance Review* 2, 173–191.

Sundaresan, Suresh, 2013, A review of mertons model of the firms capital structure with its wide applications, *Annual Review of Financial Economics* 5, 21–41.

Tahani, Nabil, 2005, Exotic options pricing under stochastic volatility, Working paper 05-01, School of Administrative Studies, York University.

Titman, Sheridan, and Roberto Wessels, 1988, The determinants of capital structure choice, *Journal of finance* 43, 1–19.

van der Ploeg, Antoine Petrus Cornelius, 2006, *Stochastic Volatility and the Pricing of Financial Derivatives*, volume 366 (Rozenberg Publishers).

Welch, Ivo, 2004, Capital structure and stock returns, *Journal of Political Economy* 112, 106–132.

Welch, Ivo, 2013, A critique of recent quantitative and deep-structure modeling in capital structure research and beyond, *Critical Finance Review* 2, 131–172.

Wu, G., 2001, The determinants of asymmetric volatility, *Review of Financial Studies* 14, 837–859.

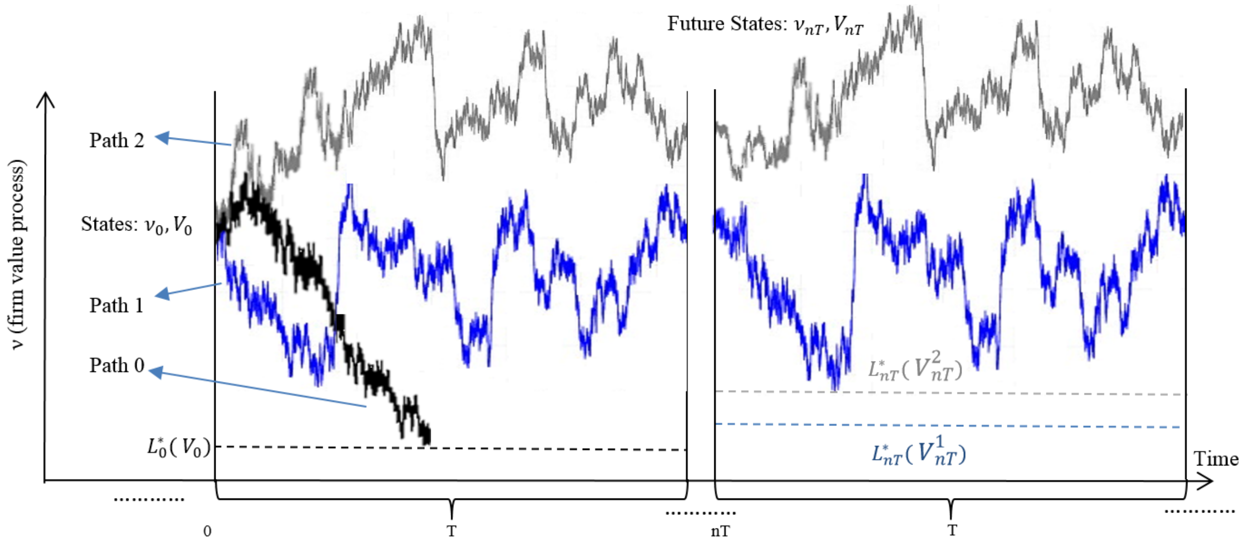
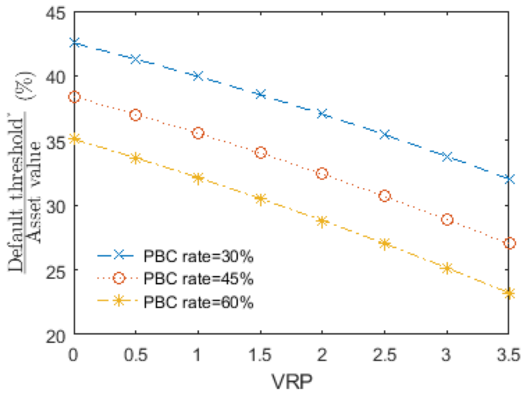
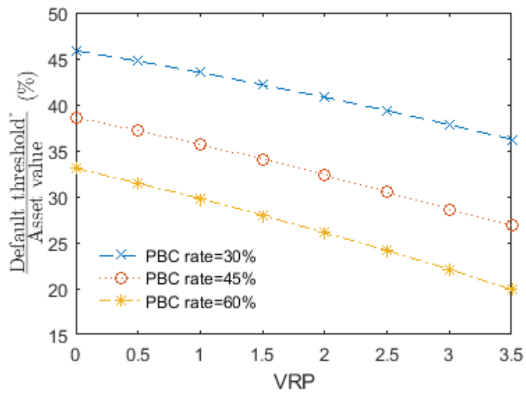


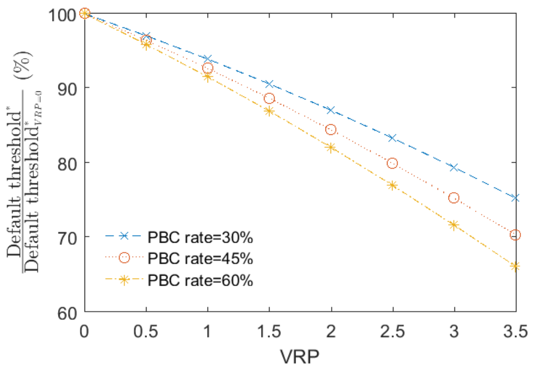
Figure 1. The setup of the model based on the state variables: all the paths start at the same state with the same default boundary. Path 0 ends in early default. Path 1 changes the boundary to L_1. Path 2 changes the boundary to L_2. Only if by coincidence $V_{nT} = V_0$ in a path, then optimal boundary for that path is $L^*_{nT} = L^*_0$, because variance is the only state variable that determines the optimal boundary.



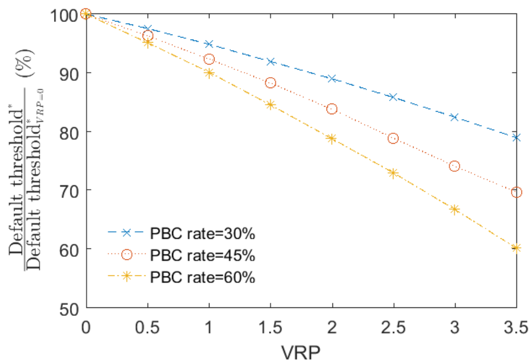
(a) Without debt roll over



(b) With debt roll over

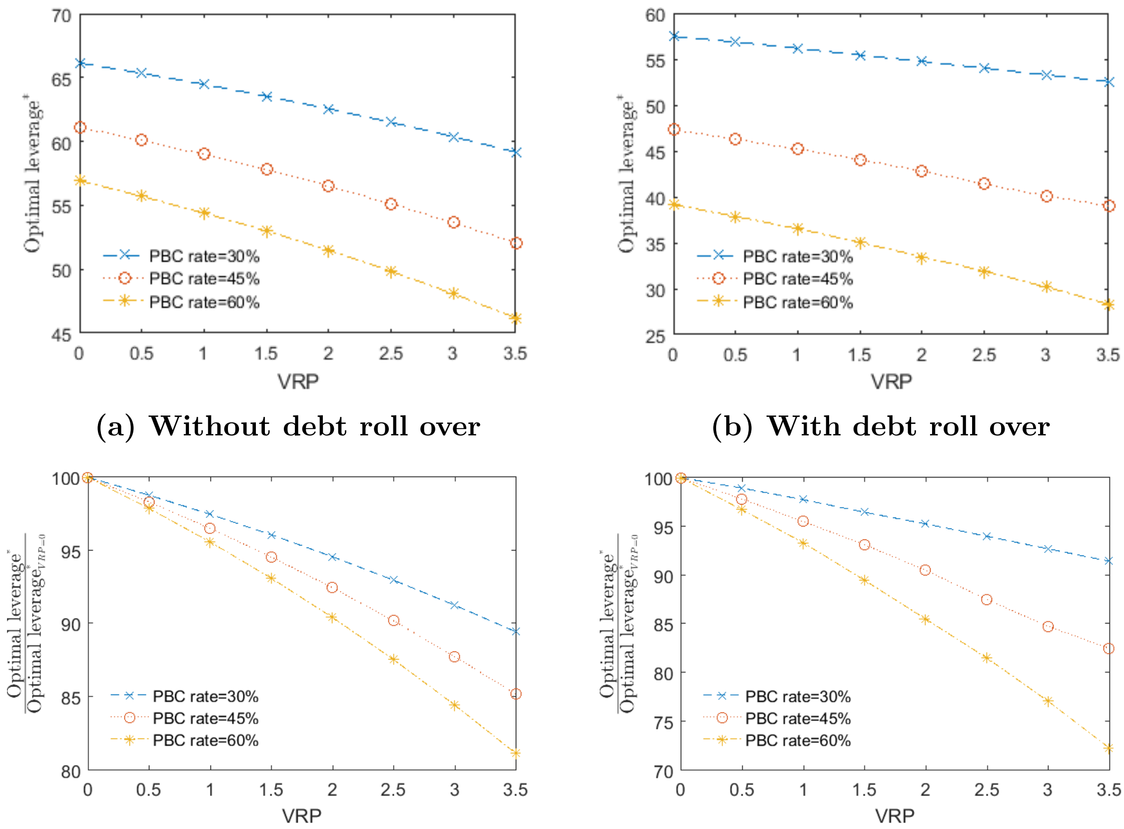


(c) Without debt roll over



(d) With debt roll over

Figure 2. Reduction in the optimal default boundary due to asset VRP: X-axis shows the Variance Risk Premium (VRP). In the first row, y-axis shows the ratio of optimal default boundary, L^*, to unlevered asset value, ν. In the second row, y-axis shows the relative optimal default boundary with and without VRP. Optimal boundary maximizes the equity value. The boundary is the unlevered asset level at which the firm files for bankruptcy (see Equation 12). The firm has optimal leverage. Debt coupon rate is set to make the debt's face value match with the market value, $c : d = p$. VRP is $|\lambda - \kappa|$. Proportional bankruptcy cost (PBC) rate, ρ, is the proportion of assets lost at default. Initial and mean variances are the same, $\theta = V_0$, and set to 4%, 0.2 squared. Initial asset value is \$100 and it is scalable. Historical variance mean-reversion speed, κ, is 4, risk-free rate, r, is 5%, asset payout rate, δ, is 3% and tax rate is 25%. Debt rollover rate, m, is 10%.



(a) Without debt roll over

(b) With debt roll over

(c) Without debt roll over

(d) With debt roll over

Figure 3. Reduction in the optimal leverage ratio due to asset VRP: X-axis shows the Variance Risk Premium (VRP). In the first row, y-axis shows the optimal market leverage, $D/(D + Eq)$. In the second row, y-axis shows the relative optimal market leverage with and without VRP. Optimal leverage maximizes the total levered firm value by choosing optimum debt, $P : \partial(Eq + D)/\partial P = 0$. Debt coupon rate is set to make the debt's face value match with the market value, $c : d = p$. The firms follow optimal default policy. VRP is $|\lambda - \kappa|$. Proportional bankruptcy cost (PBC) rate, ρ, is the proportion of assets lost at default. Initial and mean variances are the same, $\theta = V_0$, and set to 4%, 0.2 squared. Initial asset value is \$100 and it is scalable. Historical variance mean-reversion speed, κ, is 4, risk-free rate, r, is 5%, asset payout rate, δ, is 3% and tax rate is 25%. Debt rollover rate, m, is 10%.

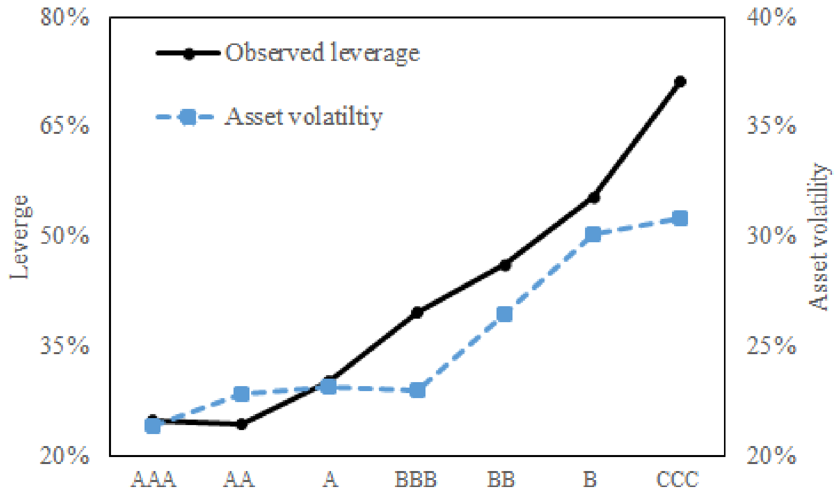


Figure 4. Leverage and calibrated historical asset volatility across risk groups (1997-2015): the figure shows the trends in observed leverage and asset volatility across the ratings as the risk group. CCC also includes ratings below CCC. The figure is based on Table 3. X-axis shows the ratings. Y-axis shows observed leverage and calibrated asset volatility. Intuitively, low historical asset volatility implies higher leverage, but the trend seems puzzling as it is reverse; firms with low business risk have also lower leverage.

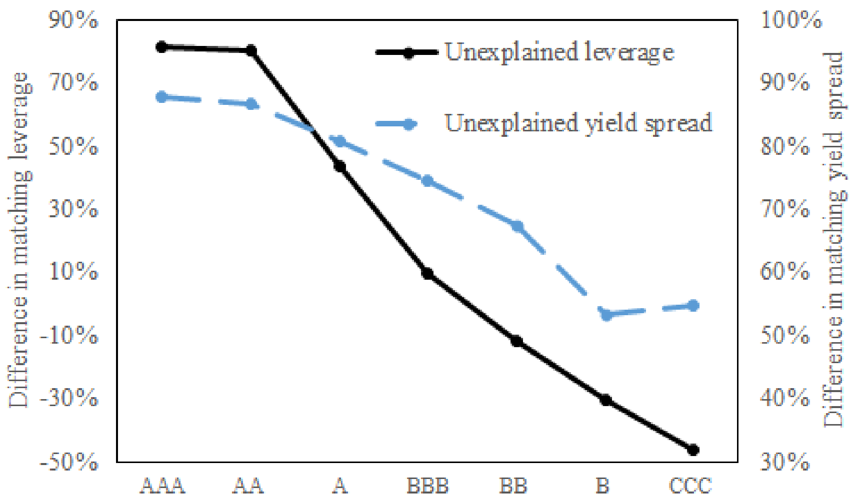
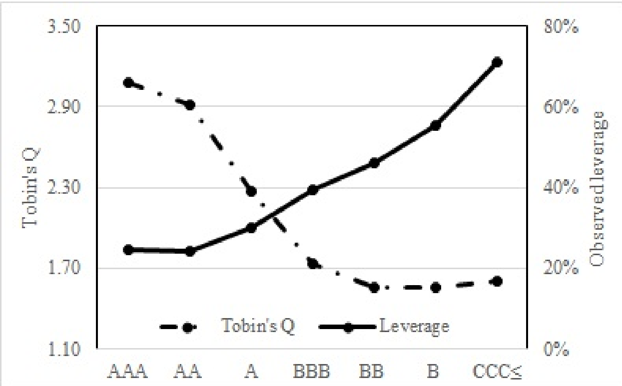
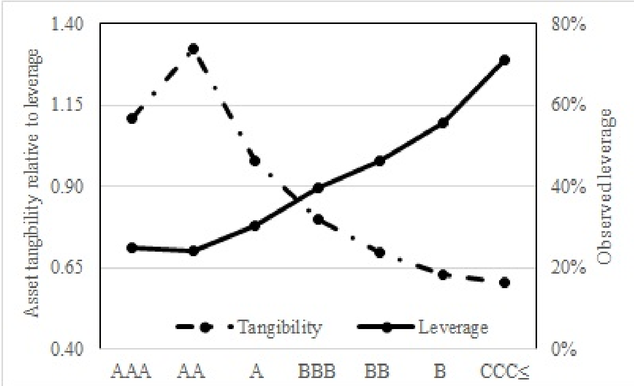


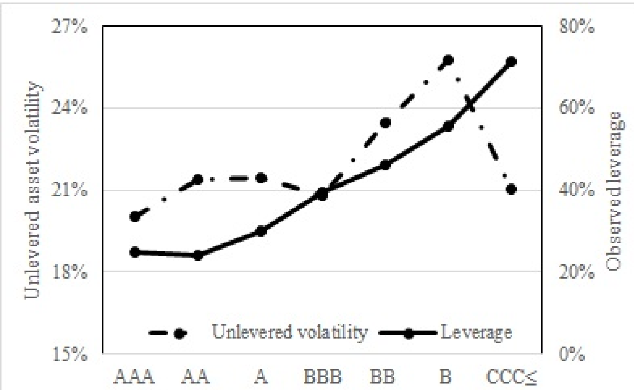
Figure 5. Model mismatch for credit spread and leverage without VRP (1997-2015): the figure shows the model error in predicting observed leverage and observed credit spreads across the risk groups after calibrating the model without asset VRP to representative firms. The figure is based on Table 2. X-axis shows the ratings. Y-axises show the relative difference in matching the observed leverage and observed yield spread in data. The unexplained portion is $\frac{emprical-\textbf{model-implied}}{empirical}$. CCC also includes ratings below CCC.



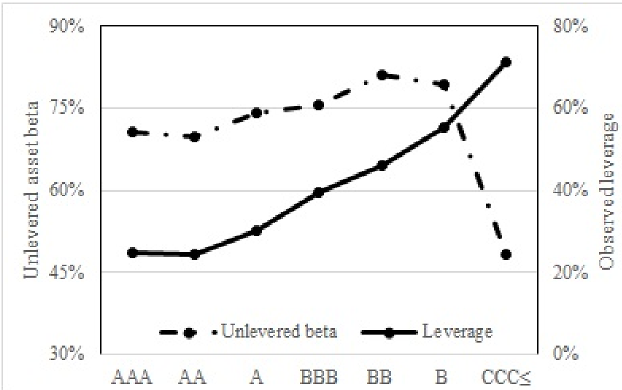
(a) **Trends in observed leverage and Tobin's Q across risk groups.**



(b) **Trends in observed leverage and tangibility across risk groups.**

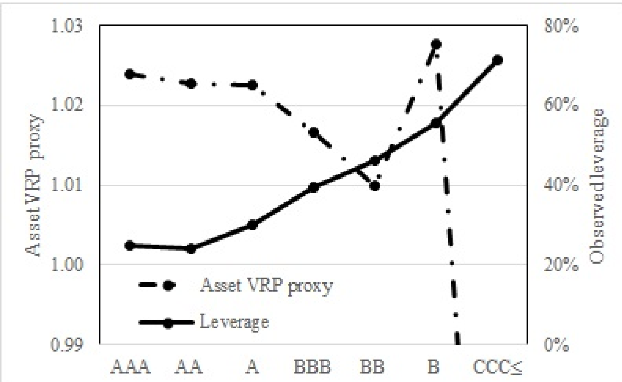


(c) **Trends in observed leverage and unlevered asset volatility across risk groups.**

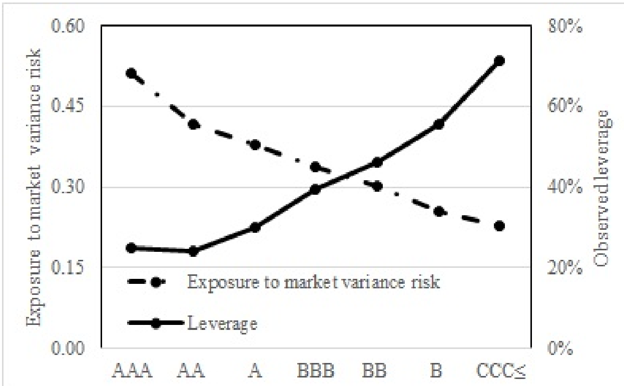


(d) **Trends in observed leverage and unlevered asset beta across risk groups.**

Figure 6. The counter-intuitive trends of classical leverage determinants and leverage (1997-2015): This figure shows the trends in observed leverage and the leverage factors. Ratings are used as they categorize the observations based on historical risk. CCC also includes ratings below CCC. Variables are defined in Appendix A and Table 4 shows the numbers. Figure 6c is comparable to Figure 4. X-axis shows the ratings. Right Y-axis shows observed leverage and left Y-axis shows the factor. Intuitively, high Tobin's Q, relatively high asset tangibility, low historical asset volatility and asset beta in the figures imply expected higher leverage for IG firms than the others. But, their reverse leverage trend implies that IG firms choose conservative leverage, which highlights them as the contributing firms to underleverage puzzle.



(a) Trends in observed leverage and asset VRP proxy across risk groups.



(b) Trends in observed leverage and exposure to variance risk across risk groups.

Figure 7. The trends of asset VRP proxy and exposure to market variance risk across ratings (1997-2015): this figure shows the trends in observed leverage and asset VRP. Ratings are used as they categorize the observations based on historical risk. CCC also includes ratings below CCC. Variables are defined in Appendix A. X-axis shows the ratings. Right Y-axis shows observed leverage and left Y-axis shows the factor. Declining asset VRP proxy across ratings is the only factor (compared to Figure 6) to imply low leverage for IG firms. This anecdotally supports the proposition that asset VRP reduces leverage for IG firms (H2).

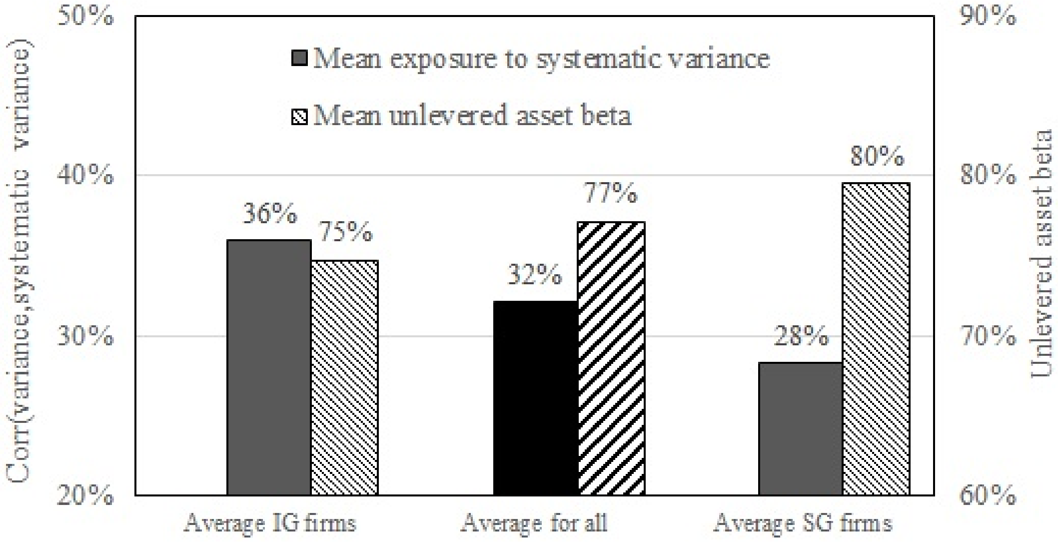


Figure 8. Exposure of variance to market variance compared to asset beta (1997-2015): Data is between 1997 and 2015 (see Table 4 in Appendix G for other sample statistics). The three columns at the top represent the average correlation and unlevered beta for all Investment-grade (IG) firm-years, all rated firm-years and all speculative-grade (SG) firm-years. Exposure of variance to market variance is 90-day average correlation between the squared VIX index and squared volatility of each firm-year. CBOE reports and calculates VIX from 30-day option-implied S&P-500 volatility. For each firm-year, the correlation is for the past 90 days from data date. The unlevered-asset beta is the unlevered beta of equity for each firm-year. The figure shows relatively higher exposure of IG firms to market-variance shocks and relatively lower exposure to market-return shocks than SG firms

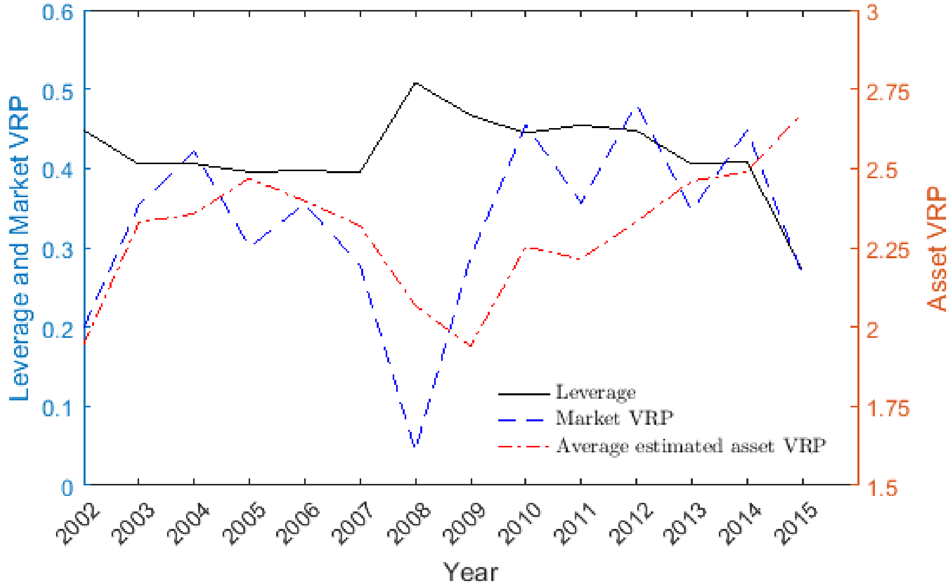


Figure 9. Time series of VRP and leverage: Leverage and asset VRP are respectively the asset-value weighted averages of observed leverage and calibrated asset VRP for all the firm-years in each year. Market VRP is the annual daily average for the difference between VIX and 30-day S&P-500 historical volatility divided by historical S&P-500 volatility. This paper's calibrated asset VRP index does not include any information content from the options' market, while VIX index is based solely on option prices. The figure shows positive correlation of the asset VRP index with market VRP and negative correlation of both with leverage in time series.

Table 1- The model's features compared to the earlier structural models of the firm: Xs determine the model features. The last row is about the model costs. In the model costs, numerical integration yields accurate results for asset prices, but makes finding optimal leverage difficult due to optimization complexity. Fast mean-reversion limits the analysis by restricting the range for variance mean-reversion speed. Zero correlation implies no asset variance asymmetry, while the model still replicates equity variance asymmetry as a stylized fact in the equity markets (see Appendix B).

Model	Leland (1994)	Leland and Toft (1996)	Hsu et al. (2010)	Ericsson et al. (2011)	McQuade (2013)	**This paper**
Endogenous leverage	X	X	-	-	-	X
Endogenous default	X	X	-	X	X	X
Stochastic variance	-	-	-	X	X	X
Time-varying boundary	-	-	X	X	X	X
Term-structure of credit spread	-	X	X	X	X	-
Model cost	No stochastic variance	No stochastic variance	No stochastic variance	Numerical integration	Zero correlation & Fast mean-reversion	Zero correlation

Table 2-Sample statistics and calibration results for representative firms across the ratings in the model without VRP (1997-2015): Table 12 in Appendix A has the details of the variable calculations. The first 2 columns on the left report the sample averages used as target values in calibration for the representative firm in each rating. Volatility is square-root of variance. Equity volatility is the standard deviation of stock returns for 365 days. Equity market cap is common shares times stock price. The last 2 columns on right report the outcomes of 2-by-2 calibration in the model without VRP to the sample averages in the first 2 columns (details are in Appendix F). Size, ν_0, is the unlevered value of the firm's assets. Mean volatility is the square-root of mean asset variance, $\sqrt{\theta}$. The two columns in the middle report yield spread and optimal leverage implied by the calibrated parameters in the last 2 columns. These middle columns are comparable with empirical leverage and yield spread in 2 columns on their left. Empirical yield spreads are the Bank-of-America Merrill-Lynch (BOA-ML) US Corporate Option-Adjusted spreads for an average rated firm. Empirical leverage is book liabilities divided by book liabilities plus equity market cap.

			1997-2015 Data statistics				Model-implied		Calibrated values in model without VRP	
Rating	Obs	statistic	Equity value	Equity volatility	Yield spread	Leverage	Optimal leverage	Yield spread	Size	Mean volatility
AAA	113	Mean	183,896	26.5%	0.8%	24.8%	45.0%	0.1%	226,943	21.3%
AA	369	Mean	82,053	28.2%	1.1%	24.2%	43.6%	0.1%	100,954	22.8%
A	1,975	Mean	26,614	30.9%	1.4%	30.2%	43.3%	0.3%	35,142	23.1%
BBB	3,681	Mean	9,308	35.1%	2.1%	39.6%	43.4%	0.5%	13,969	23.0%
IG firms	6,138	Mean	22,464	33.1%	1.6%	35.3%	43.3%	0.4%	31,725	23.1%
BB	3,741	Mean	3,160	45.2%	3.9%	46.1%	40.7%	1.3%	5,338	26.4%
B	2,321	Mean	1,551	60.3%	5.7%	55.5%	38.4%	2.7%	3,168	30.1%
CCC or below	142	Mean	1,270	87.3%	11.8%	71.2%	38.1%	5.3%	3,970	30.8%

Table 3-Sample statistics and calibration results for representative firms across the ratings in the model with VRP (1997-2015): This table is comparable to Table 2. Table 12 in Appendix A has the details of the variable calculations. The first 3 columns on the left report the sample averages used as target values in calibration for the representative firm in each rating. Volatility is square-root of variance. Equity volatility is the standard deviation of stock returns for 365 days. Equity market cap is common shares times stock price. Empirical yield spreads are the Bank-of-America Merrill-Lynch (BOA-ML) US Corporate Option-Adjusted spreads for an average rated firm. The last 3 columns on right report the outcomes of 3-by-3 calibration in the model with VRP to the sample averages in the first 3 columns. Instant volatility is set equal to mean variance (details are in Appendix F). Compared to Table 2, yield spread in the model with VRP matches its empirical value as part of calibration process. Size, ν_0, is the unlevered value of the firm's assets. Mean volatility is the square-root of the mean asset variance, $\sqrt{\theta}$. Asset VRP, $|\lambda - \kappa|$, is the price of variance risk. The two columns in the middle report the statistics for optimal leverage with and without VRP as implied by the calibrated parameters. Optimal leverage without VRP is borrowed from Table 2. These columns are comparable with actual leverage on their left. Empirical leverage is book liabilities divided by book liabilities plus equity market cap.

			1997-2015 Data statistics				Optimal leverage		Calibrated values in the model with VRP		
Rating	Obs	statistic	Equity value	Equity volatility	Yield spread	Leverage	With VRP	Without VRP	Size	Mean volatility	Asset VRP
AAA	113	Mean	183,896	26.5%	0.83%	24.8%	36.5%	45.0%	228,050	21.3%	2.96
AA	369	Mean	82,053	28.2%	1.05%	24.2%	35.5%	43.6%	101,354	22.9%	2.99
A	1,975	Mean	26,614	30.9%	1.40%	30.2%	35.7%	43.3%	35,251	23.4%	2.85
BBB	3,681	Mean	9,308	35.1%	2.10%	39.6%	35.7%	43.4%	13,947	23.7%	2.78
IG firms	6,138	Mean	22,464	33.1%	1.61%	35.3%	36.1%	43.3%	31,784	23.5%	2.66
BB	3,741	Mean	3,160	45.2%	3.85%	46.1%	34.2%	40.7%	5,222	28.3%	2.67
B	2,321	Mean	1,551	60.3%	5.74%	55.5%	33.9%	38.4%	3,037	33.1%	2.14
CCC & below	142	Mean	1,270	87.3%	11.84%	71.2%	33.1%	38.1%	3,520	37.7%	2.19

Table 4- More descriptive statistics for the sample of the rated firm-years (1997-2015): The sample is the same as in Table 3 from merged Compustat and Optionmetrics. Table 12 in Appendix A has the details of the variable calculations. Tobin's Q is market to book value of assets. Profitability is the operating income ratio. Cash is the ratio of the cash holdings. Tangibility is the ratio of the tangible assets and the proxy for bankruptcy costs. Log(Sales) is the natural log of the revenues. Asset payout is the total payout to debt and equity holders relative to size. Leverage is the total liabilities to the market cap of equity plus the liabilities. Unlevered volatility is the historical volatility of stock returns unlevered with leverage to proxy for the assets' historical volatility. VRP proxy represents the asset VRP and is the ratio of equity option-implied to historical volatilities.

Rating	statistic	Leverage	Equity value	Equity volatility	Unlevered volatility	VRP proxy	Tobin's Q	Profitability	Cash	Tangibility	Log(sales)	Asset payout
AAA	Mean	24.8%	183,896	26.5%	20.0%	1.02	3.08	19.8%	13.1%	27.5%	10.70	3.8%
113	Std	16.5%	127,046	10.1%	9.2%	0.19	1.86	8.3%	11.8%	21.2%	1.23	1.8%
AA	Mean	24.2%	82,053	28.2%	21.4%	1.02	2.92	20.1%	10.0%	32.0%	10.03	3.7%
369	Std	12.7%	85,452	10.5%	8.7%	0.20	1.72	6.6%	10.6%	16.7%	1.20	2.0%
A	Mean	30.2%	26,614	30.9%	21.5%	1.02	2.27	17.9%	9.7%	29.5%	9.09	3.8%
1975	Std	14.2%	35,312	12.4%	10.1%	0.17	1.23	7.1%	10.9%	19.2%	1.17	2.4%
BBB	Mean	39.6%	9,308	35.1%	20.8%	1.02	1.73	14.8%	8.6%	31.5%	8.49	3.9%
3681	Std	16.3%	14,043	14.5%	10.2%	0.19	1.01	7.1%	9.7%	23.3%	1.12	3.5%
IG firms	Mean	35.3%	22,464	33.1%	21.0%	1.02	2.00	16.2%	9.1%	30.8%	8.81	3.9%
6138	Std	16.3%	45,326	13.8%	10.1%	0.18	1.22	7.3%	10.2%	21.7%	1.24	3.1%
BB	Mean	46.1%	3,160	45.2%	23.5%	1.01	1.56	13.1%	9.5%	32.0%	7.56	3.5%
3741	Std	18.4%	5,436	20.4%	12.8%	0.21	0.94	8.9%	10.9%	25.1%	1.05	3.1%
B	Mean	55.5%	1,551	60.3%	25.8%	1.03	1.55	8.9%	12.9%	34.9%	6.88	3.8%
2321	Std	21.8%	3,094	27.9%	18.1%	0.31	1.20	11.0%	15.7%	25.5%	1.30	3.5%
CCC & below	Mean	71.2%	1,270	87.3%	21.1%	0.92	1.61	0.7%	12.5%	43.1%	6.74	4.6%
142	Std	25.2%	1,986	38.1%	19.1%	0.23	1.13	15.7%	10.2%	25.1%	1.81	2.6%
SG firms	Mean	50.2%	2,515	51.8%	24.3%	1.01	1.56	11.3%	10.8%	33.4%	7.29	3.6%
6204	Std	20.6%	4,703	25.7%	15.2%	0.26	1.05	10.3%	13.0%	25.3%	1.22	3.3%
All	Mean	42.8%	12,436	42.5%	22.7%	1.02	1.78	13.7%	10.0%	32.1%	8.05	3.7%
12342	Std	20.1%	33,649	22.7%	13.0%	0.22	1.16	9.2%	11.7%	23.6%	1.45	3.2%

Table 5- Regression results (1997-2015): It estimates $Lev_{i,t} = (1 - \Psi)Lev_{i,t-1} + a_0 + \sum_k \Psi a_k X_{k,i,t}$ where X has standardized independent variables and control dummies. Table 12 in Appendix A has the details of the variable calculations. Descriptive statistics are in Table 4. IG-firms sample only has all the firm-quarters rated as investment grade by S&P. SG-firms sample only has all the firm-quarters rated as none investment-grade or speculative grade by S&P. Dummies for years, and firms control for time and firm fixed effects. Average industry leverage controls fixed industry effect. Standard errors are corrected for clustered time and firm errors in parentheses. The p-values test the null hypothesis that the coefficient is zero and they are:$***p < 0.01, **p < 0.05, *p < 0.1$. Coefficients for the standardized variables show the relative importance of each variable in determining target leverage. Asset VRP has significantly negative effect on leverage (H1), especially for IG firms (H2).

Parameter	Estimated Coefficients for each statistical regression					
Model	(1)	(2)	(3)	(4)	(5)	(6)
Sample	All	IG firms	SG firms	All	IG firms	SG firms
Dependent variable	Leverage	Leverage	Leverage	Leverage	Leverage	Leverage
Lag Leverage	0.438***	0.476***	0.346***	0.380***	0.380***	0.317***
	(0.024)	(0.027)	(0.029)	(0.022)	(0.025)	(0.023)
Asset volatility	-0.0937***	-0.0826***	-0.102***	-0.0790***	-0.0645***	-0.0861***
	(0.006)	(0.009)	(0.006)	(0.005)	(0.007)	(0.005)
VRP proxy	-0.0358***	-0.0370***	-0.0336***	-0.0310***	-0.0306***	-0.0289***
	(0.005)	(0.006)	(0.006)	(0.005)	(0.005)	(0.006)
Tobin Q	-	-	-	-0.0284***	-0.0239***	-0.0363***
	-	-	-	(0.007)	(0.006)	(0.012)
Profitability	-	-	-	-0.0263***	-0.0304***	-0.0282***
	-	-	-	(0.006)	(0.005)	(0.005)
Cash	-	-	-	-0.00821***	-0.0118***	-0.00648
	-	-	-	(0.003)	(0.004)	(0.005)
Tangiblity	-	-	-	0.0105*	0.00301	0.0175**
	-	-	-	(0.006)	(0.006)	(0.009)
Log sales	-	-	-	-0.00932	0.0137	-0.00244
	-	-	-	(0.007)	(0.010)	(0.008)
Intercept	0.249***	0.248***	0.538***	0.221***	0.302***	0.489***
	(0.012)	(0.013)	(0.022)	(0.014)	(0.019)	(0.033)
Industry average leverage	yes	yes	yes	yes	yes	yes
Time and firm fixed effect	yes	yes	yes	yes	yes	yes
Time and firm clustered errors	yes	yes	yes	yes	yes	yes
R-squared	60%	57%	61%	66%	64%	67%
BIC	-30304.4	-18001.8	-14116.2	-32254.3	-19111.5	-15105.9
Obs	12342	6138	6204	12342	6138	6204

Table 6-Calibration robustness-check results to representative firms in the ratings (1997-2015): This table is comparable to Table 3. Compared to Table 3, the calibrations do not include yield spread for robustness check. This table only includes calibrations of asset volatility and value to equity volatility and value. Table 4 in Appendix G provides the statistics on firm-year data. Table 12 has the details of the variable calculations. The first five columns on the left report the sample averages used as characteristics of the representative firm in each rating for calibration. Volatility is square-root of variance. Asset VRP exposure is estimated from 90-days correlation between squared VIX index and the squared option-implied 30-day equity volatility. I choose 30-day volatility match with VIX horizon which is also 30 days. The correlation is insensitive to leverage and does not require de-levering. Asset VRP, $|\lambda - \kappa|$, is the price of variance risk, which is calculated by multiplying exposure, α, times market VRP. Market VRP is set to average 4 reported by Ait-Sahalia and Kimmel (2007). Equity volatility is the standard deviation of stock returns for 365 days. Equity market cap is common shares times stock price. The last two columns report the parameters in a 2-by-2 calibration with VRP as in Appendix F to the sample averages for equity market cap and volatility without leverage. Size, ν_0, is the unlevered value of the firm's assets. Mean volatility is the square-root of the mean asset variance, $\sqrt{\theta}$. Instant volatility is equal to mean variance. The two columns in the middle report the statistics for the optimal leverages with and without VRP as implied by the calibrated parameters. These columns are comparable with actual leverage on their left. Optimal leverage for the model without VRP is also implied by the parameters in a 2-by-2 calibration as in Appendix F.

			1997-2015 Data statistics					Optimal leverage		Calibrated values in the model with VRP	
Rating	Obs	statistic	Asset VRP exposure	Asset VRP	Equity value	Equity volatility	Leverage	Without VRP	With VRP	Size	Mean volatility
AAA	113	Mean	51.02%	2.04	183,896	26.5%	24.8%	39.5%	45.0%	227,885	21.3%
AA	369	Mean	41.84%	1.67	82,053	28.2%	24.2%	39.4%	43.6%	101,296	22.8%
A	1975	Mean	37.95%	1.52	26,614	30.9%	30.2%	39.6%	43.3%	35,271	23.1%
BBB	3681	Mean	33.85%	1.35	9,308	35.1%	39.6%	40.0%	43.4%	14,013	23.1%
IG firms	6138	Mean	35.96%	1.44	22,464	33.1%	35.3%	39.7%	43.3%	31,836	23.2%
BB	3741	Mean	30.24%	1.21	3,160	45.2%	46.1%	37.8%	40.7%	5,328	26.9%
B	2321	Mean	25.44%	1.02	1,551	60.3%	55.5%	36.2%	38.4%	3,132	31.1%
CCC or below	142	Mean	22.77%	0.91	1,270	87.3%	71.2%	35.7%	38.1%	3,855	32.9%

Table 7-Model-implied yield spread based on calibration results in Table 6 (1997-2015): This table shows model-implied yield spread in comparison with observed yield spread in data based on calibrated parameters in Table 6. Compared to Table 3 and for robustness check, the calibrations do not use yield spread. This table only includes calibrations of asset volatility and value to equity volatility and value. Appendix F has more calibration details. Table 12 in Appendix A has the details of the variable calculations. The first column on the left reports the sample averages of the representative firm in each rating for comparison. The two middle columns report the model-implied yield spread with and without asset VRP in a 2-by-2 calibration in Table 6. The second column from left replicates the classical credit premium puzzle where model without VRP produces yield premiums much lower than their observed counter parts. The last column on the right measures the improvement in explaining yield spread using asset VRP. The highest improvement belongs to IG firms caused by their high exposure to market VRP (see also Figure 8).

Rating	Observed yield spread	Yield spread without VRP	Yield spread with VRP	VRP contribution to explaining yield
AAA	0.8%	0.1%	0.4%	35%
AA	1.1%	0.1%	0.4%	24%
A	1.4%	0.3%	0.6%	23%
BBB	2.1%	0.5%	1.0%	20%
IG firms	1.6%	0.4%	0.8%	24%
BB	3.9%	1.3%	1.9%	17%
B	5.7%	2.7%	3.7%	17%
CCC or below	11.8%	5.3%	7.0%	14%

Table 8-Sample statistics and calibration results to all the rated firm-years (2002-15): This table is comparable to Table 3. The first four columns on the left show the sample statistics for each rating. Volatility is square-root of variance. Yield spread is the difference between corporate bond yield and maturity-matching risk-free rate. Equity volatility is the standard deviation of stock returns for 365 days. Equity market cap is common shares times stock price. Leverage is the total book liabilities to the market cap of equity plus the liabilities. Table 12 in Appendix A has the details of the variable calculations. The optimal leverage for the model without VRP is implied by the parameters in a 2-by-2 calibration as in Appendix F. The last five columns report the statistics for the calibrated parameters from 4-by-4 calibration with VRP as in Appendix F and the error. Asset VRP, $|\lambda - \kappa|$, is the price of variance risk. Size, ν_0, is the unlevered value of the firm's assets. Mean volatility is the square-root of the mean asset variance, $\sqrt{\theta}$. Instant volatility is the square-root of the instant asset variance, $\sqrt{V_0}$. MAPE is the mean average percentage error between model-implied and observed values in the calibration. The two columns in the middle report the statistics for the optimal leverages with and without VRP as implied by the calibrated parameters. These columns are comparable with actual leverage on their left.

		2002-2015 Data statistics				Optimal leverage		Calibrated values in the model with VRP				
Rating	statistic	Equity value	Equity volatility	Yield spread	Leverage	With VRP	Without VRP	Asset VRP	Size	Mean volatility ($\sqrt{\theta}$)	Instant volatility ($\sqrt{V_0}$)	MAPE
AAA	Mean	232,982	21.8%	0.92%	27.2%	38.8%	56.0%	2.63	331,200	26.9%	16.2%	7%
32	Std	78,960	10.8%	0.50%	19.3%	7.3%	11.6%	0.49	236,242	7.1%	8.4%	3%
AA	Mean	127,429	21.0%	1.01%	30.9%	39.4%	56.6%	2.57	191,131	25.8%	15.1%	6%
83	Std	85,303	8.4%	0.64%	14.6%	6.6%	9.7%	0.44	172,741	8.1%	6.4%	2%
A	Mean	37,169	26.0%	1.51%	33.0%	37.7%	52.1%	2.55	48,209	27.4%	18.2%	6%
365	Std	35,444	10.8%	1.02%	12.9%	4.2%	8.2%	0.46	44,943	7.7%	7.6%	2%
BBB	Mean	17,133	31.4%	2.46%	44.9%	38.6%	53.0%	2.30	28,806	27.6%	18.5%	5%
331	Std	23,291	13.3%	1.59%	14.7%	5.9%	9.3%	0.67	41,950	7.9%	7.9%	2%
IG firms	Mean	45,955	27.5%	1.82%	37.4%	38.3%	53.1%	2.46	66,083	27.3%	18.0%	5%
811	Std	64,785	12.2%	1.37%	15.5%	5.4%	9.1%	0.57	108,960	7.8%	7.7%	2%
BB	Mean	5,736	39.3%	4.19%	53.7%	37.4%	53.3%	1.96	14,562	31.7%	20.6%	5%
90	Std	7,361	15.3%	2.07%	17.6%	4.5%	11.8%	0.79	33,129	8.5%	10.4%	2%
B	Mean	4,943	63.0%	9.50%	71.9%	38.5%	57.0%	1.17	23,331	34.0%	22.6%	4%
51	Std	10,191	28.2%	11.51%	14.4%	6.3%	14.7%	0.68	57,978	14.2%	11.4%	3%
CCC	Mean	3,352	68.4%	14.98%	77.9%	35.6%	58.6%	1.34	27,726	40.7%	21.3%	3%
8	Std	1,793	21.5%	10.65%	11.2%	2.1%	11.8%	0.44	52,060	12.8%	8.8%	2%
All	Mean	39,651	30.8%	2.56%	41.1%	38.2%	53.4%	2.33	58,662	28.2%	18.5%	5%
960	Std	61,417	16.6%	3.78%	18.2%	5.3%	9.8%	0.68	103,092	8.7%	8.3%	2%

Table 9- Differences between model-implied optimal and actual leverages (2002-15): Leverage statistics are in Table 8. The differences are estimated with $Leverage_{data} - Leverage_{model} = \alpha + \epsilon_i + \epsilon_t + \epsilon$ where α is the difference, and ϵ_i and ϵ_t control for clustered firm and time errors respectively. Actual leverage is from data for each firm-year. Model-implied optimal leverage is the result of plugging the calibrated parameters into each model. The p-values test the null hypothesis that the coefficient is zero and they are:$***p < 0.01, **p < 0.05, *p < 0.1$. Negative α implies underleverage and positive implies overleverage.

Model	With VRP	Without VRP	With VRP	Without VRP	With VRP	Without VRP
Sample	IG firms	IG firms	All	All	SG firms	SG firms
$Leverage_{data} - Leverage_{model}$	-0.0091	-0.16***	0.029	-0.12***	0.24***	0.064**
p-value	(0.586)	(0.000)	(0.121)	(0.000)	(0.000)	(0.009)
Time and firm clustered errors	yes	yes	yes	yes	yes	yes
Obs	811	811	960	960	149	149

Table 10- Regression results (2002-15): The table is comparable to Table 5 with the similar independent variables, but the sample is smaller than the sample in Table 5. It estimates $Lev_{i,t} = (1-\Psi)Lev_{i,t-1} + a_0 + \sum_k \Psi a_k X_{k,i,t}$ where X has standardized independent variables and control dummies. Independent-variable statistics are described in Table 13. Table 12 in Appendix A has the details of the variable calculations. IG-firms sample only has all the firm-quarters rated as investment grade by S&P. SG-firms sample only has all the firm-quarters rated as none investment-grade or speculative grade by S&P. Dummies for years, and firms control for time and firm fixed effects. Average industry leverage controls fixed industry effect. Coefficients of these control variables are omitted for brevity. Standard errors are corrected for clustered time and firm errors in parentheses. The p-values test the null hypothesis that the coefficient is zero and they are:$***p < 0.01, **p < 0.05, *p < 0.1$. Coefficients for the standardized variables show the relative importance of each variable in determining the target leverage.

Parameter	Estimated Coefficients for each statistical regression					
Model	(1)	(2)	(3)	(4)	(5)	(6)
Sample	All	IG firms	SG firms	All	IG firms	SG firms
Dependent variable	Leverage	Leverage	Leverage	Leverage	Leverage	Leverage
Lag Leverage	0.468***	0.425***	0.436***	0.352***	0.253***	0.395***
	(0.039)	(0.051)	(0.055)	(0.040)	(0.046)	(0.060)
Asset volatility	-0.0761***	-0.0650***	-0.0861***	-0.0585***	-0.0484***	-0.0620***
	(0.009)	(0.008)	(0.024)	(0.006)	(0.005)	(0.015)
VRP proxy	-0.0405***	-0.0375***	-0.0342***	-0.0321***	-0.0297***	-0.0181***
	(0.006)	(0.006)	(0.013)	(0.005)	(0.005)	(0.005)
Tobin Q	-	-	-	-0.0427***	-0.0522***	-0.108***
	-	-	-	(0.009)	(0.009)	(0.030)
Profitability	-	-	-	-0.0300***	-0.0216***	-0.0246**
	-	-	-	(0.005)	(0.004)	(0.010)
Cash	-	-	-	-0.0066	-0.00667	-0.0119
	-	-	-	(0.006)	(0.007)	(0.018)
Tangiblity	-	-	-	0.0340**	0.0173	0.0746*
	-	-	-	(0.017)	(0.021)	(0.040)
Log sales	-	-	-	0.0331	0.0108	-0.0118
	-	-	-	(0.025)	(0.016)	(0.041)
Intercept	0.192***	0.351***	0.446***	0.229***	0.490***	0.441***
	(0.027)	(0.042)	(0.051)	(0.032)	(0.053)	(0.116)
Industry average leverage	yes	yes	yes	yes	yes	yes
Time and firm fixed effect	yes	yes	yes	yes	yes	yes
Time and firm clustered errors	yes	yes	yes	yes	yes	yes
R-squared	64%	61%	71%	74%	74%	84%
BIC	-2997.9	-2783.3	-366.3	-3294.5	-3106.9	-455.1
Obs	959	810	149	959	810	149

Appendices

A. Table of the Variables

Place Table 11 about here

Place Table 12 about here

B. Asymmetric Equity Variance Effect

Even without negative correlation between asset returns and asset variance (variance asymmetry), this simplifying assumption does not reduce the power of the model in qualitatively replicating asymmetric variance observed at the equity level. It is a stylized fact that equity returns and return variance have negative correlation.[19]

In Figure 10, model-implied equity volatility is asymmetric in this paper and has negative correlation with equity returns. Volatility has one-on-one relation with variance by square-root transformation. Equations 35 and 36 in Appendix F show the equity process which creates Figure 10. Without debt, the firm is all equity and there is no correlation between the returns and volatility by the model assumption. As the fraction of debt in the capital structure increases, a negative shock to equity return raises equity volatility due to financial distress costs. Hence, there exists a negative correlation between equity return and its volatility. The higher is the leverage ratio of the firm, the higher is the asymmetric volatility.

Insert Figure 10 about here.

[19]See for example Ait-Sahalia, Fan, and Li (2013), Bekaert and Wu (2000), Figlewski and Wang (2001), and Wu (2001).

C. Change of Variable

Let's define $\zeta = ln(\nu/L)$. By Ito's lemma, it follows:

$$d\zeta = (r - \delta - \frac{1}{2}V)dt + \sqrt{V}dW \tag{15}$$

Integration over the equation above results in the simple solution of the differential equation:

$$\zeta_T - \zeta_0 = (r - \delta)T - \frac{1}{2}\int_0^T V dt + \int_0^T \sqrt{V}dW \tag{16}$$

Defining $\hat{V} = (\int_0^T V_s ds)/T$ transfers the above equation into:

$$\zeta_T - \zeta_0 = (r - \delta - \frac{1}{2}\hat{V})T + \int_0^T \sqrt{V}dW \tag{17}$$

The variable ζ_T follows a Brownian motion with mean $\zeta_0 + (r - \delta - \frac{1}{2}\hat{V})T$. The variance for ζ_T is $\int_0^T V dt$ which is also equal to $\hat{V}T$. Therefore, the differential of the process in equation 17 is:

$$d\zeta_T = (r - \delta - \frac{1}{2}\hat{V})dt + \sqrt{\hat{V}}dW \tag{18}$$

D. Derivation of Security Formulas

1. Debt formula

Since debt value will be conditioned on variance, first I analyze the debt value when the variance is constant in Equation 4:

$$\begin{aligned} d(\nu_0) = \quad & E^Q\Big(\tfrac{c+m\,p}{m+r} + (1 - I_{\tau<T})\Big[e^{-rT}(D(\nu_T) - \tfrac{c+m\,p}{m+r})\Big] + \\ & I_{\tau<T}\Big[e^{-r\tau}((1-\rho)mL - \tfrac{c+m\,p}{m+r})\Big]\Big) \end{aligned} \tag{19}$$

With constant variance, debt value satisfies the following PDE and Dirichlet conditions, which is similar to the Black-Scholes PDE for contingent claim pricing:

$$\begin{cases} \frac{1}{2}V\nu^2 d_{\nu\nu} + (r-\delta)\nu d_\nu - (r+m)d + (C+mP) = 0, \\ d(L) = (1-\rho)mL, \qquad d(\nu \to +\infty) = \frac{c+mp}{m+r} \end{cases} \tag{20}$$

With constant variance, the default boundary is also constant and the debt value has a closed-form solution:

$$\begin{aligned} d &= \tfrac{c+mp}{m+r} + e^{-H_b\zeta_0}\Big((1-\rho)mL - \tfrac{c+mp}{m+r}\Big) \\ h &= \tfrac{r-\delta-\frac{1}{2}\hat{V}}{\sqrt{\hat{V}}} \qquad H_b = \tfrac{\sqrt{h^2+2(r+m)}+h}{\sqrt{\hat{V}}} \end{aligned} \tag{21}$$

where future variance, $\hat{V}$, is equal to variance, V, because variance is constant.

I use conditioning on future variance. In Equation 4, new state variables ζ and $\hat{V}$ replace ν and V:

$$\begin{aligned} d(\zeta_0, \hat{V}_0) = \quad & E^Q\Big[E\Big(\tfrac{c+mp}{m+r} + (1-I_{\tau<T})\Big[e^{-rT}(d(\zeta_T, \hat{V}_T) - \tfrac{c+mp}{m+r})\Big] + \\ & \qquad I_{\tau<T}\Big[e^{-r\tau}((1-\rho)mL - \tfrac{c+mp}{m+r})\Big]|\hat{V}\Big)\Big] \end{aligned} \tag{22}$$

Note that $d(\zeta_0, \hat{V}_0)$ does not have the same form as $d(\nu_0, V_0)$, because it is not a linear transformation. Conditioning on $\hat{V}$ makes the expression within the parentheses similar to Equation (19), which has a solution in the form of Equation (21). Variance is constant from the conditioning and the default boundary is decided and fixed at the start of the period:

$$d = \frac{c+mp}{m+r} + \left((1-\rho)\, mL - \frac{c+mp}{m+r}\right) E(e^{-H_b\zeta_0}) \tag{23}$$

The expectation term is presented in Appendix E. A similar method generates formulas for tax benefits, bankruptcy costs, and equity.

2. Tax savings and bankruptcy cost formulas

Similar to debt, the derivation uses conditioning on variance and, first, I show the valuation when variance is constant. With constant variance, any claim, F, making continuous payments C satisfies the following PDE and the solution depends on Dirichlet conditions:

$$\frac{1}{2}V\nu^2F_{\nu\nu} + (r-\delta)\nu F_\nu - rF + C = 0, \tag{24}$$

With constant variance, the default boundary is also constant. Tax savings' formula is:

$$\begin{aligned} &TB(L) = 0, \quad TB(\nu \to +\infty) = tax.C/r \\ &TB(\zeta_0, \hat{V}_0) \quad = E\Big(tax.\tfrac{C}{r} + (1 - I_{\tau<T})[e^{-rT}(TB(\zeta_T, \hat{V}_T) - tax.\tfrac{C}{r})] + \\ &\qquad I_{\tau<T}[e^{-r\tau}(0 - tax.\tfrac{C}{r})]\Big) \\ &\qquad\qquad = tax \times \tfrac{C}{r} - tax \times \tfrac{C}{r}e^{-H.\zeta_0} \end{aligned} \tag{25}$$

For bankruptcy costs, the formula is:

$$\begin{aligned} &BC(L) = \rho L, \quad BC(\nu \to +\infty) = 0 \\ &BC(\zeta_0, \hat{V}_0) \quad = E\Big((1 - I_{\tau<T})[e^{-rT}BC(\zeta_T, \hat{V}_T)] + I_{\tau<T}[e^{-r\tau}\rho L]\Big) \\ &\qquad\qquad = \rho L e^{-H.\zeta_0} \end{aligned} \tag{26}$$

A similar method conditioning on future variance as in debt valuation (see Equation 22) generates formulas for tax benefits and bankruptcy costs:

$$TB(\zeta_0, \hat{V}_0) = tax \times \frac{C}{r} - tax \times \frac{C}{r}E\Big(e^{-H.\zeta_0}\Big) \tag{27}$$

$$BC(\zeta_0, \hat{V}_0) = \rho L E\Big(e^{-H.\zeta_0}\Big) \tag{28}$$

The expectation term is derived in Appendix E.

E. Discounted RN Default Probability and the Moment Generating Function

1. Taylor expansion

The following derivations apply to the expectation terms in equations 6, 8, and 9. Since the debt rollovers with rate m, the expectation term in Equation 6 has m as an extra parameter compared to the other two. Regardless of the form, Taylor expansion of any function within the expectation term up to the second degree is:

$$
\begin{aligned}
E\Big\{F(\zeta_0,\hat{V}_0)\Big\} \simeq\ & E\Big\{F\Big(\zeta_0,E[\hat{V}_0]\Big)+F'\Big(\zeta_0,E[\hat{V}_0]\Big).\Big(\hat{V}_0-E[\hat{V}_0]\Big)+ \\
& \tfrac{1}{2}F"\Big(\zeta_0,E[\hat{V}_0]\Big).\Big(\hat{V}_0-E[\hat{V}_0]\Big)^2\Big\} \\
& = F\Big(\zeta_0,E[\hat{V}_0]\Big)+\tfrac{1}{2}F"\Big(\zeta_0,E[\hat{V}_0]\Big).E\left[\Big(\hat{V}_0-E[\hat{V}_0]\Big)^2\right]
\end{aligned}
\tag{29}
$$

where $F(\zeta_0,\hat{V}_0)=exp(-\zeta_0.H_b)$ is in Equation 6, and $F(\zeta_0,\hat{V}_0)=exp(-\zeta_0.H)$ is in equation 8 and 9. I only show the derivation for $F(\zeta_0,\hat{V}_0)=exp(-\zeta_0.H)$. The other derivations are similar by replacing H with H_b. Since the function is exponential, deriving its higher order derivatives is straightforward:

$$
\begin{aligned}
& E\Big\{exp(-\zeta_0.H)\Big\} \simeq exp\Big(-\zeta_0.\hat{H}\Big)\Big[1-\tfrac{1}{2}(A\zeta_0-B\zeta_0^2)\Big] \\
& A=\hat{H}''.E\Big[(\hat{V}_0-E[\hat{V}_0])^2\Big], \qquad B=\hat{H}'^2.E\Big[(\hat{V}_0-E[\hat{V}_0])^2\Big] \\
& \hat{H}=H|_{\hat{V}_0=E[\hat{V}_0]}, \quad \hat{H}'=\tfrac{\partial H}{\partial \hat{V}_0}|_{\hat{V}_0=E[\hat{V}_0]}, \quad \hat{H}''=\tfrac{\partial^2 H}{\partial \hat{V}_0^2}|_{\hat{V}_0=E[\hat{V}_0]}
\end{aligned}
\tag{30}
$$

where $E[\hat{V}_0]$ and $E[(\hat{V}_0-E[\hat{V}_0])^2]$ are presented next.

2. Future variance moments

To calculate the moments of future variance, this paper uses the Moment Generating Function (MGF) derived from the Feynman-Kac formula and the method recommended by Tahani (2005). The MGF is:

$$MGF(\hat{V}, x) = E[exp(x\hat{V})] = F(V_0, x) \tag{31}$$

The expected value follows Feynman-Kac PDE as function of variance, V, under $\mathbb{Q}$:

$$\frac{\partial F}{\partial t} + (\kappa\theta - \lambda V)\frac{\partial F}{\partial V} + \frac{1}{2}\sigma^2 V\frac{\partial F}{\partial V} + xVF = 0 \tag{32}$$

The solution is exponential-linear in V. Hence, the MGF for $\hat{V}$ is log-linear in V:

$$\begin{aligned}
&MGF(\hat{V}, x) = exp(V_0 M(T, x) + N(T, x))\\
&M(T, x) = \frac{2x(1 - e^{-\omega T})}{T(\lambda+\omega)(1+\phi e^{-\omega T})}\\
&N(T, x) = \tfrac{-2\kappa\theta}{\sigma^2}[log(\tfrac{1+\phi e^{-\omega T}}{1+\phi}) - \tfrac{(\omega-\lambda)T}{2}]\\
&\omega = \sqrt{\lambda^2 - \tfrac{2x\sigma^2}{T}} \quad ; \quad \phi = \tfrac{\omega-\lambda}{\omega+\lambda}
\end{aligned} \tag{33}$$

From the MGF, the first and the second moments are calculated as $MGF'|_{x=0}$ and $MGF''|_{x=0}$:

$$\begin{aligned}
&E(\hat{V}_0) = V_0 M_1 + N_1, \qquad E\Big([\hat{V}_0 - E(\hat{V}_0)]^2\Big) = V_0 M_2 + N_2\\
&M_1 = \tfrac{1-exp(-T\lambda)}{T\lambda}, \qquad M_2 = \tfrac{\sigma^2}{T^2\lambda^3}\Big[1 - 2T\lambda exp(-T\lambda) - exp(-2T\lambda)\Big]\\
&N_1 = \tfrac{\kappa\theta}{\lambda}\Big[1 - M_1\Big], \qquad N_2 = \tfrac{\kappa\theta\sigma^2}{T\lambda^3}\Big[1 + exp(-T\lambda) - 2M_1\Big] - \tfrac{\kappa\theta}{2\lambda}M_2
\end{aligned} \tag{34}$$

F. Calibration Procedure

Applying Ito's lemma on Equation 11 generates the model-implied equity process:

$$\begin{aligned}\frac{dEq}{Eq} &= \mu_{Eq}dt + a_1 dW_1^p + b_1 dW_2^p \\ a_1 &= \frac{\partial Eq}{\partial \nu_0}\frac{\nu_0}{Eq}\sqrt{V_0}, \quad b_1 = \frac{\partial Eq}{\partial V_0}\frac{\sigma}{Eq}\sqrt{V_0}\end{aligned} \tag{35}$$

where μ_{Eq} is the drift for equity-return process. Variance of equity return, σ_r^{2Eq}, is:

$$\begin{aligned}\sigma_r^{2Eq} &= a_1^2 + b_1^2 \\ d\sigma_r^{2Eq} &= \mu_r^{Eq}dt + a_2 dW_1^p + b_2 dW_2^p \\ a_2 &= \frac{\partial \sigma_r^{2Eq}}{\partial \nu_0}\nu_0\sqrt{V_0}, \quad b_2 = \frac{\partial \sigma_r^{2Eq}}{\partial V_0}\sigma\sqrt{V_0}\end{aligned} \tag{36}$$

Empirical calibrations use equity value and equity variance.

1. Firm-year calibration

For the model with VRP, the calibration uses a 4-by-4 equation: i) the instant variance, V_0, ii) long-run variance mean, θ, iii) unlevered asset value, ν_0, and iv) VRP, $|\lambda - \kappa|$, are set so that the model implied values match with i) equity value, Eq, ii) historical volatility of equity return, $\sqrt{\sigma_r^{2Eq}}$, iii) corporate yield spread, ysp, and iv) the leverage of the firm, Lev, for each firm-year. In the model without VRP, the stochastic variance parameters, VRP and θ, do not exist. The process without VRP follows the standard in the literature and the calibration collapses into solving a 2-by-2 equation: unlevered asset value, ν, and variance,

V, calibrate to match the leverage ratio and historical equity volatility:

$$\text{(37)} \qquad \text{With VRP} \begin{cases} Eq(V_0, \theta, \nu_0, \lambda - \kappa) = Eq_{data} \\ \sqrt{\sigma_r^{2Eq}(V_0, \theta, \nu_0, \lambda - \kappa)} = \sigma_{data}^{Eq} \\ ysp(V_0, \theta, \nu_0, \lambda - \kappa) = ysp_{data} \\ Lev(V_0, \theta, \nu_0, \lambda - \kappa) = Lev_{data} \end{cases} , \text{ Without VRP} \begin{cases} \sqrt{\sigma_r^{2Eq}(V_0, \nu_0)} = \sigma_{data}^{Eq} \\ Lev(V_0, \nu_0) = Lev_{data} \end{cases}$$

where Eq_{data} is the equity market cap, σ_{data}^{Eq} is the historical 365-day equity volatility, ysp_{data} is the yield spread for the longest bond maturity, and Lev_{data} is the leverage of the firm-year in data.

With VRP, model-implied equity value and variance are in Equations 11 and 36. Model-implied market leverage is $D/(D+Eq)$ where $D = d/m$ and d is in Equation 7. Model-implied yield spread is calculated using the following equation:

$$\text{(38)} \qquad ysp = \frac{C_s}{D(C_s)} - r, \quad C_s : P = D(C_s)$$

To solve the equations, the calibration process minimizes the Mean Absolute Percentage Error (MAPE). Without VRP, model-implied value for equity is available by plugging Equations 21, 25, and 26 from Appendix D into Equation 11. From the equity value, equity volatility is easy to derive similar to Equation 36. Debt from Equation 21 divided by equity plus debt also result in model-implied leverage. There is no yield spread in the model without VRP because this model cannot match the observed yield spreads in data as a stylized fact (Eom et al. (2004)).

Some other model parameters are set to match the data as well and Table 12 in Appendix A has the details of the calculations for each variable: outstanding debt, P, is set equal to total liabilities.[20] Outstanding debt's coupon rate is the continuously compounded

[20]Debt and leverage calculations are similar to Elkamhi et al. (2012). Trade credits of the firm from its suppliers or customers are affected by the risk of the firm. There is also evidence that some firms substitute

annual interest expenses relative to the liabilities. Asset payout rate is 3% to match the average of the total asset payout. Variance volatility for each firm-year matches volatility for all the firm-years' unlevered volatility in the rating. Other model parameters are the same as the values used in the comparative statics to match the empirical trends (m=0.1, r=0.05, ρ=0.45, κ=4, T=1). The model without VRP does not have variance volatility, physical mean-reversion speed, and the decision period.

2. Representative-firm calibration

Since the calibrations are done to the representative firms instead of the firm-years, the process slightly changes compared to the firm-year calibrations: instant variance is set equal to the long-run mean and the decision period is two years. Hence, the last equation with leverage and instant variance from the 4-by-4 calibration in Equation 37 is dropped and the calibration becomes a 3-by-3 match. Equity value, volatility, leverage, outstanding debt, outstanding debt's coupon rate, asset payout rate, and variance volatility match with the empirical averages similar to the earlier calibration. Other model parameters are the same as the values used in the comparative statics (m=0.1, r=0.05, ρ=0.45, κ=4) which match the empirical reports. The model without VRP lacks variance volatility, physical mean-reversion speed, and the decision period and its 2-by-2 calibration matches equity value and volatility by adjusting asset variance and size. Plugging the estimated parameters into each model yields the model-implied optimal leverage.

In section C.1, the calibration is similar with one slight change. Asset VRP and yield spread is dropped in the model calibration with VRP. Hence, both models are calibrated in 2-by2 where asset variance and size are adjusted so that model-implied equity value and volatility match their empirical values. In model with asset VRP, asset VRP is simply the asset variance exposure times market VRP.

borrowing with trade credits (Petersen and Rajan, 1997; Murfin and Njoroge, 2015).

G. Sample statistics for data between 2002 and 2015

Place Table 13 about here

H. Robust estimation with bias correction and control for small sample properties

The inferences are robust when I control for small sample properties with the dynamic-panel regressions and bootstraped errors. Table 14 shows the results where VRP is more significant for IG firms than SG firms. The estimated coefficients and the inferences are also close to the results in Table 10 in magnitude. Estimations are done using the method described in Bruno (2005), which uses bias-corrected least-squares dummy variable (LSDV) estimators (it is embedded in Stata's LSDVC). The procedure automatically includes lagged variable while corrects for biases. There are 200 iterations using estimates from the method recommended by Arellano and Bond (1991) as the initial points. Therefore, the dynamic regression does not require control for the industry effect and drops the control variables. This method not only resolves the issue with small sample properties, but also addresses possible concerns raised by the classical Nickell (1981)'s critique.

Place Table 14 about here

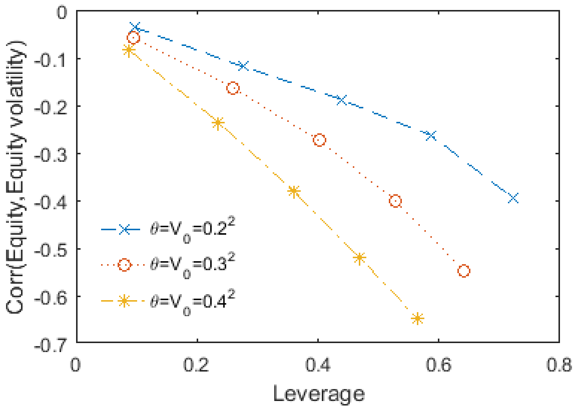


Figure 10. Replicated volatility asymmetry at the equity level implied by the model: X-axis shows the market leverage ratio of the firm. Y-axis shows negative correlation between equity returns and equity volatility, known as stock volatility asymmetry. Volatility is the standard deviation of equity returns. Coupon for outstanding debt is the risk-free rate times the face-value of debt, $C = rP$. Initial and mean variances are the same, $\theta = V_0$. Initial asset value is \$100 and it is scalable. Historical variance mean-reversion speed, κ, is 4, VRP, $|\lambda - \kappa|$, is 2, risk-free rate, r, is 5%, asset payout rate, δ, is 3%, PBC rate, ρ, is 45%, tax rate is 25%, and debt rollover rate, m, is 10%.

Table 11-The list of the variables in the model

ν	unlevered asset value,	$\sqrt{V}$	volatility,
V	variance of unlevered asset returns	T	period length for following optimal default policy
λ	Risk-Neutral (RN) speed of mean-reversion	$\hat{V}$	future variance
κ	mean-reversion speed under physical measure	$I_{\tau<T}$	is 1, if default happens prior to time T
$\lambda - \kappa$	Variance Risk Premium (VRP)	δ	assets' payout rate
r	risk-free rate	σ	variance volatility
θ	mean variance (long-run or long-term variance)	y	debt yield
θ^*	RN mean variance	ysp	credit spread, yield spread or credit premium
tax	tax rate	C	debt's coupon (interest) payment
μ	return drift under physical measure $\mathbb{P}$	P	outstanding face value of the firm's debt
D	debt value	Eq	equity value
L^*	optimal default boundary	ρ	proportional bankruptcy cost (PBC) rate
d	new debt's value	c	newly issued debt's coupon
p	newly issued debt's face value	M	average debt maturity (=1/m)
α	Asset variance exposure to market VRP	$\sqrt{b}$	asset beta
W_i^p	independent Brownian motions under physical measure $\mathbb{P}$, i={1,2}	W_i	independent Brownian motions under RN measure $\mathbb{Q}$, i={1,2}

Table 12-The list of the empirical variables with Compustat codes for the calculation of each variable.

Equity market cap	Total common shares times stock price (CSHO × PRCC_F)
Leverage	The total book value of liabilities to the market cap of equity plus the book liabilities (Lev=LT/(LT+CSHO × PRCC_F))
Historical equity volatility	Historical standard deviation of equity returns for 365 days from Optionmetrics (RV)
Historical equity beta	Historical covariance of equity returns and CRSP value-weighted index divided by its variance with minimum 60 days and maximum 250 days of returns (β_{equity})
Option-implied equity volatility	Daily average for 365-day-option implied equity standard deviation across all the strike prices in volatility surface from Optionmetrics (IV)
Asset VRP proxy	The ratio of the historical annual average of option-implied equity volatility to the historical equity volatility based on the same period ($Mean_{365day}(IV)/RV$)
Market VRP proxy	The ratio of the difference between CBOE VIX and historical 30-day S&P volatility to historical volatility ($(VIX - Vol)/Vol$)
Exposure to market variance	90-day correlation between squared option-implied 30-day volatility and squared VIX ($Corr_{90day}(IV^2_{30day}, VIX^2)$)
Asset volatility proxy	Unlvered historical equity volatility ((1-Lev) × RV)
Asset beta proxy	Unlvered historical equity beta ((1-Lev) $\times\beta_{equity}$)
Corporate yield spread	Credit spread, Yield of longest maturity bond less the maturity-matching risk-free rate
Asset payout	Payout to shareholders (dividends and share repurchases) plus interest payments divided to assets ((DVPSP_F*CSHO +PRSTKC +XINT)/(LT+Equity))
Variance volatility	The standard deviation of unlevered equity volatility for all the firm-years in the rating
Tobin's Q	Equity market cap, liabilities and preferred shares less taxes divided by the book assets ((CSHO × PRCC_F + LT + PSTKL - TXDITC)/AT)
Tangibility	Proxy for bankruptcy costs, the ratio of the property, plant and equipment to the assets (PPENT/AT)

Profitability	Operating income divided by the book assets (OIBDP/AT)	Cash	The ratio of cash and equivalents to the book assets (CHE/AT)
Log(Sales)	The natural log of the revenues (log(SALE))	Risk-free rate	US treasury notes rates
Coupon rate, C	Continuously compounded coupon rate ($ln(1 + XINT/LT)$)	Outstanding debt, P	Total liabilities (LT)

Table 13- More descriptive statistics for the sample of the rated firm-years used in the regressions (2002-15): The sample is the same as in Table 8 from merged Compustat and Optionmetrics. Table 12 in Appendix A has the details of the variable calculations. Tobin's Q is market to book value of assets. Profitability is the operating income ratio. Cash is the ratio of the cash holdings. Tangibility is the ratio of the tangible assets and the proxy for bankruptcy costs. Log(Sales) is the natural log of the revenues. Asset payout is the total payout to debt and equity holders relative to size. Leverage is the total liabilities to the market cap of equity plus the liabilities. Unlevered volatility is the historical volatility of stock returns unlevered with the leverage to proxy for the assets' historical volatility. VRP proxy represents the asset VRP and is the ratio of equity option-implied to historical volatilities.

Rating	statistic	Tobin's Q	Profitability	Cash	Tangibility	Log(sales)	Unlevered volatility	VRP proxy	Asset payout
AAA	Mean	2.34	18.9%	23.0%	12.9%	11.17	15.5%	1.12	4.7%
32	Std	0.80	7.2%	16.2%	4.2%	0.44	8.2%	0.23	1.6%
AA	Mean	2.23	16.8%	10.1%	28.5%	10.85	14.3%	1.14	4.0%
83	Std	0.84	5.7%	6.4%	15.9%	1.21	6.2%	0.18	1.8%
A	Mean	2.11	17.4%	8.5%	32.2%	9.75	17.1%	1.07	4.0%
365	Std	0.87	6.4%	7.9%	20.8%	0.96	7.2%	0.17	2.0%
BBB	Mean	1.48	13.2%	8.8%	33.3%	9.35	16.7%	1.03	4.3%
331	Std	0.50	4.9%	7.8%	24.6%	0.94	7.1%	0.16	4.8%
IG firms	Mean	1.87	15.7%	9.3%	31.5%	9.75	16.6%	1.06	4.1%
811	Std	0.80	6.2%	8.7%	22.0%	1.10	7.1%	0.17	3.4%
BB	Mean	1.27	10.5%	10.6%	28.7%	8.63	17.7%	1.03	4.0%
90	Std	0.35	5.4%	8.6%	15.8%	1.13	8.9%	0.18	3.9%
B	Mean	1.35	8.1%	11.9%	31.9%	9.06	16.2%	1.02	3.5%
51	Std	0.35	7.0%	8.6%	16.4%	1.25	9.0%	0.22	1.8%
CCC	Mean	1.24	4.0%	13.2%	57.8%	8.95	13.7%	1.00	4.3%
8	Std	0.25	4.8%	3.5%	16.1%	1.31	6.3%	0.22	0.6%
All	Mean	1.78	14.7%	9.6%	31.5%	9.61	16.6%	1.06	4.1%
960	Std	0.78	6.6%	8.6%	21.3%	1.16	7.4%	0.18	3.4%

Table 14- Regression results with small-sample controls: the results are similar to Table 10. Stata's LSDVC procedure estimates bias-corrected least-squares dummy variable (LSDV) model. The procedure automatically includes lagged variable and corrects its inclusion bias. The model is $Lev_{i,t} = (1 - \Psi)Lev_{i,t-1} + a_0 + \sum_k \Psi a_k X_{k,i,t}$ where X has standardized independent variables and control dummies. Independent variables are described in Table 13. Table 12 in Appendix A has the details of the variable calculations. IG-firms sample only has all the firm-quarters rated as investment grade by S&P. SG-firms sample only has all the firm-quarters rated as none investment-grade or speculative grade by S&P. Dummies for years and firms control for time and firm fixed effects. Standard errors are corrected for small sample properties with bootstrapping 200 iterations and they are reported below the estimates. The p-values test the null hypothesis that the coefficient is zero:$***p < 0.01, **p < 0.05, *p < 0.1$. Coefficients for the standardized variables show the relative importance of each variable in determining the target leverage.

Parameter	Estimated Coefficients for each statistical regression		
Model	(1)	(2)	(3)
Sample	All	IG firms	SG firms
Dependent variable	Leverage	Leverage	Leverage
Lag Leverage	0.415***	0.317***	0.504***
	(0.026)	(0.033)	(0.092)
Asset volatility	-0.0617***	-0.0504***	-0.0608***
	(0.004)	(0.004)	(0.011)
VRP proxy	-0.0316***	-0.0304***	-0.0162
	(0.003)	(0.003)	(0.010)
Tobin Q	-0.0420***	-0.0544***	-0.0605
	(0.005)	(0.005)	(0.040)
Profitability	-0.0269***	-0.0186***	-0.0259
	(0.004)	(0.005)	(0.017)
Cash	-0.0029	-0.00364	-0.0193
	(0.004)	(0.004)	(0.019)
Tangiblity	0.0659***	0.0383***	0.085
	(0.011)	(0.013)	(0.068)
Log sales	0.0328***	0.00124	0.0134
	(0.010)	(0.012)	(0.046)
Time and firm fixed effect	yes	yes	yes
Obs	814	683	131

I. Internet appendices

1. From EBIT to unlevered value

Operating incomes (EBIT) from assets follow Geometric Brownian motion with stochastic variance under both measures:

$$(39) \qquad \mathbb{P}: \begin{cases} \frac{dEBIT}{EBIT} = (\mu - \delta)dt + \sqrt{V}dW_1^p \\ dV = \kappa(\theta - V)dt + \sigma\sqrt{V}dW_2^p \end{cases} \qquad \mathbb{Q}: \begin{cases} \frac{dEBIT}{EBIT} = (r - \delta)dt + \sqrt{V}dW_1 \\ dV = \lambda(\theta^* - V)dt + \sigma\sqrt{V}dW_2 \end{cases}$$

The EBIT and payout rate are strictly positive to assure unlevered asset value is always positive, where I borrow the assumption from Goldstein et al. (2001). The value of the unlevered assets, ν, that generate the income is the present value of all the future cashflows from operating income. Since $E^Q(EBIT_s|EBIT_t) = EBIT_t e^{(r-\delta)\times(s-t)}$, the value is :

$$(40) \qquad \nu_t = E^Q\Big(\int_t^{\infty}(1-\tau)e^{-r(s-t)}EBIT_s ds\Big) = \frac{(1-tax)}{\delta}EBIT_t$$

By applying Ito's lemma to Equation 40, the unlevered-asset return process also follows Geometric Brownian with stochastic variance under both measures as in Equation 1.

2. Economic Assumptions

In this economy, assets are traded and their premium can be determined. But, variance is not traded and in order to determine the instant variance premium, Heston (1993) assumes that the instant premium is proportional to variance itself following Cox et al. (1985). Therefore, the stochastic discount factor, SDF, and the premiums of the returns follow:

$$\frac{dSDF}{SDF} = -rdt - \frac{\mu-r}{\sqrt{V}}dW_1^p - \frac{(\lambda-\kappa)\sqrt{V}}{\sigma}dW_2^p, \quad d\langle W_1^p, W_2^p\rangle = 0,$$
$$\begin{cases} E^{\mathbb{P}}[\frac{d\nu}{\nu}] - E^{\mathbb{Q}}[\frac{d\nu}{\nu}] = -E^P[\frac{dSDF}{SDF}\frac{d\nu}{\nu}] = (\mu-r)dt = AP.dt \\ E^{\mathbb{P}}[\frac{dV}{V}] - E^{\mathbb{Q}}[\frac{dV}{V}] = -E^P[\frac{dSDF}{SDF}\frac{dV}{V}] = (\lambda-\kappa)dt = VRP.dt \end{cases} \tag{41}$$

where AP is the asset premium. When variance, V, is constant, the model collapses into classical Black-Scholes economy with classical stochastic discount factor:

$$dW_1^p = \frac{\mu-r}{\sqrt{V}}dt - dW_1, \quad AP = \frac{\mu-r}{\sqrt{V}} \times \sqrt{V} = \mu - r, \quad \frac{dSDF}{SDF} = -rdt - \frac{AP}{\sqrt{V}}dW_1^p \tag{42}$$

In another representation, Barras and Malkhozov (2016) define the following as the premiums where the expected values are based on Equation 1:

$$E^{\mathbb{P}}[\tfrac{d\nu}{\nu}] - E^{\mathbb{Q}}[\tfrac{d\nu}{\nu}] = (\mu-r)dt = AP.dt,$$
$$\begin{cases} E^{\mathbb{P}}[dV] - E^{\mathbb{Q}}[dV] = (\lambda-\kappa).Vdt = VRP.Vdt, \\ \frac{E^{\mathbb{P}}[dV]-E^{\mathbb{Q}}[dV]}{V} = VRPdt = (\lambda-\kappa)dt \end{cases} \tag{43}$$

In this setup which is used in options literature, $\lambda - \kappa$ is the relative difference between RN and historical instant variances. The results are similar to the assumptions in Heston (1993).

3. Example of different exposures to market shocks and market variance risk

Let's consider two firms: one firm has high exposure to market variance, α, with low beta, $\sqrt{b}$, and idiosyncratic variance, V_i. But, the other has low exposure to market variance and high beta and idiosyncratic variance. The first is relatively more exposed to variance shocks and the later is relatively more exposed to return shocks. Low-VRP firm has high asset beta and total variance, but it has low VRP because it has relatively small proportion of systematic variance compared to the high-VRP firm. High-VRP firm, on the other hand, has low asset beta and total variance, but its asset VRP is high because of small idiosyncratic variance and relatively large systematic variance. While it has no effect on asset beta, idiosyncratic variance reduces the impact of market variance shocks and premium to be transferred to the firm.

Table 15 shows the numerical example. Total volatility and asset VRP for high-VRP firm are within estimated ranges for IG firms and total volatility and asset VRP for low-VRP firm are within the range for B firms in Table 8. Market variance risk exposure for high-VRP firm is within the range for IG firms and the exposure for low-VRP fimr is within the range for B firm in Table 6.

Place Table 15 about here

4. Proof for Proposition1

All state variables are Markov. Optimal default boundary, L^*, is chosen based on Markov state variables, which also makes L^* a Markov variable.

Assumption 1. *Equity value at each point of time is monotonic and increasing in the firm's value, given all other parameters being constant ($\nu_1 < \nu_2 \rightarrow Eq(\nu_1) \leq Eq(\nu_2)$)*

The assumption is plausible because if $Eq(\nu_1) > Eq(\nu_2)$, then shareholders will destroy part of the firm's value to move it to ν_1. Therefore, the equity value at ν_2 cannot be smaller than ν_1, and the equity value at ν_2 is at least as large as the equity value at ν_1.

Assumption 2. *The equity function is a continuous function of the unlevered firm value for all parameters and other state variables.*

PROPOSITION 1. *The optimal default triggering boundary L^* is independent of the firm's current value, if the firm's value is above the boundary.*

The proposition is analogical to the optimal exercise policy of an American put option;

the optimal exercise boundary of the put is independent from the underlying asset's value as long as the put is not exercised.

Proof: The rationale of the proof is about showing that the optimal boundary is the same for two different firm values. For the firm's value below default boundary, the firm defaults, equity is valued at zero, and debt holders are in control of the firm. Hence, I only consider the values above the boundary. Based on the rationality of investors equity value is positive for any asset value above the boundary, L^*. Shareholders maximize equity value, $Eq(\{\nu, \Sigma\}; \Theta)$, where Θ={all model parameters} and $\{\nu$,$\Sigma\}$={all state variables including current unlevered firm-value}. Based on the smooth pasting condition, the optimal control variable must satisfy $\frac{\partial Eq(\Theta;\{\nu,\Sigma\})}{\partial \nu}|_{(\nu=L^*)} = 0$. From Assumption 1 and Assumption 2, the equity function is monotonic and continuous. Therefore, the solution to the smooth pasting condition is unique. *Ceteris paribus*, this result implies that the optimal default policy is the same for two completely similar firms with only different unlevered assets' values $(L^*[\Theta; \{\nu_1, \Sigma\}] = L^*[\Theta; \{\nu_2, \Sigma\}])$.

5. Approximation Errors

The approximation errors are small and closed-form debt value with Taylor expansion to the second degree is close to the value from the simulations. The closed-form formula slightly overestimates the value of debt and leverage compared to the simulation. Hence, using the closed-form is more parsimonious because debt value is even slightly lower with VRP based on simulations, which implies stronger negative effect of VRP on leverage.

For the simulations, I draw 100,000 paths of both variance and unlevered asset value with weekly steps, $\Delta t = 1/50$, under RN measure :

$$\begin{aligned} \nu_t &= \nu_{t-\Delta t}\, exp\Big((r - \delta - \tfrac{V_{t-\Delta t}}{2})\Delta t + \sqrt{\Delta t}\sqrt{V_{t-\Delta t}} z_1\Big) \\ V_t &= V_{t-\Delta t}\, exp\Big(\tfrac{[\lambda(\frac{\kappa\theta}{\lambda} - V_{t-\Delta t}) - \frac{\sigma^2}{2}]\Delta t + \sqrt{\Delta t}\sigma\sqrt{V_{t-\Delta t}} z_2}{V_{t-\Delta t}}\Big) \end{aligned} \tag{44}$$

where z_1 and z_2 are respectively standard normal shocks to the asset return and variance under RN measure. Random numbers are all antithetic to increase convergence in the results. Although the path is discrete, the process is time continuous. At the end of each year, $T = 1$, I calculate and update the default boundary, L, as in Equation 12. For each path, if asset value hits the boundary, firm defaults and creditors control the firm. To value their debt, I discount the firm's value less the default costs plus all the coupons up to the default. Otherwise, I discount debt's market value, considering the bond sold after 50 years for the constant-variance price, plus all the coupons during the 50 years. This method allows me to calculate the debt value under the perpetual process. Although this violates the model assumption about stochastic volatility, it has a minor effect on the results because the discounting time is after 50 years, $e^{-50r} \approx 0.08$. If the path does not hit default, I calculate the market value of debt using Equation 21 on year 50 and discount it to time 0.

Simulating perpetual stochastic variance processes has a main obstacle that is having an infinite time dimension. It is not yet possible to simulate this process with an infinite horizon. Therefore, I assume the variance to stay constant after 50 years at the mean level under the RN measure. At the end of 50 years with this assumption, there exists a closed-form formula and it is the closest value to estimate the terminal debt value.

Figures 11 and 12 show the results. Average and median of the debt value from all the paths provides the simulated value for the perpetual debt. VRP is 2 within the range of empirically estimated values. Instant and long-run variances are equal to 0.04, 0.2 squared. The rest of the parameters are also similar to calibrations. The results for the tax benefits and bankruptcy costs are similar because they follow a similar derivation.

Insert Figure 11 about here.

Insert Figure 12 about here.

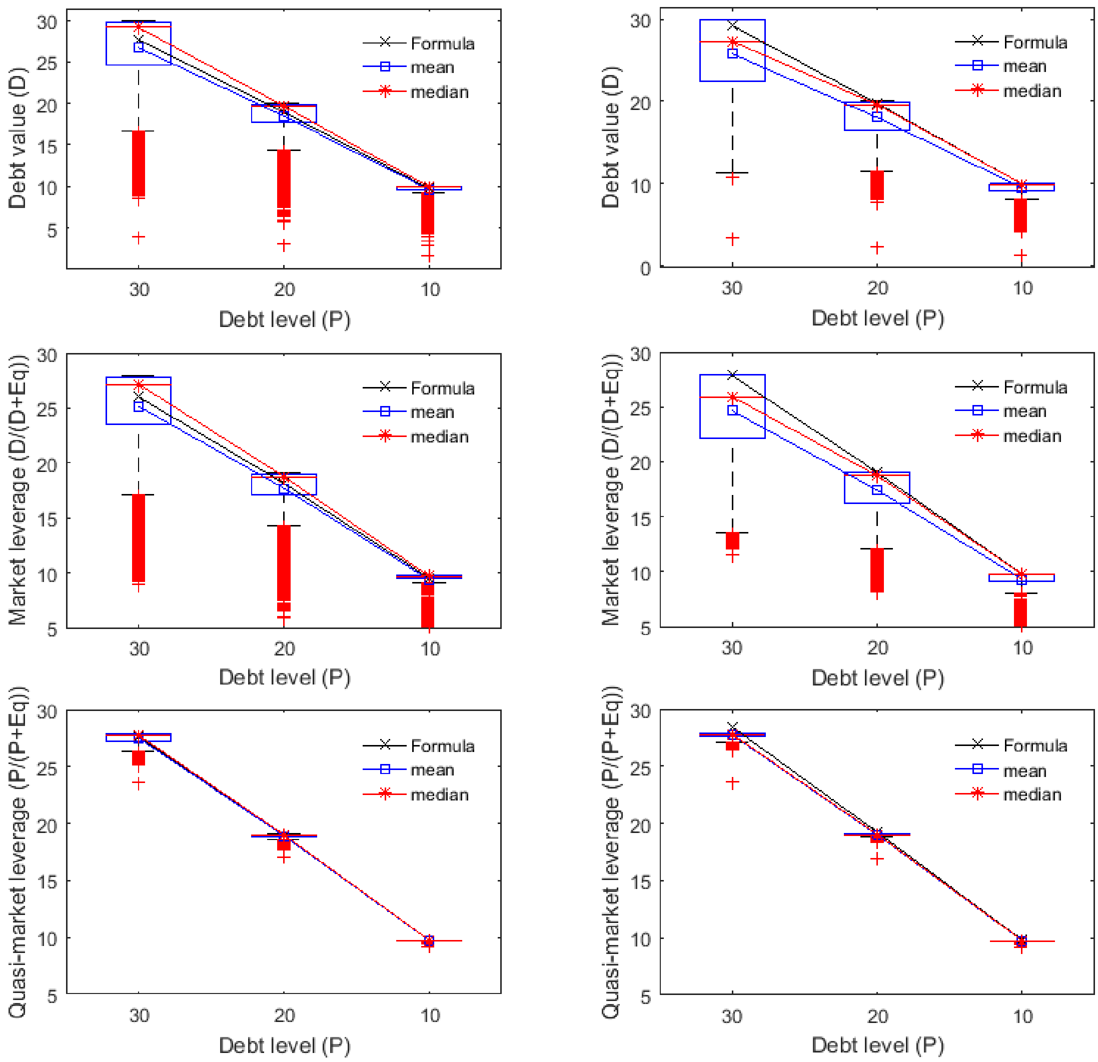
6. 3-by-3 calibration statistics for data (2002-15)

The results are robust to dropping leverage in the calibrations. In the model with VRP, dropping the leverage from the calibration reduces the equations into 3-by-3 match where I drop instant variance and assume it is equal to the long-run mean. This is a more restrictive assumption because it assumes that the variance term structure is flat and the asset VRP is more important. The 2-by-2 calibration for the model without VRP also misses the leverage and the equity value replaces leverage in the calibration. Table 16 shows the results. The comparison between optimal leverage from each model shows that the model with VRP implies lower leverage closer to the observed leverage, especially for IG firm-years.

Place Table 16 about here

Table 15-Examples of two firms with different exposure to market risk and market variance risk: This table shows that a firm with low asset beta can have high asset VRP and vice versa. Asset beta measures the exposure of the assets to market return premium and return shocks. Asset VRP measures the exposure of the assets to market VRP and variance shocks. Proportional variance from market exposure is also correlation of total asset variance with market variance. Market variance is set to 4% and market VRP is set to 4 reported by Ait-Sahalia and Kimmel (2007). The correlations are in the range of the empirical values in Table 6. Total volatility and asset VRP are within ranges in Table 8 . All the numbers are calculated based on Equation 3.

	High-VRP firm		Low-VRP firm
Positive constant, b_1	1	b_2	1.2
Idiosyncratic variance, V_{i1}	$(24\%)^2$	V_{i2}	$(45\%)^2$
Proportional variance from market exposure, α_1	50%	α_2	20%
Total volatility, $\sqrt{V_1}$	32%	$\sqrt{V_2}$	50%
Asset beta, β_1	1.00	β_2	1.10
Total asset variance, V_1	10%	V_2	25%
VRP, $\lvert\lambda_1 - \kappa_1\rvert$	2.00	$\lvert\lambda_2 - \kappa_2\rvert$	0.8



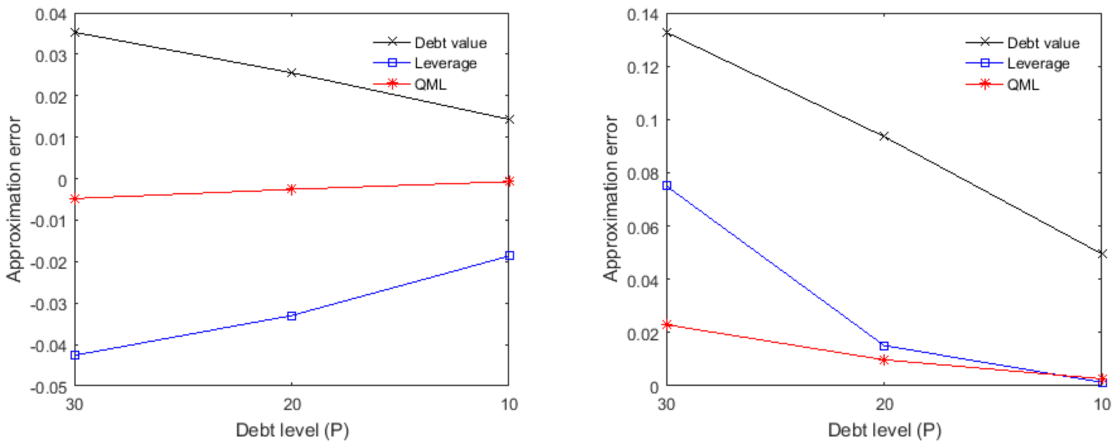
(a) Without debt roll over

(b) With debt roll over

Figure 11. Comparing the approximate closed-form values with the simulations: X-axis shows face value of debt, P. Y-axis shows the value of the variables of interest: the first row is for debt value, the second row is the market leverage ratio, $D/(D + Eq)$, and the third row is the quasi-market leverage (QML), $P/(P + Eq)$. Coupon for outstanding debt is the risk-free rate times the face-value of debt, $C = rP$. Initial and mean variances are the same, $\theta = V_0$, and set to 0.04, 0.2 squared. Initial asset value is \$100 and it is scalable. Historical variance mean-reversion speed, κ, is 4, VRP, $|\lambda - \kappa|$, is 2, risk-free rate, r, is 5%, asset payout rate, δ, is 3%, PBC rate, ρ, is 45% and tax rate is 25%. Debt rollover rate, m, is 10%. There are 100,000 simulation paths. For each path, I discount the value of debt to time zero, either from default or after 50 years at estimated perpetual value, and also measure leverage and QML. The box is between 25 and 75 percentiles of the simulated values.

Table 16-Results for calibrations without leverage on the rated firm-years (2002-15): The table is comparable to Table 8. Table 12 in Appendix A has the details of the variable calculations. The last four columns report the statistics for the parameters in a 3-by-3 calibration with VRP to the firm-years as in Equation 37 and the error without having leverage. Asset VRP, $|\lambda - \kappa|$, is the price of variance risk. Size, ν_0, is the unlevered value of the firm's assets. Mean volatility is the square-root of the mean asset variance, $\sqrt{\theta}$. MAPE is the mean average percentage error between model-implied and observed values in the calibration. The first two columns report the statistics for the optimal leverages with and without VRP as implied by the calibrated parameters. These columns are comparable with actual leverage. The optimal leverage for the model without VRP is implied by the parameters in a 2-by-2 calibration as in Equation 37 where equity value replaces leverage.

		Optimal leverage		Calibrated values in the model with VRP			
Rating	statistic	With VRP	Without VRP	Asset VRP	Size	Mean volatility	MAPE
AAA	Mean	44.2%	54.5%	3.45	340,754	17.2%	20%
32	Std	9.9%	11.2%	0.92	230,505	8.2%	15%
AA	Mean	42.9%	54.9%	3.66	188,772	16.3%	22%
83	Std	7.0%	9.1%	0.87	172,226	6.4%	16%
A	Mean	40.1%	50.5%	3.47	49,312	19.3%	16%
365	Std	5.2%	7.8%	1.04	44,886	7.4%	15%
BBB	Mean	40.6%	51.0%	3.48	29,539	19.6%	13%
331	Std	6.9%	9.1%	0.99	42,168	7.9%	13%
IG firms	Mean	40.7%	51.3%	3.49	67,014	19.0%	16%
811	Std	6.5%	8.8%	1.00	108,907	7.6%	15%
BB	Mean	39.7%	51.1%	3.56	14,649	21.6%	13%
90	Std	7.4%	11.2%	0.85	32,627	10.5%	13%
B	Mean	41.3%	54.0%	3.09	23,087	22.3%	15%
51	Std	9.2%	14.6%	1.36	58,314	10.9%	23%
CCC	Mean	40.2%	53.4%	3.65	34,160	21.4%	21%
8	Std	10.2%	7.8%	0.64	68,392	9.2%	28%
All	Mean	40.7%	51.4%	3.48	59,283	19.5%	15%
960	Std	6.8%	9.4%	1.01	102,979	8.2%	15%



(a) **Without debt roll over** (b) **With debt roll over**

Figure 12. Median approximation errors: X-axis shows face value of debt P. Y-axis shows the approximation error for each value. All the parameters are the same as Figure 11.

Imprint and acknowledgements

I am indebted to my co-supervisors, Jan Ericsson and Aytek Malkhozov, and PhD thesis defense committee. I am also grateful to Redouane Elkamhi, Sergei Sarkissian, Dirk Hackbarth, Stefan Nagel, Chris Parsons, Georges Dionne, Christian Dorion, Sebastien Betermier, Nabil Tahani, Hugues Langlois, Timothy Trombley, Januj Juneja, Marie Lachance, Nikhil Varaiya, Jaemin Kim, Mehdi Salehizadeh, Tolga Cenesizoglu, Amir Akbari, Edward Lawrence, Nelson Heintz, Ali Jafari, participants and discussants in McGill University seminar, San Diego State University seminar, Mathematical Finance Days 2014, Financial Management Association meeting 2015, Midwest Finance Association meeting 2015, and Global Finance Conference 2016 for their helpful comments. Part of this research is conducted in CyberLab at IUPUI. Financial support is provided by the Institut de Finance Mathèmatique de Montreal (*IFM*^{2}). All errors are mine.

Babak Lotfaliei
Fowler College of Business, San Diego State University, San Diego, United States;
email: **babak.lotfaliei@sdsu.edu**

Postal address 60640 Frankfurt am Main, Germany
Telephone +49 69 1344 0
Website **www.esrb.europa.eu**

ISSN 2467-0677 (pdf)
ISBN 978-92-9472-022-1 (pdf)
DOI 10.2849/840761 (pdf)
EU catalogue No DT-AD-18-008-EN-N (pdf)