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Corrective regulation with imperfect instruments

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Abstract

This paper studies optimal second-best corrective regulation, when some agents/activities cannot be perfectly regulated. We show that policy elasticities and Pigouvian wedges are sufficient statistics to characterize the marginal welfare impact of regulatory policies in a large class of environments. We show that a subset of policy elasticities, *leakage elasticities*, determine optimal second-best policy, and characterize the marginal value of relaxing regulatory constraints. We apply our results to scenarios with unregulated agents/activities, uniform regulation across agents/activities, and costly regulation. We illustrate our results in applications to financial regulation with environmental externalities, shadow banking, behavioral distortions, asset substitution, and fire sales.

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1 Introduction

Many economic policies are motivated by the desire to correct externalities. However, the instruments available to policymakers are often imperfect. Financial regulation is a prime example of this phenomenon. In particular, in the aftermath of the 2007–2009 financial crisis and guided by theories of corrective policy in the presence of a diverse set of market failures — including fire-sale externalities and distortive government subsidies (e.g., Lorenzoni, 2008; Bianchi, 2016; Farhi and Werning, 2016; Dávila and Korinek, 2018) — most economies have expanded the set and scope of regulations faced by the financial sector. At the same time, many agents and activities in the financial system remain *unregulated*, and regulators are frequently forced to impose *uniform* regulations across heterogeneous agents and activities. In addition, recent policy discussions have focused on how to employ financial regulation to address environmental externalities, a view that must deal with significant restrictions on the set of feasible policy instruments.

These policy imperfections are often viewed as generating "unintended consequences" (e.g., Adrian and Ashcraft, 2016; Hachem, 2018).[1] Hence, a natural normative question is how regulators should proceed once they are aware of previously unintended consequences. The associated second-best policy problem appears daunting because, as we have outlined, there are many possible market failures to consider and many seemingly disparate imperfections in policy instruments.

This paper characterizes, for a broad class of economies, how the presence of imperfect regulatory instruments affects the design of optimal corrective regulation. Our goal is to identify a set of unifying economic principles for regulation in a second-best world in which regulation is costly and/or subject to constraints. Therefore, our results build on and complement the existing theoretical literature, which focuses on the properties of particular types of market failures and regulatory imperfections.

We initially consider a general model in which multiple investors have access to a rich set of investment and financing opportunities, which may induce externalities.[2] A regulator can, in principle, impose corrective Pigouvian taxes/subsidies on each decision to address these externalities. However, the regulator has to choose such regulations from a constrained set. Our general results impose minimal structure on the nature of regulatory constraints. We show four results in this general framework before considering its various applications.

First, we characterize the marginal welfare impact of varying any given regulatory instrument. We show that the marginal welfare effects of varying corrective regulations are determined by two sets of statistics: i) policy elasticities and ii) Pigouvian wedges. Policy elasticities correspond to the general equilibrium responses of financing and investment decisions, both across and within investors. Pigouvian wedges correspond to the difference between the existing corrective regulation

[1]In the context of financial regulation, there are concerns about leakage of activity to the unregulated financial sector in the US and China in recent years, and about asset substitution, whereby institutions tilt their portfolios towards the riskier end of each asset category defined by regulatory "risk weights".

[2]This general model, phrased in terms of investment and financing decisions, allows us to directly explore different dimensions of financial regulation. Focusing on financial regulation is natural, since financial activity is inherently hard to regulate (Arseneau et al., 2022). However, our results apply beyond the sphere of financial regulation. Indeed, we provide a formal counterpart of our results using classical consumer theory in Section E of the Online Appendix.

that directly affects a given activity and the actual marginal distortion (externality) generated by that activity. These wedges capture the extent to which different activities are regulated too strictly or too leniently for any given set of corrective policies.

Second, as a benchmark, we characterize the optimal first-best policy, in which the regulator has access to an unconstrained set of regulations, and note that the Pigouvian principle — also called the principle of targeting — applies.[3] In that case, the optimal regulation is chosen so that all Pigouvian wedges are exactly equal to zero, with Pigouvian regulations set to equal marginal distortions. Hence, policy elasticities do not form part of the first-best policy. In other words, policy elasticities are only inherently important for corrective regulation in second-best scenarios, in which the set of regulatory instruments is imperfect.

Third, we characterize optimal second-best policy. We show that the second-best regulation of perfectly regulated decisions (i.e., decisions for which regulatory constraints are not binding) is given by the sum of i) the associated marginal distortion, guided by the first-best Pigouvian principle, and ii) a second-best correction. Two sets of sufficient statistics determine the sign of this correction and, therefore, whether an activity should be overregulated relative to the Pigouvian principle (super-Pigouvian regulation) or underregulated (sub-Pigouvian regulation). The first set of statistics contains the Pigouvian wedges associated with all imperfectly regulated decisions (i.e., all decisions associated with a binding regulatory constraint). The second is a subset of policy elasticities, which we refer to as *leakage elasticities,* and which measure the responses of imperfectly regulated decisions to the changes in the regulation of perfectly regulated decisions. Intuitively, under the second-best policy, regulators want to discourage imperfectly regulated activities that are underregulated (with a negative Pigouvian wedge), and encourage those that are overregulated (with a positive wedge). The leakage elasticities measure how these activities respond to the regulator's unconstrained policy choices. In particular, we demonstrate that the nature of the second-best correction depends crucially on whether perfectly and imperfectly regulated decisions are gross substitutes or gross complements. In our applications, we discuss natural examples of both cases.

Fourth, the last of our general results characterizes the marginal welfare effect of relaxing regulatory constraints. This is a relevant object in light of recent policy proposals that aim to extend the scope of institution- or activity-level regulations (e.g., Gorton, Metrick, Shleifer and Tarullo, 2010; Adrian and Ashcraft, 2016). This object is also important to understand the optimal regulation of imperfectly regulated decisions. We decompose this marginal welfare effect into two terms. The first is the direct effect, which is determined by the policy elasticities of imperfectly regulated decisions and the associated Pigouvian wedges. For example, welfare is improved most by relaxing a constraint if doing so discourages severely underregulated activities, with large negative wedges. The second effect is a feedback effect that incorporates the welfare impact of the responses of perfectly regulated decisions to the changes in the regulation of imperfectly regulated decisions under the second-best policy — this is a form of reverse leakage. Interestingly, we show that this

[3]Throughout the paper, we use the term first-best regulation to refer to the benchmark in which a planner can freely correct every individual decision, while also respecting individual/technological constraints.

second effect systematically dampens the welfare benefit of relaxing constraints whenever decisions are *either* substitutes or complements. While the logic behind this result is reminiscent of the Le Chatelier principle (e.g., Milgrom and Roberts, 1996), we find the opposite qualitative conclusion: when we let our system adjust further by accounting for the impact of relaxing a constraint on the perfectly regulated decisions under the second-best policy, the shadow welfare gains from regulation are typically dampened, not amplified.[4]

Next, we specialize these general results to characterize in detail three classes of imperfections regarding the set of policy instruments, given their practical relevance. First, we consider the case in which some investors or activities are entirely unregulated. In this case, the optimal second-best regulation is given by a weighted sum of distortions in both the regulated and unregulated segments, with the sign and magnitude of the appropriate weights determined by the leakage elasticities. This part of our analysis generalizes the well-known Tinbergen (1952) rule by deriving the optimal policy when the number of policy instruments is less than the number of targets. Second, we consider the case of *uniform* regulation, where the same regulations must apply to different investors or activities, even if they impose externalities of different magnitudes. We derive the optimal second-best uniform regulation in a general environment, in which other (non-uniform) regulations may remain freely adjustable. The optimal uniform regulation, which generalizes insights from Diamond (1973), takes the form of a weighted average of distortions, with weights that account for the responses of perfectly and imperfectly regulated decisions according to the Le Chatelier/reverse leakage adjustment discussed above. Third, we consider the case in which a subset of regulations is subject to smooth, quadratic costs. In this case, we show that the optimal regulation is optimally attenuated. This case also underlines the more general idea that the presence of perfectly regulated decisions — once again via the Le Chatelier adjustment — is a force that contributes to attenuating the optimal regulation of imperfectly regulated decisions.

Finally, to demonstrate the usefulness of these general principles, we consider a suite of applications. In our headline application, we leverage our general results to provide new insights into the question of financial regulation in the presence of environmental externalities. This question has only recently received interest in academic and policy circles, and remains underexplored. For this application, we develop a canonical model of modern banking/leveraged investment, in which investors choose the scale of their risky investment, the composition of their portfolios, and their leverage.

The planner in our headline application controls a risk-weighted capital requirement, which we show to be equivalent to imperfect corrective taxes. The planner can effectively regulate investors' leverage and portfolio ratios, but the overall scale of investment is a free, unregulated choice. Moreover, regulated and unregulated activities (e.g., leverage and the scale of risky investment) are gross complements, implying an incentive to over-regulate the regulated decisions. Following the

[4] In its simplest form, the Le Chatelier principle states that whenever choices are either complements or substitutes, the long-run response of a system is larger than its short-term response — see Milgrom and Roberts (1996) for a modern treatment. More generally, as described by Milgrom (2006), all versions of the Le Chatelier principle explain how the direct effect of a parameter change is typically amplified by feedbacks in a system.

current policy debate on climate finance, we then compare optimal policy under a narrow/financial mandate that only considers externalities related to financial stability, and a broad mandate that considers the impact of financial regulation on environmental externalities. Crucially, we demonstrate that the nature of optimal regulation is substantially different once we account for the imperfections inherent in current regulatory regimes. One implication of our approach is that it is natural to adjust risk weights, as opposed to leverage caps, when regulators become concerned with broader environmental mandates.

In four further applications, we show how our results can be employed in common regulatory scenarios, each with different kinds of regulatory instruments and constraints. These applications also illustrate how our results apply to different rationales for regulation. The economic insights from our further applications can be summarized as follows:

1. *Shadow Banking/Unregulated Investors:* We study a model with two types of leveraged investors that can be interpreted as regulated banks and unregulated (shadow) banks. Regulation is imperfect in the sense that shadow banks cannot be subject to any corrective regulation. We derive optimal second-best leverage regulation in a setting where the government provides ex-post bailouts without commitment. We find that the optimal policy in the regulated segment is commonly *sub-Pigouvian.* Concretely, the optimal policy imposes regulations below marginal distortions whenever i) leverage imposes negative externalities, and ii) leverage choices between regulated and unregulated investors are gross substitutes. Existing direct measurements of leakage elasticities (e.g., Irani, Iyer, Meisenzahl and Peydro, 2021) suggest that the substitutes case is the empirically relevant one. Our results further clarify how optimal second-best policy responds to potential changes in marginal distortions that arise from unregulated activities in general equilibrium. We also illustrate how the Le Chatelier/reverse leakage adjustment affects the welfare gains of being able to regulate unregulated investors.

2. *Behavioral Distortions/Unregulated Activities*: This application demonstrates how our general method and, in particular, the notions of Pigouvian wedges and policy/leakage elasticities, can be employed to analyze economies with behavioral distortions. We consider a model in which macro-prudential regulation is motivated by a type of internality, namely, distortions in investors' and creditors' beliefs about investment returns. We derive optimal policy under the assumption that the planner can regulate investors' leverage, i.e., the ratio of borrowing to risky investment, but not the overall scale of investment. In this situation, regulated and unregulated activities (e.g., leverage and the scale of risky investment) are gross complements, and the second-best optimal policy is *super-Pigouvian.*

3. *Asset Substitution/Uniform Activity Regulation:* We consider an environment where investors choose between two types of risky investment, but where regulation is imperfect in that the regulator imposes a uniform regulation across both types of investments. Regulation in this application is motivated by the fact that investors are "too big to fail" and enjoy an implicit

government subsidy. This case leads to novel insights into the classical "asset substitution" problem in financial economics (e.g., Jensen and Meckling, 1976). The optimal second-best regulation is a weighted average of the downside distortions imposed by different types of investment, with weights proportional to the policy elasticities of investment. Our general formula also leads us to a deeper characterization of the optimal weights, which reveals that they are closely related to the elasticity of the probability of receiving government support.

4. *Pecuniary Externalities with Heterogeneous Investors/Uniform Investor Regulation:* Finally, we consider a model of excessive credit booms along the lines of Lorenzoni (2008) in which the investment decisions of investors/entrepreneurs are associated with distributive pecuniary/fire-sale externalities. While most of the related literature focuses on characterizing constrained-efficient allocations, often assuming that a planner has access to investor-specific regulations, we assume that all investors must face the same regulation. Consistent with our general results, we show that the optimal second-best regulation is a weighted average of the induced distortions (pecuniary externalities), which in this case are given by differences in marginal valuations, net trade positions, and price sensitivities. This application is of independent interest, since it shows that even when a planner does not have access to investor-specific regulations, it may still be desirable to set corrective regulation to address pecuniary externalities.

In each application, we provide numerical illustrations of the optimal second-best policy, and how it compares to the first-best policy. When possible, we discuss how the existing empirical findings can be used to guide the optimal policy.

Our paper is directly related to the literature on imperfect regulation. In particular, the issue of regulatory arbitrage and shadow banking has been widely studied in recent years. Within the theoretical literature, Plantin (2015), Farhi and Tirole (2017), Huang (2018), and Martinez-Miera and Repullo (2019) study the impact of capital requirements on banking activity and financial stability. Hachem and Song (2021) explore how increased liquidity requirements can generate credit booms when banks are heterogeneous. Grochulski and Zhang (2019) show, in an environment in which regulation is motivated by a pecuniary externality as in Farhi, Golosov and Tsyvinski (2009), how regulation is constrained by the presence of shadow banks. Gennaioli, Shleifer and Vishny (2013) and Moreira and Savov (2017) develop theories that highlight the fragile nature of shadow banking arrangements. Ordoñez (2018) shows how shadow banking enables better-informed banks to avoid blunt regulations. Bengui and Bianchi (2018), building on Bianchi (2011), provide a theoretical and quantitative analysis of macroprudential policy with imperfect instruments based on a collateral pecuniary externality. Dávila and Korinek (2018) briefly discuss the impact of specific regulatory constraints on policy in a setup with pecuniary externalities, while Korinek (2017) provides a systematic study of optimal corrective policy in environments with multiple regulators. Clayton and Schaab (2021) study regulatory policy in the presence of shadow banks when there are pecuniary externalities. Korinek, Montecino and Stiglitz (2022) study the role of technological innovation as regulatory arbitrage. Begenau and Landvoigt (2021) provide a quantitative general equilibrium assessment of regulating commercial banks for financial stability and macroeconomic outcomes in the presence of ex-post subsidies — see Dempsey (2020) for a related quantitative assessment. Xiao (2020) characterizes monetary policy transmission in an environment with shadow banks. One can view monetary policy as an example of uniform corrective regulation with potentially heterogeneous responses. There is also a growing empirical literature on regulatory arbitrage and shadow banking, which includes the work of Acharya, Schnabl and Suarez (2013), Demyanyk and Loutskina (2016), and Buchak, Matvos, Piskorski and Seru (2018*a*,*b*), among others.

More broadly, our results are connected to the public economics literature that deals with imperfect corrective regulation. Along this dimension, one contribution of this paper is to show that several classic results that have been treated as independent can be derived and expanded upon using a common approach. For instance, the uniform corrective taxation result derived in Diamond (1973) is seemingly distinct from the characterization of second-best policy in Lipsey and Lancaster (1956), but we show that both are corollaries of Proposition 1 in this paper. To our knowledge, we provide the first general, systematic treatment of corrective regulation with imperfect instruments. Other contributions in this literature, often focused on whether indirect regulation is effective or even more desirable than direct regulation, include Baumol (1972), Sandmo (1975), Green and Sheshinski (1976), Balcer (1980), Wijkander (1985), and Cremer, Gahvari and Ladoux (1998) — see also the textbook treatment of Salanié (2011) and the lecture notes of Werning (2012).

Our results are also related to Hendren (2016), from whom we adopt the terminology "policy elasticity". We identify the special role that a subset of policy elasticities, leakage elasticities, play when studying second-best regulation. Finally, second-best corrective regulation is often discussed in the context of environmental policy and congestion — see Bovenberg and Goulder (2002) for a comprehensive review of that literature — as well as rent-seeking. Rothschild and Scheuer (2014, 2016) study optimal taxation with both corrective and redistributive motives in environments with rent-seeking, highlighting the importance of general equilibrium effects.

Outline The structure of the paper is as follows. Section 2 introduces our general framework and characterizes its equilibrium. Section 3 characterizes the general marginal effects that determine the optimal regulation and presents their implications for optimal regulation. Sections 4 and 5 provide concrete illustrations of the general results in a set of tractable applications. Section 6 concludes. All proofs and derivations are in the Appendix.

2 General Framework

This section lays out our general framework, which is broad enough to capture a wide range of scenarios, but sufficiently tractable to yield precise insights and highlight the channels at work. We consider an environment in which a group of agents (investors) make multiple financing and investment decisions that can be subject to a potentially rich set of regulations.

In this section, we assume that the decisions made by an investor directly induce externalities on others, providing a rationale for corrective regulation. In Sections 4 and 5, we provide concrete applications of our results, which illustrate how our general formulation encompasses various rationales for regulation.

2.1 Environment

There are two dates $t \in \{0, 1\}$ and a single consumption good, which serves as numeraire. At date 1, there is a continuum of possible states of nature $s \in S$, where $S = [\underline{s}, \overline{s}]$. The state s is a random variable with cumulative distribution function $F(s)$.

There are two sets of agents: investors and creditors. There is a finite number of investor types (investors, for short), with each type in unit measure and indexed by $i, j, \ell \in \mathcal{I}$, where $\mathcal{I} = \{1, 2, \ldots, |\mathcal{I}|\}$.[5] There is a unit measure of representative/identical creditors, indexed by C. Finally, there is also a social planner/regulator/government, who sets regulatory policy.

At date 0, investors have access to a set of financing opportunities, given by $\mathcal{B} = \{1, 2, \ldots, |\mathcal{B}|\}$, and a set of investment opportunities, given by $\mathcal{K} = \{1, 2, \ldots, |\mathcal{K}|\}$. We denote the financing and investment choices of investor i by $\boldsymbol{b}^i \in \mathbb{R}_+^{|\mathcal{B}|}$ and $\boldsymbol{k}^i \in \mathbb{R}_+^{|\mathcal{K}|}$, respectively. We collect the financing and investment choices/decisions/activities of an investor i in a vector

$$\boldsymbol{x}^i = \left(\boldsymbol{b}^i, \boldsymbol{k}^i\right).$$

When needed, we denote the investors' choice set by $\mathcal{X} = \mathcal{B} \cup \mathcal{K}$, so $|\mathcal{X}| = |\mathcal{B}| + |\mathcal{K}|$ and $\boldsymbol{x}^i \in \mathbb{R}_+^{|\mathcal{X}|}$. We emphasize that our approach does not depend on interpreting $\mathcal{X}$ through a financing/investment lens. We choose this formulation because it is directly applicable to the study of regulation in economies with financial frictions. However, our general results also apply in the context of classical consumer theory — see Online Appendix E.

At date 1, once s is realized, investors receive the return on their investments and pay back (fully or partially) their financial obligations. Creditors provide financing to investors at date 0 and receive (full or partial) repayments from investors at date 1. We define these repayments in detail below.

Investors. Investor i's preferences are of the form:

$$u^i \left(c_0^i, \left\{c_1^i(s)\right\}_{s \in S}, \left\{\overline{\boldsymbol{x}}^j\right\}_{j \in \mathcal{I}}\right), \tag{1}$$

where $u^i(\cdot)$ is a function of c_0^i and $c_1^i(s)$, which denote the consumption of investor i at date 0 and at date 1 in state s, as well as $\overline{\boldsymbol{x}}^j$, which denotes the balance-sheet choices of type j investors as a whole. In equilibrium, as explained below, it will be the case that $\boldsymbol{x}^j = \overline{\boldsymbol{x}}^j$, $\forall j \in \mathcal{I}$. Importantly, an individual type i investor, being infinitesimal, does not account for the impact on $\overline{\boldsymbol{x}}^i$ when choosing $\boldsymbol{x}^i$.

[5]The notion of investor used in this paper is meant to be understood broadly. Investors could be households, firms, or (financial) intermediaries. We could also have referred to investors as experts or entrepreneurs.

Investor i faces the following budget constraints:

$$c_0^i \leq n_0^i + Q^i\left(\boldsymbol{x}^i\right) - \Upsilon^i\left(\boldsymbol{x}^i\right) - \boldsymbol{\tau}^i \cdot \boldsymbol{x}^i + T_0^i \tag{2}$$

$$c_1^i(s) \leq n_1^i(s) + \rho_i\left(\boldsymbol{x}^i, s\right), \quad \forall s, \tag{3}$$

where we use $\cdot$ to denote the inner product between two vectors. At date 0, investor i is initially endowed with n_0^i dollars. We denote the amount of financing raised by investor i by $Q^i\left(\boldsymbol{x}^i\right)$, whose determination in equilibrium is described below. Moreover, the balance-sheet decisions made by investor i are associated with a cost $\Upsilon^i\left(\boldsymbol{x}^i\right) \geq 0$. This term can capture, among other forces, the technological adjustment costs associated with investment, or represent financing frictions.

Importantly, each investor faces investor-specific taxes/subsidies on balance-sheet decisions, via the vector $\boldsymbol{\tau}^i \in \mathbb{R}^{|\mathcal{X}|}$, $\forall i$. In Section 3, our main results consider alternative regulatory scenarios by imposing constraints on $\boldsymbol{\tau}^i$. Finally, investor i receives a lump-sum transfer $T_0^i \geq 0$ at date 0, as described below. Notice that, in principle, individual- and decision-specific regulation with $\boldsymbol{\tau}^i \neq \boldsymbol{\tau}^j$ is possible in this environment.

At date 1, investor i is endowed with $n_1^i(s)$ dollars when state s is realized. We denote the final return on the investments of investor i in state s, net of any financial obligations contained in the balance-sheet $\boldsymbol{x}^i$, by $\rho_i\left(\boldsymbol{x}^i, s\right)$. As shown in the Appendix, this general formulation of $\rho_i(\cdot)$ can accommodate the possibility of default by investors, as we also illustrate in our applications.

Creditors. Creditors' preferences are of the form:

$$u^C\left(c_0^C, \left\{c_1^C(s)\right\}_{s\in S}, \left\{\overline{\boldsymbol{x}}^j\right\}_{j\in\mathcal{I}}\right), \tag{4}$$

where $u^C(\cdot)$ is a function of c_0^C and $c_1^C(s)$, which denote the consumption of creditors at date 0 and at date 1 in state s, as well as $\overline{\boldsymbol{x}}^j$, which denotes the balance-sheet choices of type j investors as a whole.

Creditors face the following budget constraints:

$$c_0^C \leq n_0^C - \sum_{i\in\mathcal{I}} h_i^C Q^i\left(\overline{\boldsymbol{x}}^i\right) \tag{5}$$

$$c_1^C(s) \leq n_1^C(s) + \sum_{i\in\mathcal{I}} h_i^C \rho_i^C\left(\overline{\boldsymbol{x}}^i, s\right), \quad \forall s. \tag{6}$$

At date 0, creditors are initially endowed with n_0^C dollars. They choose to fund a share h_i^C of each investor i's financing needs $Q^i(\cdot)$, although, in equilibrium, $h_i^C = 1$, as we explain below. At date 1, when state s is realized, creditors are endowed with $n_1^C(s)$ dollars and receive repayments $\rho_i^C\left(\overline{\boldsymbol{x}}^i, s\right)$ from investor i. As we show in the Appendix and illustrate through our applications, this general formulation of $\rho_i^C(\cdot)$ can accommodate deadweight losses associated with the possibility of default by investors.

Regulation with imperfect instruments. As explained when introducing the investors' problem, the regulator has access to investor-specific taxes/subsidies on all balance-sheet decisions. Formally, the regulator controls the vector $\boldsymbol{\tau} \in \mathbb{R}^{|\mathcal{X}||\mathcal{I}|}$, given by stacking the investor-specific vectors $\boldsymbol{\tau}^i \in \mathbb{R}^{|\mathcal{X}|}$, as follows:

$$\boldsymbol{\tau} = \begin{pmatrix} \boldsymbol{\tau}^1 \\ \vdots \\ \boldsymbol{\tau}^i \\ \vdots \\ \boldsymbol{\tau}^{|\mathcal{I}|} \end{pmatrix}, \quad \text{where} \quad \boldsymbol{\tau}^i = \begin{pmatrix} \tau_1^i \\ \vdots \\ \tau_n^i \\ \vdots \\ \tau_{|\mathcal{X}|}^i \end{pmatrix}, \tag{7}$$

where τ_n^i denotes the regulation that directly affects the balance-sheet decision n of investor i.

Any revenue raised by the regulator is returned back to investors in the form of lump-sum transfers $\{T_0^i\}_{i \in \mathcal{I}}$, whose sum across investors must satisfy

$$\sum_{i \in \mathcal{I}} T_0^i = \sum_{i \in \mathcal{I}} \boldsymbol{\tau}^i \cdot \boldsymbol{x}^i. \tag{8}$$

Our results are valid for any policy $\{\boldsymbol{\tau}^i, T_0^i\}_{i \in \mathcal{I}}$ that satisfies Equation (8).[6] In particular, note that Equation (8) trivially holds when the set of transfers $\{T_0^i\}_{i \in \mathcal{I}}$ satisfies the more restrictive condition $T_0^i = \boldsymbol{\tau}^i \cdot \boldsymbol{x}^i$, $\forall i$. In this special case, any revenue raised from type i investors is returned to themselves, which allows us to interpret the choice of $\boldsymbol{\tau}$ as quantity regulation — see Section 4 for an example.

The main focus of this paper is on situations in which the set of instruments available to the regulator is imperfect. We flexibly model such imperfections by assuming that the regulator chooses taxes/subsidies $\boldsymbol{\tau}$ subject to $M \geq 0$ predetermined constraints, which we write as

$$\boldsymbol{\Phi}(\boldsymbol{\tau}) \leq 0,$$

where the vector-valued function $\boldsymbol{\Phi} : \mathbb{R}^{|\mathcal{X}||\mathcal{I}|} \to \mathbb{R}^M$ defines the set of feasible regulations. This general specification captures many scenarios of interest for regulators. For instance, when $\boldsymbol{\Phi}(\cdot) \equiv 0$, then the regulator is unconstrained and can achieve the first-best policy, which we characterize in Section 3.3.

Alternatively, it is natural to consider the case of linear constraints, in which $\boldsymbol{\Phi}(\boldsymbol{\tau})$ takes the form

$$\boldsymbol{\Phi}(\boldsymbol{\tau}) \equiv \boldsymbol{A}\boldsymbol{\tau} - \boldsymbol{c}, \tag{9}$$

where $\boldsymbol{A}$ denotes a matrix of dimension $M \times |\mathcal{X}| |\mathcal{I}|$ and $\boldsymbol{c}$ is an M-dimensional vector. As we formally study in Section 3.6, the linear constraint case captures second-best scenarios in which the

[6] If the regulator finds it optimal to implement subsidies, revenue raised and the associated lump-sum transfers can be negative. Note that the distribution of transfers $\{T_0^i\}_{i \in \mathcal{I}}$ can affect the actual optimal policy, however, it only does so through the sufficient statistics that we identify in Proposition 1.

regulator i) can only regulate the activities of a particular subset of investors, ii) can only regulate a specific subset of activities, iii) faces constraints on the level that some particular regulation must take, perhaps because it is set by a different authority, or iv) must impose uniform regulations across heterogeneous investors and activities, among others. These different scenarios simply correspond to different specifications of $\boldsymbol{A}$ and $\boldsymbol{c}$.

Even though, to simplify the exposition, we mostly focus on scenarios in which a regulator faces hard constraints, our results apply almost unchanged to the case in which imposing regulations on different agents and/or activities is subject to costs, as we describe in Section 3.4. In that case, it is natural to consider quadratic costs of adjusting regulations, which effectively correspond to assuming that $\boldsymbol{\Phi}(\boldsymbol{\tau})$ takes the form

$$\boldsymbol{\Phi}(\boldsymbol{\tau}) \equiv \frac{1}{2}\boldsymbol{\tau}'\boldsymbol{B}\boldsymbol{\tau}, \tag{10}$$

as we further describe in Section 3.6.3. In addition, it is worth noting that the case of unregulated activities, emphasized in some of our applications, can arise endogenously if regulation is subject to a cost function that induces sparsity, such as the L1 norm of the tax vector.[7]

Finally, it is worth highlighting that the main general results of this paper, in Propositions 1 through 4, are valid for any specification of constraints/costs $\boldsymbol{\Phi}(\cdot)$.

Equilibrium definition. An *equilibrium,* given corrective taxes/subsidies $\{\boldsymbol{\tau}^i\}_{i\in\mathcal{I}}$ and lump-sum transfers $\{T_0^i\}_{i\in\mathcal{I}}$, consists of consumption bundles $\{c_0^i, c_1^i(s)\}_{i\in\mathcal{I}}$ and $\left\{c_0^C, c_1^C(s)\right\}$, investors' decisions $\{\overline{\boldsymbol{x}}^i\}_{i\in\mathcal{I}} = \left\{\overline{\boldsymbol{b}}^i, \overline{\boldsymbol{k}}^i\right\}_{i\in\mathcal{I}}$, creditors' funding decisions $\left\{h_i^C\right\}_{i\in\mathcal{I}}$, financing schedules $\{Q^i(\boldsymbol{x}^i)\}_{i\in\mathcal{I}}$, investors' investment returns net of repayments $\{\rho_i(\boldsymbol{x}^i, s)\}_{i\in\mathcal{I}}$, and creditors' received repayments $\left\{\rho_i^C(\overline{\boldsymbol{x}}^i, s)\right\}_{i\in\mathcal{I}}$ given investors' default decisions such that i) investors maximize utility, Equation (1), subject to budget constraints (2) and (3); ii) creditors maximize utility, Equation (4), subject to budget constraints (5) and (6); iii) any revenue raised is transferred back to investors, satisfying Equation (8); iv) financing decisions satisfy market clearing so that $h_i^C = 1$, $\forall i$; and v) investors' balance-sheet decisions are consistent in the aggregate, that is, $\boldsymbol{x}^i = \overline{\boldsymbol{x}}^i$, $\forall i$.

Our notion of equilibrium, in which investors internalize that their balance-sheet decisions can affect their cost of financing in equilibrium, is standard in models that allow for default (e.g., Dubey, Geanakoplos and Shubik, 2005). Until we introduce our applications in Section 5, we proceed as if the environment considered here is well-behaved. We discuss the necessary regularity conditions for this to be the case within each of our applications.

Remarks. Before characterizing the equilibrium of the model, we conclude the description of the environment with three remarks.

First, assuming that the utility functions of investors or creditors depend directly on the choices of others — in Equations (1) and (4) — immediately justifies the desirability of corrective regulation. Given that the main insights of this paper do not rely on the exact rationale behind the corrective regulation, we adopt this formulation since it is the simplest. In Section 5, we show how our

[7]See Tibshirani (1996) and Gabaix (2014) for examples of cost functions based on the L1 norm in different contexts.

formulation encompasses widely studied rationales for regulation, including bailouts, pecuniary externalities, and internalities.

Second, note that we model investors and creditors as distinct groups of agents mostly for tractability. One can interpret creditors in our model as a type of investor who is only allowed to fund other investors, and does so without generating welfare-relevant externalities. Therefore, as we show in Section 3.3, it is sufficient to regulate the balance-sheet decisions of investors to reach the first-best outcome of the model.[8]

Finally, note that by suitably interpreting the utility of creditors, our model captures non-pecuniary benefits that may accrue to creditors from some particular form of financing. For instance, the liabilities of some investors are often seen as special, featuring a convenience yield, which is consistent with our framework — see, in related contexts, Stein (2012), Sunderam (2015), or Begenau and Landvoigt (2021).

2.2 Equilibrium Characterization

For given corrective taxes/subsidies and lump-sum transfers, we now succinctly characterize the equilibrium conditions of the model. First, we present the optimality conditions associated with creditors' optimal funding decisions, which are given by

$$Q^i\left(\boldsymbol{x}^i\right) = \int m^C(s)\, \rho_i^C\left(\boldsymbol{x}^i, s\right) dF(s)\,, \ \forall i, \tag{11}$$

where $m^C(s)$ denotes creditors' stochastic discount factor. Equation (11), which is an Euler equation for creditors, will determine the financing conditions that investors face. Note that the stochastic discount factor $m^C(s)$ is an equilibrium object, which depends on the choices of all investors in the model and the regulatory policy. Hence, regulating an investor j impacts investor i in equilibrium through $Q^i(\boldsymbol{x}^i)$, via changes in creditors' stochastic discount factor.

Next, we present the optimality conditions associated with investors' optimal balance-sheet decisions, which are given by

$$-\frac{\partial Q^i(\boldsymbol{x}^i)}{\partial \boldsymbol{x}^i} + \frac{\partial \Upsilon^i(\boldsymbol{x}^i)}{\partial \boldsymbol{x}^i} + \boldsymbol{\tau}^i = \int m^i(s)\, \frac{\partial \rho_i(\boldsymbol{x}^i, s)}{\partial \boldsymbol{x}^i} dF(s)\,, \ \forall i, \tag{12}$$

where $m^i(s)$ denotes the stochastic discount factor of investor i. Note that Equation (12) represents the $|\mathcal{X}|$ optimality conditions that determine the optimal balance-sheet of investors. These conditions are Euler equations for both financing and investment. Given Equations (11) and (12), which fully characterize the equilibrium of the model once $\boldsymbol{x}^i = \overline{\boldsymbol{x}}^i$, we can now study the optimal corrective regulation.

[8]In an earlier version of this paper, we allowed for investors to invest in each other's liabilities and for creditors' decisions to also be associated with welfare-relevant externalities. Since the main insights are identical in both formulations, we adopt the current formulation, which substantially simplifies the notation.

3 Optimal Corrective Regulation

In this section, which contains the main contributions of this paper, we study the problem of a planner who can set the optimal corrective regulation under different constraints on the set of regulatory instruments $\boldsymbol{\tau}$. First, we provide a general characterization of the marginal welfare effect of adjusting corrective regulation. Subsequently, we characterize the optimal first-best and second-best regulations. In Subsection 3.1, we preemptively introduce the notation and definitions necessary to formulate our results.

3.1 Notation and Definitions

We denote by $\{V^i(\boldsymbol{\tau})\}_{i\in\mathcal{I}}$ and $V^C(\boldsymbol{\tau})$ the indirect utilities of investors and creditors, as a function of the full set of regulatory instruments $\boldsymbol{\tau} \in \mathbb{R}^{|\mathcal{X}||\mathcal{I}|}$. In order to abstract from redistributional concerns and focus on the corrective nature of the regulation, we assess the aggregate welfare gains/losses of a marginal change in regulation by aggregating money-metric utility changes across all agents. This approach can be interpreted as selecting equal-weighted generalized social marginal welfare weights, using the terminology of Saez and Stantcheva (2016). In the Appendix, we describe how to allow for traditional social welfare weights and how to account for redistributional considerations in our framework.

Formally, we express the change in social welfare induced by a marginal change in a given variable (or vector) z, denoted by $\frac{dW}{dz}$, as follows:

$$\frac{dW}{dz} = \sum_{i\in\mathcal{I}} \frac{dV_m^i}{dz} + \frac{dV_m^C}{dz}, \tag{13}$$

where $\frac{dV_m^i}{dz} = \frac{dV^i}{dz}/\lambda_0^i$ and $\frac{dV_m^C}{dz} = \frac{dV^C}{dz}/\lambda_0^C$ denote the money-metric change in indirect utility for investors and creditors, and where λ_0^i and λ_0^C denote the marginal value of a dollar at date 0 for investors and creditors.[9] In particular, we will characterize the *marginal welfare effect* of varying the set of balance-sheet regulations $\boldsymbol{\tau}$, given by

$$\frac{dW}{d\boldsymbol{\tau}} = \begin{pmatrix} \frac{dW}{d\boldsymbol{\tau}^1} \\ \vdots \\ \frac{dW}{d\boldsymbol{\tau}^j} \\ \vdots \\ \frac{dW}{d\boldsymbol{\tau}^{|\mathcal{I}|}} \end{pmatrix}, \quad \text{where} \quad \frac{dW}{d\boldsymbol{\tau}^j} = \begin{pmatrix} \frac{dW}{d\tau_1^j} \\ \vdots \\ \frac{dW}{d\tau_n^j} \\ \vdots \\ \frac{dW}{d\tau_{|\mathcal{X}|}^j} \end{pmatrix}, \tag{14}$$

and where $\frac{dW}{d\boldsymbol{\tau}} \in \mathbb{R}^{|\mathcal{X}||\mathcal{I}|}$ and $\frac{dW}{d\boldsymbol{\tau}^j} \in \mathbb{R}^{|\mathcal{X}|}$. Each element of $\frac{dW}{d\boldsymbol{\tau}^j}$ denotes the marginal welfare effect of varying the regulation that investor j faces. By vertically stacking $\frac{dW}{d\boldsymbol{\tau}^j}$, we collect the set of

[9] We use the same notation for partial derivatives, that is, $\frac{\partial V_m^i}{\partial z} = \frac{\frac{\partial V^i}{\partial z}}{\lambda_0^i}$ and $\frac{\partial V_m^C}{\partial z} = \frac{\frac{\partial V^C}{\partial z}}{\lambda_0^C}$. The sub-index m stands for money-metric.

marginal welfare effects associated with varying each of the elements of $\boldsymbol{\tau}$ in the vector $\frac{dW}{d\boldsymbol{\tau}}$.

We also define the vectors of investors' balance-sheets $\boldsymbol{x} \in \mathbb{R}^{|\mathcal{X}||\mathcal{I}|}$ and *marginal distortions/externalities* $\boldsymbol{\delta} \in \mathbb{R}^{|\mathcal{X}||\mathcal{I}|}$, given by stacking the vectors $\boldsymbol{x}^i \in \mathbb{R}^{|\mathcal{X}|}$ and $\boldsymbol{\delta}^i \in \mathbb{R}^{|\mathcal{X}|}$, as follows:

$$\boldsymbol{x} = \begin{pmatrix} \boldsymbol{x}^1 \\ \vdots \\ \boldsymbol{x}^i \\ \vdots \\ \boldsymbol{x}^{|\mathcal{I}|} \end{pmatrix} \text{ and } \boldsymbol{\delta} = \begin{pmatrix} \boldsymbol{\delta}^1 \\ \vdots \\ \boldsymbol{\delta}^i \\ \vdots \\ \boldsymbol{\delta}^{|\mathcal{I}|} \end{pmatrix}, \quad \text{where} \quad \boldsymbol{x}^i = \begin{pmatrix} x_1^i \\ \vdots \\ x_n^i \\ \vdots \\ x_{|\mathcal{X}|}^i \end{pmatrix} \text{ and } \boldsymbol{\delta}^i = \begin{pmatrix} \delta_1^i \\ \vdots \\ \delta_n^i \\ \vdots \\ \delta_{|\mathcal{X}|}^i \end{pmatrix}, \tag{15}$$

where x_n^i denotes the balance-sheet decision n of investor i and δ_n^i corresponds to the money-metric aggregate of marginal externalities associated with balance-sheet decision n of investor i, given by

$$\delta_n^i = -\left(\sum_{\ell \in \mathcal{I}} \frac{1}{\lambda_0^\ell} \frac{\partial u^\ell}{\partial \overline{x}_n^i} + \frac{1}{\lambda_0^C} \frac{\partial u^C}{\partial \overline{x}_n^i}\right). \tag{16}$$

Note that an activity generates negative externalities when $\frac{\partial u^\ell}{\partial \overline{x}_n^i}$ or $\frac{\partial u^C}{\partial \overline{x}_n^i}$ is negative, making δ_n^i positive. Conversely, an activity generates positive externalities when $\frac{\partial u^\ell}{\partial \overline{x}_n^i}$ or $\frac{\partial u^C}{\partial \overline{x}_n^i}$ is positive, making δ_n^i negative.

We define the Jacobian matrix of investors' balance-sheets $\boldsymbol{x}$ with respect to $\boldsymbol{\tau}$, of dimension $|\mathcal{X}|\,|\mathcal{I}| \times |\mathcal{X}|\,|\mathcal{I}|$, as follows:

$$\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}} = \begin{pmatrix} \frac{d\boldsymbol{x}^1}{d\boldsymbol{\tau}^1} & \cdots & \frac{d\boldsymbol{x}^{|\mathcal{I}|}}{d\boldsymbol{\tau}^1} \\ \vdots & \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} & \vdots \\ \frac{d\boldsymbol{x}^1}{d\boldsymbol{\tau}^{|\mathcal{I}|}} & \cdots & \frac{d\boldsymbol{x}^{|\mathcal{I}|}}{d\boldsymbol{\tau}^{|\mathcal{I}|}} \end{pmatrix}, \quad \text{where} \quad \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} = \begin{pmatrix} \frac{dx_1^i}{d\tau_1^j} & \cdots & \frac{dx_{|\mathcal{X}|}^i}{d\tau_1^j} \\ \vdots & \frac{dx_n^i}{d\tau_{n'}^j} & \vdots \\ \frac{dx_1^i}{d\tau_{|\mathcal{X}|}^j} & \cdots & \frac{dx_{|\mathcal{X}|}^i}{d\tau_{|\mathcal{X}|}^j} \end{pmatrix}, \tag{17}$$

where $\frac{dx_n^i}{d\tau_{n'}^j}$ denotes how the balance-sheet decision n of investor i changes when regulating the balance-sheet decision n' of investor j. Following Hendren (2016), we refer to the elements of $\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}$, which represent the equilibrium responses of balance-sheets $\boldsymbol{x}$ to changes in regulation $\boldsymbol{\tau}$, as *policy elasticities*.

Finally, we define *Pigouvian wedges* $\boldsymbol{\omega} \in \mathbb{R}^{|\mathcal{X}||\mathcal{I}|}$ between corrective regulations $\boldsymbol{\tau}$ and marginal distortions $\boldsymbol{\delta}$ as follows:

$$\boldsymbol{\omega} = \boldsymbol{\tau} - \boldsymbol{\delta}. \tag{18}$$

As we show in Proposition 2, Pigouvian wedges are zero at the first-best, so they define the distance between a given set of regulations and the first-best regulation. Outside of the first-best, Pigouvian wedges can be positive or negative. If a wedge ω_n^i is positive, then decision n of investor i is overregulated, in the sense that it is welfare-improving to increase the level of the associated x_n^i.

Alternatively, if a wedge ω_n^i is negative, the balance-sheet decision n of investor i is underregulated, in the sense that it is welfare-improving to reduce the level of the associated x_n^i. Therefore, we say that:

$$\omega_n^i > 0 \Rightarrow \text{Overregulation} \left(\text{increasing } x_n^i \text{ is welfare-improving}\right)$$
$$\omega_n^i < 0 \Rightarrow \text{Underregulation} \left(\text{decreasing } x_n^i \text{ is welfare-improving}\right).$$

Our results below demonstrate that both over- and underregulation can arise as part of the optimal second-best policy, depending on the nature of the constraints faced by the planner.

3.2 Marginal Welfare Effects of Corrective Regulation

Given these definitions, we are ready to present Proposition 1, which characterizes the marginal welfare effects of varying the set of balance-sheet regulations, $\frac{dW}{d\boldsymbol{\tau}}$. Proposition 1 highlights that $\frac{dW}{d\boldsymbol{\tau}}$ can be characterized in terms of two sets of sufficient statistics: policy elasticities and Pigouvian wedges.

Proposition 1. *[Marginal Welfare Effects of Corrective Regulation: Policy Elasticities and Pigouvian Wedges] The marginal welfare effects of varying the set of balance-sheet regulations* $\boldsymbol{\tau}$, $\frac{dW}{d\boldsymbol{\tau}}$, *are given by*

$$\frac{dW}{d\boldsymbol{\tau}} = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}(\boldsymbol{\tau} - \boldsymbol{\delta}) = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}\boldsymbol{\omega}, \tag{19}$$

where $\frac{dW}{d\boldsymbol{\tau}}$ *is a vector of dimension* $|\mathcal{X}|\,|\mathcal{I}| \times 1$, *defined in Equation* (14); $\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}$ *is the square Jacobian matrix of policy elasticities of dimension* $|\mathcal{X}|\,|\mathcal{I}| \times |\mathcal{X}|\,|\mathcal{I}|$, *defined in Equation* (17); *and* $\boldsymbol{\tau}$ *and* $\boldsymbol{\delta}$ *are vectors of dimension* $|\mathcal{X}|\,|\mathcal{I}| \times 1$, *where the vector of regulations* $\boldsymbol{\tau}$ *is defined in Equation* (7) *and the vector of marginal distortions* $\boldsymbol{\delta}$ *is defined in Equation* (15). *Therefore, the marginal welfare effects* $\frac{dW}{d\boldsymbol{\tau}}$ *can be exclusively characterized in terms of two sets of sufficient statistics: policy elasticities,* $\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}$, *and Pigouvian wedges,* $\boldsymbol{\omega}$.[10]

Proposition 1 shows that in order to characterize the welfare impact of any change in regulation it is sufficient to understand i) how the decisions of all investors react in equilibrium to such a change, via the matrix of policy elasticities, and ii) the size of the marginal uncorrected externalities associated with each individual balance-sheet decisions, via the vector of Pigouvian wedges.[11]

The first set of sufficient statistics are the *policy elasticities*, $\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}$, defined in Equation (17), which capture the general equilibrium responses of the balance-sheet decisions of investor i to changes in

[10] Note that Equation (19) implies that the marginal welfare effects of varying the balance-sheet regulations that directly affect agent j can also be expressed as follows:

$$\frac{dW}{d\boldsymbol{\tau}^j} = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}^j}\boldsymbol{\omega} = \sum_{i\in\mathcal{I}} \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\boldsymbol{\omega}^i = \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{X}} \frac{dx_n^i}{d\boldsymbol{\tau}^j}\left(\tau_n^i - \delta_n^i\right).$$

[11] Note that the planner accounts for the welfare impact of policy changes on equilibrium prices. As we show in the Appendix, the impact of changes in equilibrium prices is zero-sum on aggregate. Using the language of Dávila and Korinek (2018), the distributive pecuniary impact of a policy change nets out on aggregate, which simplifies the characterization of $\frac{dW}{d\boldsymbol{\tau}}$.

the regulation that affect every other investor. Note that policy elasticities both across investors, e.g., $\frac{dx_n^i}{d\tau_n^j}$, and across balance-sheet activities within the same investor, e.g., $\frac{dx_n^i}{d\tau_{n'}^i}$, are relevant. Equation (19) implies that, for a given Pigouvian wedge, whether balance-sheet activities are gross substitutes ($\frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} > 0$) or gross complements ($\frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} < 0$) becomes critical to determine the welfare impact of policy changes. The distinction between substitutes and complements is central to the design of optimal second-best regulation.

The second set of sufficient statistics are the *Pigouvian wedges* between corrective taxes/subsidies and marginal distortions. For any given set of regulations, these wedges capture the extent to which different balance-sheet activities are regulated too strictly or too leniently. For example, if $\omega_n^i = \tau_n^i - \delta_n^i < (>)\, 0$, then the corrective regulation on the balance-sheet activity n of investor i is smaller (larger) than the marginal distortion that this activity creates. Therefore, negative wedges imply that the private marginal cost of an activity is smaller than social marginal cost, while positive wedges imply that the private marginal cost exceeds the social marginal cost.

Moreover, Equation (19) shows that the marginal welfare effects of any regulatory change can be interpreted as a linear transformation of wedges, with the matrix $\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}$ of policy elasticities acting as a transformation matrix. Intuitively, welfare will increase if a policy reform discourages (encourages) activities, e.g., $\frac{dx_n^i}{d\tau_n^j} < (>)\, 0$, that are currently regulated too leniently (strongly), e.g., $\omega_n^i < (>)\, 0$, or vice versa. The overall marginal welfare effect corresponds to adding up the products of leakage elasticities and Pigouvian wedges.

The marginal welfare effects presented in Proposition 1 are useful to characterize the form of the optimal regulation in alternative regulatory scenarios. In the remainder of this section, we show how to employ Proposition 1 to characterize the optimal first-best and second-best regulations.

3.3 First-Best Regulation: Benchmark

Under the first-best regulation, a planner is allowed to set arbitrary corrective regulations $\boldsymbol{\tau} \in \mathbb{R}^{|\mathcal{X}||\mathcal{I}|}$ for all investors. Note that we use the term first-best regulation to refer to the benchmark in which a planner can freely correct every individual decision, while also respecting individual/technological constraints.[12] In that case, Proposition 2 provides a well-known characterization of the first-best policy, which provides a benchmark against which we evaluate the optimal second-best policy.

Proposition 2. *[First-Best Regulation/Pigouvian Principle] If the planner can freely regulate all investors without constraints, and the matrix of policy elasticities has full rank, then the first-best regulation satisfies:*

$$\boldsymbol{\omega} = 0 \iff \boldsymbol{\tau}^\star = \boldsymbol{\delta}.$$

[12] We could have alternatively refer to this benchmark as the constrained efficient benchmark (Diamond, 1967; Hart, 1975; Geanakoplos and Polemarchakis, 1986). We avoid using the terminology constrained efficient to avoid confusion between the use of the term constrained in constrained efficiency, which refers to individual/technological constraints, and the focus of our paper, which are constraints on the planner's instruments.

Therefore, the first-best regulation does not depend directly on the magnitude of the policy elasticities.

This is an instance of the Pigouvian principle, i.e., the "polluter pays" (Pigou, 1920; Sandmo, 1975).[13] The first-best regulation on investors is set to perfectly align private and social incentives across every activity undertaken by each agent. In terms of the Pigouvian wedges defined in Equation (18), the optimal regulation is such that all wedges are set to zero. Proposition 2 directly implies that an economy without externalities, i.e., one for which $\frac{\partial u^j}{\partial x_n^i} = 0$ and $\frac{\partial u^C}{\partial x_n^i} = 0$, $\forall i, j \in \mathcal{I}$, is efficient.

An important consequence of Proposition 2 is that the first-best regulation does not directly depend on the magnitude of the policy elasticities. It is exclusively a function of the Pigouvian wedges. Intuitively, the first-best regulation must satisfy $\frac{dW}{d\boldsymbol{\tau}} = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}\boldsymbol{\omega} = 0$, which defines a system of homogeneous linear equations in $\boldsymbol{\omega}$. If the matrix of policy elasticities $\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}$ is invertible (i.e., has full rank), then the only solution to this system is the trivial solution, in which $\boldsymbol{\omega} = 0$ and $\boldsymbol{\tau}^{\star} = \boldsymbol{\delta}$.

Importantly, while Proposition 2 characterizes the optimal first-best regulation, it does not provide a solution in terms of primitives unless the marginal distortions $\boldsymbol{\delta}$ are invariant to the level of regulation (this will be the case in several of our applications). Whenever the marginal distortions are endogenous to the level of the regulation, our claims here become statements about the form of the optimal policy formulas. The same caveat applies to our discussions of Propositions 3 through 4.

3.4 Second-Best Regulation

Now we consider scenarios in which the planner faces a set of predetermined constraints on the set of instruments $\boldsymbol{\tau}$, providing a novel general characterization of the optimal second-best policy. Formally, the optimal second-best policy is given by

$$\boldsymbol{\tau}^{\star\star} = \arg\max_{\boldsymbol{\tau}} W(\boldsymbol{\tau}) \quad \text{s.t.} \quad \boldsymbol{\Phi}(\boldsymbol{\tau}) \leq 0,$$

where the vector-valued function $\boldsymbol{\Phi}(\cdot) : \mathbb{R}^{|\mathcal{X}||\mathcal{I}|} \to \mathbb{R}^M$ defines a set of $M \geq 0$ constraints on the set of instruments $\boldsymbol{\tau}$. This general specification of $\boldsymbol{\Phi}(\cdot)$ allows us to consider a wide range of regulatory constraints, which we further describe in Section 3.6. For instance, as discussed above, when the planner cannot regulate agent j, the appropriate constraints are $\tau_n^j = 0$, $\forall n \in \mathcal{X}$. Similarly, when the planner cannot regulate a particular activity n, the appropriate constraints are $\tau_n^i = 0$, $\forall i \in \mathcal{I}$. Alternatively, when all agents are regulated at the same rate or when all activities are regulated at the same rate, the appropriate constraints are $\tau_n^i = \overline{\tau}_n$, $\forall i \in \mathcal{I}$, or $\tau_n^i = \overline{\tau}^i$, $\forall n \in \mathcal{X}$. Many other scenarios of practical relevance can be interpreted as combinations of these.

[13]It is common to also refer to the Pigouvian principle as the principle of targeting, see e.g., Dixit (1985); Rothschild and Scheuer (2016).

Consequently, the second-best regulation must satisfy

$$\frac{dW}{d\boldsymbol{\tau}} - \frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}}\boldsymbol{\mu} = 0, \tag{20}$$

where $\frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}}$ denotes the Jacobian of the constraints — a matrix of dimension $|\mathcal{X}|\,|\mathcal{I}| \times M$ — and where $\boldsymbol{\mu} \in \mathbb{R}^M$ denotes the vector of Lagrange multipliers associated with the constraints, formally given by

$$\frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}} = \begin{pmatrix} \frac{d\Phi^1}{d\boldsymbol{\tau}^1} & \cdots & \frac{d\Phi^M}{d\boldsymbol{\tau}^1} \\ \vdots & \frac{d\Phi^m}{d\boldsymbol{\tau}^j} & \vdots \\ \frac{d\Phi^1}{d\boldsymbol{\tau}^{|\mathcal{I}|}} & \cdots & \frac{d\Phi^M}{d\boldsymbol{\tau}^{|\mathcal{I}|}} \end{pmatrix} \quad \text{where} \quad \frac{d\Phi^m}{d\boldsymbol{\tau}^j} = \begin{pmatrix} \frac{d\Phi^m}{d\tau_1^j} \\ \vdots \\ \frac{d\Phi^m}{d\tau_{|\mathcal{X}|}^j} \end{pmatrix}, \quad \text{and} \quad \boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_M \end{pmatrix}.$$

At this point, we make a distinction between i) perfectly regulated decisions and ii) imperfectly regulated decisions. We say that a balance-sheet decision of a given investor is *perfectly regulated* when all constraints associated with that decision are slack, and *imperfectly regulated* when its regulation is subject to a binding constraint. Formally, we denote the mutually exclusive sets of perfectly regulated ($\mathcal{R}$) and imperfectly regulated ($\mathcal{U}$) decisions by[14]

$$\begin{aligned} \mathcal{R} &= \left\{(j,n) : j \in \mathcal{I}, n \in \mathcal{X}, \eta_n^j = 0\right\} \;\Rightarrow\; \text{Perfectly Regulated}, \\ \mathcal{U} &= \left\{(j,n) : j \in \mathcal{I}, n \in \mathcal{X}, \eta_n^j \neq 0\right\} \;\Rightarrow\; \text{Imperfectly Regulated}, \end{aligned} \tag{21}$$

where $\boldsymbol{\eta} \in \mathbb{R}^{|\mathcal{X}||\mathcal{I}|}$ is defined as the $|\mathcal{X}|\,|\mathcal{I}| \times 1$ vector

$$\boldsymbol{\eta} = \frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}}\boldsymbol{\mu}.$$

The vector $\boldsymbol{\eta}$ quantifies, for each regulatory instrument in $\boldsymbol{\tau}$, the shadow cost of increasing the regulation associated with the regulatory constraints. Notice that, because we have a general specification of constraints, the shadow costs in $\boldsymbol{\eta}$ can be negative. For example, if one of the constraints embedded in $\boldsymbol{\Phi}(\boldsymbol{\tau})$ imposes a binding lower bound on a tax/subsidy τ_n^j, then the corresponding shadow cost is $\eta_n^j < 0$. It is important not to confuse this property of $\boldsymbol{\eta}$ with the Lagrange multipliers $\boldsymbol{\mu}$, which must be non-negative when appropriately defined.

Accordingly, we define the values of the perfectly and imperfectly regulated decisions by $\boldsymbol{x}^R = \{x_n^j\}_{(j,n)\in\mathcal{R}}$ and $\boldsymbol{x}^U = \{x_n^j\}_{(j,n)\in\mathcal{U}}$, and similarly partition other vectors such as $\boldsymbol{\tau} = \left\{\boldsymbol{\tau}^R, \boldsymbol{\tau}^U\right\}$, $\boldsymbol{\delta} = \left\{\boldsymbol{\delta}^R, \boldsymbol{\delta}^U\right\}$, and $\boldsymbol{\omega} = \left\{\boldsymbol{\omega}^R, \boldsymbol{\omega}^U\right\}$. The Jacobian matrix $\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}$ of policy elasticities, introduced in Equation (17), can also be decomposed into smaller Jacobian matrices: $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}$, $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}$, $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}$, and $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$, as described in the Appendix. For the remainder of this section, we will assume that the matrices of own-regulatory effects $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}$ and $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$ are invertible.

In this paper, we introduce the notion of *leakage elasticities* to refer to the elements of the Jacobian matrix $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}$, which capture the responses of imperfectly regulated decisions to changes

[14] We choose $\mathcal{U}$ to denote the set of imperfectly regulated decisions since "unregulated" decisions are a leading case of imperfectly regulated decisions.

in regulation. Below, we will refer to the elements of the Jacobian matrix $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}$ as reverse leakage elasticities. In Proposition 3, we show that leakage elasticities are a key determinant of the second-best policy.

Proposition 3. *[Second-Best Regulation: Perfectly Regulated Decisions] The optimal second-best regulation satisfies*

$$\boldsymbol{\tau}^R = \boldsymbol{\delta}^R + \left(-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1} \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\boldsymbol{\omega}^U, \tag{22}$$

where $\boldsymbol{\delta}^R$ is a vector of distortions of dimension $|\mathcal{R}| \times 1$, $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$ and $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}$ are Jacobian matrices of dimension $|\mathcal{R}| \times |\mathcal{R}|$ and $|\mathcal{R}| \times |\mathcal{U}|$, respectively, and $\boldsymbol{\omega}^U = \boldsymbol{\tau}^U - \boldsymbol{\delta}^U$ is a vector of Pigouvian wedges of dimension $|\mathcal{U}| \times 1$. Therefore, the optimal second-best regulation only depends directly on a subset of all policy elasticities: $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$ and, importantly, $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}$ (leakage elasticities).

Proposition 3 provides direct insights into the form of the optimal second-best policy. Since the first-best solution is given by $\boldsymbol{\tau}^R = \boldsymbol{\delta}^R$, whether the optimal second-best policy overregulates or underregulates perfectly regulated decisions is determined by the sign of $-\left(\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1} \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\boldsymbol{\omega}^U$, which we refer to as the second-best correction.[15] First, we provide a heuristic interpretation of the general characterization in Equation (22), which explains the most relevant economic effects. Next, we provide formal insights in the context of two illustrative examples.

At a heuristic level, as long as perfectly regulated activities decrease when their regulation is tightened ($\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$ is "negative"), the sign of the second-best correction becomes a product of the leakage elasticities and the Pigouvian wedges of imperfectly regulated choices. Under the natural presumption that the constraints are such that imperfectly regulated activities are indeed underregulated ($\boldsymbol{\tau}^U < \boldsymbol{\delta}^U$ or, equivalently, $\boldsymbol{\omega}^U < 0$), whether the optimal second-best policy overregulates or underregulates an activity becomes a function of whether such an activity is a gross substitute or a gross complement with respect to imperfectly regulated decisions. Therefore, it is optimal to underregulate the regulated relative to the first-best ($\boldsymbol{\omega}^R < 0$), when regulated and unregulated are gross substitutes ($\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} > 0$). Alternatively, it is optimal to overregulate the regulated relative to the first-best ($\boldsymbol{\omega}^R > 0$) when regulated and unregulated are gross complements ($\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} < 0$). Indeed, our applications below demonstrate that both gross substitutes and gross complements are common in standard scenarios, depending on which activities are imperfectly regulated.

To provide formal insights, it is useful to study two special cases. First, we consider a scenario in which there is a single fully regulated decision. Second, we consider a scenario in which the responses of perfectly regulated activities to changes in regulation are independent of one another. In both cases, the formulas for second-best regulation simplify because we do not have to account

[15] As explained when describing the first-best regulation, Equation (22) does not characterize the optimal second-best regulation in terms of primitives. When the set of marginal distortions $\boldsymbol{\delta} = \left\{\boldsymbol{\delta}^R, \boldsymbol{\delta}^U\right\}$ is invariant to the level of regulation, any statement on whether the second-best policy overregulates or underregulates a decision relative to the first-best is an exact directional statement — this will be the case in several of our applications. Whenever the marginal distortions are endogenous to the level of the regulation, our claims here become statements about the form of the optimal policy formulas.

for the responses of different regulated activities on one another.

Example 1. [Single Decision] Consider the simple scenario in which there are two investors $|\mathcal{I}| = 2$, and each investor has a single decision $|\mathcal{X}| = 1$. Assume that only investor 1 can be regulated, with regulatory constraints dictating that $\tau^2 \equiv 0$. In that case, it follows from Proposition 3 that the optimal regulation for the regulated is simply given by

$$\tau^1 = \delta^1 - \left(-\frac{dx^1}{d\tau^1}\right)^{-1} \frac{dx^2}{d\tau^1}\delta^2. \tag{23}$$

This case clearly shows the relationships discussed above. The optimal regulation on investor type 1 is equal to the first-best equivalent δ^1 minus a correction that accounts for the distortion imposed by the other unregulated agent. Assume, for instance, that the distortion by the unregulated agent satisfies $\delta^2 > 0$. The weight on the distortion by the unregulated agent is negative, implying that it pushes τ^1 towards underregulation, whenever i) the regulated agent responds negatively to increased regulation (the "regular" case with $\frac{dx^1}{d\tau^1} < 0$), and ii) the associated leakage elasticity indicates gross substitutes with $\frac{dx^2}{d\tau^1} > 0$.[16]

Example 2. [Diagonal Case] Assume that $\frac{dx^R}{d\tau^R}$ is a diagonal matrix. Then, the second-best regulation on choice $(j, n) \in \mathcal{R}$ is

$$\tau_n^j = \delta_n^j + \left(-\frac{dx_n^j}{d\tau_n^j}\right)^{-1} \sum_{(j',n')\in\mathcal{U}} \frac{dx_{n'}^{j'}}{d\tau_n^j}\omega_{n'}^{j'}.$$

The simplified formula again shows the importance of leakage elasticities, which are weighted by wedges and summed across all unregulated activities $(j', n') \in \mathcal{U}$. It is clear in this case that it is optimal to underregulate the regulated ($\tau_n^j < \delta_n^j$) if each of the imperfectly regulated activities is underregulated ($\omega_{n'}^{j'} < 0$) and is a gross substitute to the regulated activity ($\frac{dx_{n'}^{j'}}{d\tau_n^j} > 0$). In addition, the formula shows that, even when not every activity satisfies gross substitutes, it is optimal to underregulate the regulated when a weighted average of leakage elasticities — with the weights proportional to the associated wedges — is positive.

We conclude the discussion of Proposition 3 with two remarks. Our first remark explains how our characterization of the optimal second-best policy relates to the classic results in Lipsey and Lancaster (1956). Our second remark highlights the duality between considering regulatory costs or constraints.

[16] While this is the simplest example for building intuition, note that the same insight extends to any economy with a single regulated decision $\mathcal{R} = \{(j', n')\}$, and with an arbitrary set of unregulated decisions $\mathcal{U}$ for which taxes/subsidies are forced to be zero. In this more general example, the optimal policy formula becomes

$$\tau^R = \delta^R - \sum_{(j,n)\in\mathcal{U}} \left(-\frac{dx^R}{d\tau^R}\right)^{-1} \frac{dx_n^j}{d\tau^R}\delta_n^j.$$

Remark 1. *[Connection to Lipsey and Lancaster (1956)]* Equation (22) immediately implies that $\boldsymbol{\omega}^R \neq 0$ as long as $\boldsymbol{\omega}^U \neq 0$ and $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} \neq 0$. This insight is related to the discussion of second-best policy in Lipsey and Lancaster (1956), who show that price distortions for one good imply optimal distortions in other goods in the context of second-best tariff and monopoly regulation. The results of that paper are often perceived as implying that there is little structure to the problem of the second-best.[17] While it is true that over and underregulation relative to the first-best are possible, our results show that there is significant structure on how the optimal second-best regulation must be conducted. Our formal general results show that leakage elasticities and the distinction between gross complements and substitutes are critical to determine the optimal second-best regulation. Lipsey and Lancaster (1956) are also often credited with the insight that social welfare can decrease when relaxing a constraint. We revisit this argument after Proposition 4, which comes next and characterizes the welfare effects of relaxing regulatory constraints.

Remark 2. *[Regulatory constraints/costly regulation: dual interpretation]* It is worth highlighting that Proposition 3 applies unchanged to the case in which imposing regulations to a subset of agents and/or activities is costly for a planner, even when there are no hard regulatory constraints. In that case, the vector $\boldsymbol{\mu}$ is simply a primitive of the planning problem, instead of having the interpretation of a multiplier, while the definitions of the sets $\mathcal{R}$ and $\mathcal{U}$ in Equations (21) and (22) remain unchanged.

3.5 Imperfectly Regulated Decisions/Welfare Effects of Relaxing Regulatory Constraints

Next, we proceed to characterize the marginal welfare impact of relaxing regulatory constraints or, equivalently, the shadow value of regulating imperfectly regulated choices $\boldsymbol{x}^U$, under the optimal second-best regulation. This result is important for two reasons. First, a regulator may genuinely be interested in understanding the welfare effect of extending the scope of institution- or activity-level regulation. Second, our characterization of $\frac{dW}{d\boldsymbol{\tau}^U}$ under the optimal second-best regulation is informative to understanding how the set of imperfectly regulated decisions ($\mathcal{U}$) is optimally regulated. Formally, in the case of regulatory constraints, the optimality conditions $\frac{dW}{d\boldsymbol{\tau}^U} = \frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}^U}\boldsymbol{\mu}$ along with $\boldsymbol{\Phi}\left(\boldsymbol{\tau}^U\right) = 0$ can be used to characterize $\boldsymbol{\tau}^U$ and $\boldsymbol{\mu}$ (see, e.g., Section 3.6.2). In the case of regulatory costs, $\frac{dW}{d\boldsymbol{\tau}^U} = \frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}^U}$ is sufficient to characterize $\boldsymbol{\tau}^U$ (see, e.g., Section 3.6.3).

Proposition 4 evaluates the marginal welfare effects of hypothetical (i.e., unconstrained) adjustments to the constrained taxes/subsidies $\boldsymbol{\tau}^U$ under the optimal second-best regulation.

Proposition 4. *[Second-best Regulation: Imperfectly Regulated Decisions/Welfare Effects of Relaxing Regulatory Constraints] The marginal welfare effects of regulating the set of imperfectly*

[17] Lipsey and Lancaster (1956) explicitly write:

> *"It is important to note that in general, nothing can be said about the direction or the magnitude of the secondary departures from optimum conditions made necessary by the original non-fulfillment of one condition".*

regulated decisions, $\boldsymbol{\tau}^U$, under the optimal second-best regulation, are given by

$$\frac{dW}{d\boldsymbol{\tau}^U} = \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I} - \boldsymbol{L}\right)\boldsymbol{\omega}^U, \tag{24}$$

where $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}$ is a Jacobian matrix of dimension $|\mathcal{U}| \times |\mathcal{U}|$, $\boldsymbol{I}$ is the identity matrix of dimension $|\mathcal{U}| \times |\mathcal{U}|$, $\boldsymbol{L}$ is a matrix of dimension $|\mathcal{U}| \times |\mathcal{U}|$, given by

$$\boldsymbol{L} = \left(\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\right)^{-1}\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}\left(\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}, \tag{25}$$

and where $\boldsymbol{\omega}^U = \boldsymbol{\tau}^U - \boldsymbol{\delta}^U$ is a vector of dimension $|\mathcal{U}| \times 1$.

Equation (24) decomposes the value of regulating $\boldsymbol{x}^U$ into two parts. First, we have the direct welfare effect of adjusting $\boldsymbol{\tau}^U$, which would prevail in a scenario in which all perfectly regulated decisions in the set $\mathcal{R}$ remained unchanged. By Proposition 1, this direct effect is equal to the product $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\boldsymbol{\omega}^U$ of policy elasticities and Pigouvian wedges on all imperfectly regulated decisions. Second, Equation (24) makes an adjustment for the indirect policy effect, that is, for the welfare effect associated with the response of the perfectly regulated decisions to relaxing regulatory constraints, $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}$, which is a form of reverse leakage. Proposition 4 shows that, under the second-best policy, the appropriate adjustment is given by $-\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\boldsymbol{L}\boldsymbol{\omega}^U$. Interestingly, this adjustment due to the reverse leakage tends to *attenuate* the welfare effect of regulating the imperfectly regulated, regardless of whether unregulated and regulated choices are substitutes or complements, as we describe next.

To illustrate this effect most clearly, we revisit the simple case from Example 1:

Example 3. [Single Decision, cont.] Assume, as in Example 1, that there are two agents, each of whom makes a single decision, and only agent 1 is regulated, with $\tau^2 = 0$. Then, substituting the optimal second-best regulation from Equation (23), the welfare effect of marginally increasing τ^2 above zero is

$$\frac{dW}{d\tau^2} = -\frac{dx^2}{d\tau^2}\left(1 - \underbrace{\frac{\frac{dx^2}{d\tau^1}\frac{dx^1}{d\tau^2}}{\frac{dx^1}{d\tau^1}\frac{dx^2}{d\tau^2}}}_{=L}\right)\delta^2.$$

To interpret this expression, assume that the distortion $\delta^2 > 0$, and that we are in the "regular" case where the own-regulatory responses are negative with $\frac{dx^1}{d\tau^1} < 0$ and $\frac{dx^2}{d\tau^2} < 0$. First, consider the substitutes case, in which $\frac{dx^2}{d\tau^1} < 0$ and $\frac{dx^1}{d\tau^2} < 0$. We have $L > 0$, so that the welfare gain from increasing τ^2 is smaller than it would be in the absence of an indirect effect on agent type 1. Intuitively, regulating the unregulated pushes distorted activity back to the regulated sector, which dampens the direct welfare gains. Second, consider the complements case, in which $\frac{dx^2}{d\tau^1} > 0$ and $\frac{dx^1}{d\tau^2} > 0$. Once again, we have $L > 0$. This result arises from the nature of the second-best regulation of agent type 1, which in the case of complements involves overregulation ($\omega^1 > 0$; see

Example 1). Raising τ^2 in this scenario reduces the activity of agent 1 which, due to the initial overregulation, dampens the associated welfare benefit.[18]

Notice that Proposition 4 generalizes this reasoning to the case with multiple decisions. Heuristically, suppose that we are in the regular case where the own-regulatory responses are "negative" ($\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R} < 0$, $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} < 0$). Then, the adjustment matrix $\boldsymbol{L}$ is "positive", both in the case of gross substitutes ($\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U} < 0$, $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} < 0$) and gross complements ($\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U} > 0$, $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} > 0$), implying once again that the welfare effect of regulating the imperfectly regulated choices $\boldsymbol{x}^U$ is dampened. These insights bear a connection to the Le Chatelier principle, which we discuss in the following remark.

Remark 3. *[Connection to Le Chatelier principle (Samuelson, 1948; Milgrom and Roberts, 1996)]* A prominent result in economic theory that is similarly invariant to complementarity versus substitutability is the Le Chatelier principle. In its simplest form, it states that whenever choices are either complements or substitutes, the long-run response of a system is larger than its short-term response — see Milgrom and Roberts (1996) for a modern treatment. More generally, as noted by Milgrom (2006), all versions of the Le Chatelier principle explain how the direct effect of a parameter change is typically amplified by feedback in a system. In our case, the feedback occurs because the relaxation of a regulatory constraint also impacts social welfare through the response of the perfectly regulated decisions. Interestingly, even though the logic behind Proposition 4 is reminiscent of the logic behind the Le Chatelier principle, we find the exact *opposite* implication for welfare. When we let our system adjust further by accounting for the welfare impact of relaxing a constraint on the perfectly regulated decisions under the second-best policy, the welfare gains from regulation are typically dampened, not amplified.

It is worth making two final observations. First, note that, under the second-best regulation, the planner does not always wish to push regulations for imperfectly regulated choices towards their first-best value. As noted above in our discussion of Lipsey and Lancaster (1956), even when $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\boldsymbol{\omega}^U > 0$, it is possible to finds scenarios in which $\frac{dW}{d\boldsymbol{\tau}^U} < 0$. This result shows that loosening some regulations can be welfare-decreasing in our environment. Proposition 4 shows that this reversal is more likely when the matrix of Le Chatelier/reverse leakage adjustments $\boldsymbol{L}$ plays an important role. Second, note that the fact that the welfare effect of regulating imperfectly regulated decisions is typically attenuated when some decisions are perfectly regulated implies that imperfectly regulated decisions are typically lower in magnitude relative to first-best. We provide a clear example of this phenomenon in Section 3.6.3.

3.6 Common Scenarios of Regulatory Constraints/Costs

Finally, before illustrating our general results in the context of the applications, we specialize our results in the case of three natural scenarios in which regulation is subject to constraints or costs. First, we consider the case in which some investors or activities cannot be regulated at all.

[18] Note that it is conceivable to construct environments in which decisions are neither complements nor substitutes. In this example, this would correspond to $\frac{dx^1}{d\tau^2}$ and $\frac{dx^2}{d\tau^1}$ having opposite signs. Our characterization also applies to these cases, which are rare in common economic applications — see Milgrom and Roberts (1996).

Second, we consider the case in which the same corrective regulation must apply to all activities and/or investors. Third, we consider the case in which setting regulatory instruments is subject to quadratic costs.

3.6.1 Unregulated investors/activities

A particular type of regulatory constraint that is highly relevant in practice is when some investors or activities cannot be regulated at all. Formally, here we assume that the planner faces a constraint of the form

$$\boldsymbol{\Phi}\left(\boldsymbol{\tau}\right)=\boldsymbol{\tau}^{U}=0,$$

so a subset of investors/activities are not subject to regulation at all.[19] In that case, a specialized version of Equation (22) applies.

Proposition 5. *[Second-Best Regulation: Unregulated Investors/Activities] When some investors and/or activities cannot be regulated at all, i.e., $\boldsymbol{\Phi}\left(\boldsymbol{\tau}\right)=\boldsymbol{\tau}^{U}=0$, the optimal second-best regulation satisfies*

$$\boldsymbol{\tau}^{R}=\boldsymbol{\delta}^{R}-\left(-\frac{d\boldsymbol{x}^{R}}{d\boldsymbol{\tau}^{R}}\right)^{-1}\frac{d\boldsymbol{x}^{U}}{d\boldsymbol{\tau}^{R}}\boldsymbol{\delta}^{U},\tag{26}$$

where $\boldsymbol{\delta}^{R}$ and $\boldsymbol{\delta}^{U}$ are vectors of distortions of dimensions $|\mathcal{R}|\times 1$ and $|\mathcal{U}|\times 1$, respectively, and $\frac{d\boldsymbol{x}^{R}}{d\boldsymbol{\tau}^{R}}$ and $\frac{d\boldsymbol{x}^{U}}{d\boldsymbol{\tau}^{R}}$ are Jacobian matrices of dimensions $|\mathcal{R}|\times|\mathcal{R}|$ and $|\mathcal{R}|\times|\mathcal{U}|$.

As in Proposition 3, whether the regulated and unregulated decisions are gross complements or substitutes is critical for the determination of the optimal second-best policy. In the case in which some activities are unregulated, $\boldsymbol{\omega}^{U}=-\boldsymbol{\delta}^{U}$, so the planner only relies on the value of the distortion of the unregulated $\boldsymbol{\delta}^{U}$, instead of the value of the Pigouvian wedge of the unregulated.

Remark 4. [Connection to the Tinbergen (1952) rule] Proposition 5 relates to the classical analysis of policy instruments in Tinbergen (1952). The Tinbergen (1952) rule states that first-best policy must have access to as many instruments as it has targets. A concordant interpretation of Equation (26) is that a second-best planner must use the $|\mathcal{R}|$ instruments contained in the free taxes/subsidies $\boldsymbol{\tau}^{R}$ (on the left-hand side of the equation) to target $|\mathcal{R}|+|\mathcal{U}|$ distortions contained in $\boldsymbol{\delta}^{R}$ and $\boldsymbol{\delta}^{U}$ (on the right-hand side). It is immediate from the equation that first-best cannot be achieved unless $\boldsymbol{\delta}^{U}=0$, consistent with the Tinbergen rule. The characterization of second-best regulation in the equation offers a further refinement of the Tinbergen rule: with insufficient policy instruments, the optimal tax/subsidy equals a weighted sum of all distortions in the economy, whose weights are linked directly to leakage elasticities.

3.6.2 Uniform regulation

A second type of regulatory constraints that is highly relevant in practice is when the same corrective regulation must apply to all activities and/or investors, despite the fact that each activity and/or

[19] Slightly more generally, one could consider the case in which $\boldsymbol{\Phi}\left(\boldsymbol{\tau}\right)=\boldsymbol{\tau}^{U}-\overline{\boldsymbol{\tau}}^{U}=0$, so same regulations are fixed at a predetermined value $\overline{\boldsymbol{\tau}}^{U}$.

investor may be associated with externalities of different magnitudes. Formally, here we assume that the planner is forced to set the same regulation for a subset $\mathcal{U}$ of choices, that is, the planner faces constraints of the following form:[20]

$$\tau_n^j = \tau_{n'}^{j'}, \forall (j,n), (j',n') \in \mathcal{U}.$$

It follows immediately that $\frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}}$ for the subset of regulated decisions, which we denote by $\frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}^U}$, is given by

$$\frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}^U} = \underbrace{\begin{pmatrix} 1 & & \cdots & 0 \\ -1 & 1 & & \vdots \\ & \ddots & \ddots & \\ \vdots & & -1 & 1 \\ 0 & \cdots & & -1 \end{pmatrix}}_{|\mathcal{U}|\times(|\mathcal{U}|-1)}.$$

We say that in this case the planner's regulation is imperfectly targeted. In Proposition 6, we show that Equation (22) can be specialized to conclude that the second-best policy is given by a weighted average of distortions.

Proposition 6. *[Second-Best Regulation: Imperfect Targeting] When some investors and/or activities must be regulated at the same rate, i.e.,* $\tau_n^j = \tau_{n'}^{j'}, \forall (j,n), (j',n') \in \mathcal{U}$, *the optimal second-best regulation satisfies*

$$\tau_n^j = \bar{\tau}^U = \frac{\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} (\boldsymbol{I} - \boldsymbol{L}) \boldsymbol{\delta}^U}{\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} (\boldsymbol{I} - \boldsymbol{L}) \boldsymbol{\iota}}, \forall (j,n) \in \mathcal{U}, \tag{27}$$

where $\boldsymbol{\iota}$ *is a vector of ones with dimension* $|\mathcal{U}| \times 1$, $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}$ *is a Jacobian matrix of dimension* $|\mathcal{U}| \times |\mathcal{U}|$, *and* $\boldsymbol{L}$ *is the matrix of dimensions* $|\mathcal{U}| \times |\mathcal{U}|$ *defined in Proposition 4, with*

$$\boldsymbol{L} = \left(\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\right)^{-1} \frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U} \left(\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1} \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}.$$

Unlike our previous characterizations, Proposition 6 provides an explicit formula for taxes/subsidies on *imperfectly* regulated activities, leveraging the special case where regulation must be uniform on a subset of activities. Equation (27) demonstrates that the optimal second-best uniform regulation $\bar{\tau}^U$ is a weighted average of the distortions $\boldsymbol{\delta}^U$ generated by the associated activities. Notice that, if distortions are symmetric across activities with $\boldsymbol{\delta}^U = \boldsymbol{\iota}\bar{\delta}$, then Equation (27) implies that the planner should set the first-best regulation $\bar{\tau}^U = \bar{\delta}$. However, if there is any asymmetry, then the first-best cannot be achieved with uniform regulation.

To build further intuition for this result, it is useful to first consider the special case where *all* activities are subject to uniform regulation ($\boldsymbol{x}^U = \boldsymbol{x}$). In that case, it follows from Proposition 6

[20] Note that all choices in $\mathcal{U}$ will be generically associated with a binding constraint, so this notation is consistent with the way we introduced the set $\mathcal{U}$.

that the optimal uniform regulation is given by

$$\bar{\tau}^U = \frac{\sum_{j\in\mathcal{I}} \sum_{n\in\mathcal{X}} \frac{dx_n^j}{d\bar{\tau}^U} \delta_n^j}{\sum_{j\in\mathcal{I}} \sum_{n\in\mathcal{X}} \frac{dx_n^j}{d\bar{\tau}^U}}, \tag{28}$$

where we have re-written the *total* response of activity x_n^j to the uniform regulation as

$$\frac{dx_n^j}{d\bar{\tau}^U} = \sum_{j'\in\mathcal{I}} \sum_{n'\in\mathcal{X}} \frac{dx_n^j}{d\tau_{n'}^{j'}}.$$

This optimal regulation is a weighted average of distortions generated by imperfectly regulated activities, and the weights are equal to the total policy elasticities of each activity. Intuitively, the optimal regulation is large when activities with large (positive) distortions are most responsive to uniform regulation.

Equation (27) generalizes this idea to the case where there may also be *perfectly* regulated activities $\boldsymbol{x}^R$, on which the planner can set regulation freely, in addition to the uniformly regulated activities $\boldsymbol{x}^U$. In the general case, the optimal weights are adjusted for the endogenous responses of perfectly regulated activities $\boldsymbol{x}^R$. Interestingly, the necessary adjustment is captured by the same matrix $\boldsymbol{L}$ that features in the value of relaxing regulatory constraints in Proposition 4. In the special case where either $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U} = 0$ or $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} = 0$, we have $\boldsymbol{L} = 0$, and we recover the expression in Equation (28).

We close this section by relating these results to the classical analysis of uniform corrective taxation in Diamond (1973):

Remark 5. *[Connection to Diamond (1973)]* The insight that uniform regulation of heterogeneous externalities is given by a weighted average of the distortions can be traced back to Diamond (1973). Indeed, the special case where all activities are subject to uniform regulation in our model yields Equation (28), which corresponds to Diamond's result that the optimal weights on different distortions are equal to policy elasticities. The general analysis in Proposition 6 further shows that when there are policy instruments that are freely adjustable, the optimal weights on different distortions for the optimal uniform regulation must account for the Le Chatelier/reverse leakage adjustments that we introduce in Section 3.5.

3.6.3 Quadratic costs of regulation

A third relevant scenario is one in which setting some regulations is costly. In particular, we consider the tractable case in which setting a subset $\mathcal{U}$ of regulations is subject to quadratic costs, defined in Equation (10). This formulation can be interpreted as allowing for the restrictions on the regulatory toolbox to arise endogenously, in the sense that adjusting one regulation may make it easier or harder to adjust another.

Formally, the optimal regulation of the subset of instruments subject to quadratic costs must satisfy

$$\frac{dW}{d\boldsymbol{\tau}^{U}} = \frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}^{U}} = \boldsymbol{B}\boldsymbol{\tau}^{U}, \tag{29}$$

where $\frac{dW}{d\boldsymbol{\tau}^{U}}$ is defined in Equation (24) and $\boldsymbol{B}$ is defined in Equation (10). In Proposition 7, we show that the optimal policy in the presence of quadratic adjustment costs is given by a scaled down version of the first-best policy.

Proposition 7. *[Second-Best Regulation: Attenuation under Quadratic Costs of Regulation] When adjusting the regulations of some investors and/or activities faces quadratic adjustment costs, the optimal second-best regulation satisfies*

$$\boldsymbol{\tau}^{U} = \left(\boldsymbol{B} + \boldsymbol{K}\right)^{-1} \boldsymbol{K}\boldsymbol{\delta}^{U}, \tag{30}$$

where $\boldsymbol{K}$ is given by

$$\boldsymbol{K} = \left(-\frac{d\boldsymbol{x}^{U}}{d\boldsymbol{\tau}^{U}}\right)\left(\boldsymbol{I} - \boldsymbol{L}\right),$$

and $\boldsymbol{L}$ is the matrix of dimensions $|\mathcal{U}| \times |\mathcal{U}|$ defined in Proposition 4.

Interestingly, as in the uniform regulation case, the adjusted product of policy elasticities $\frac{d\boldsymbol{x}^{U}}{d\boldsymbol{\tau}^{U}}\left(\boldsymbol{I} - \boldsymbol{L}\right)$ is a key input for the optimal policy. The larger $\frac{d\boldsymbol{x}^{U}}{d\boldsymbol{\tau}^{U}}\left(\boldsymbol{I} - \boldsymbol{L}\right)$ is relative to $\boldsymbol{B}$, the smaller the deviation of the optimal policy with quadratic costs from the first-best policy. Alternatively, as expected, when the costs vanish, as $\boldsymbol{B} \to 0$, the optimal policy converges to the first-best, so $\boldsymbol{\tau}^{U} \to \boldsymbol{\delta}^{U}$. When $\boldsymbol{B} \neq 0$, Equation (30) can be interpreted as stating that the optimal policy is simply an attenuated version of the first-best policy, in which $\boldsymbol{\tau}^{U} = \boldsymbol{\delta}^{U}$. As discussed in Section 3.5, note that the presence of perfectly regulated decisions — via a large $\boldsymbol{L}$ — is a force that contributes to attenuating the optimal choice of $\boldsymbol{\tau}^{U}$.

In general, the correction relative to the first-best policy is given by $\left(\boldsymbol{B} + \boldsymbol{K}\right)^{-1} \boldsymbol{K}$, which has the interpretation of an attenuation matrix. For instance, in a scenario with a single agent, $|\mathcal{I}| = 1$, and a single activity, $|\mathcal{X}| = 1$, Equation (30) becomes

$$\tau^{U} = \frac{\left(-\frac{dx^{U}}{d\tau^{U}}\right)}{b + \left(-\frac{dx^{U}}{d\tau^{U}}\right)}\delta^{U}, \tag{31}$$

where b is a non-negative scalar that modulates the cost. In the well-behaved case in which $\frac{dx}{d\tau} < 0$, it follows that the optimal regulation is simply a scaled down version of the first best-regulation.[21]

This result concludes the characterization of second-best policy with imperfect instruments in our general model. In the remainder of the paper, we discuss the usefulness of these general results in the context of specific applications. We begin with our headline application to macro-prudential

[21] In this case, note that the perfectly regulated decisions in turn must satisfy:

$$\boldsymbol{\tau}^{R} = \boldsymbol{\delta}^{R} + \left(-\frac{d\boldsymbol{x}^{R}}{d\boldsymbol{\tau}^{R}}\right)^{-1} \frac{d\boldsymbol{x}^{U}}{d\boldsymbol{\tau}^{R}} \left(\left(\boldsymbol{B} + \boldsymbol{K}\right)^{-1} \boldsymbol{K} - \boldsymbol{I}\right) \boldsymbol{\delta}^{U}.$$

regulation in the presence of environmental externalities, and then consider a sequence of further applications.

4 Application: Financial Regulation with Environmental Externalities

Central banks and macro-prudential regulators have increasingly become interested in accounting for environmental concerns. There are two possible motivations for this. First, there are links between the financial system, a primary target of macro-prudential regulation, and climate-related risks, as evidenced by a growing literature on climate finance (e.g., Giglio, Kelly and Stroebel, 2021). For instance, the safety and soundness of financial institutions may be at risk if they are heavily exposed to climate-related risks. Second, some believe that prudential regulation should take into account its effect on the broader societal goal of sustainable investment.[22] For instance, the European Central Bank's bond purchase program has taken the latter motivation into account by introducing preferential treatment for bonds associated with "green" technologies (Piazessi, Papoutsi and Schneider, 2021).

A nascent academic literature studies the welfare implications of financial regulatory reforms when there are environmental concerns (e.g., Oehmke and Opp, 2022; Rola-Janicka and Döttling, 2022). In this section, we use our general results to characterize optimal policy with environmental externalities and *imperfect macro-prudential regulation*. This is a particularly important question because regulators are already discussing potential imperfections and unintended consequences of policies in the presence of environmental externalities.[23]

In this application, we first show that imperfections are inherent to the primary mode of financial regulation in advanced economies, namely, risk-weighted leverage constraints. Indeed, these requirements constrain only relative quantities on institutions' balance sheets but leave the overall scale of investment as a free variable. Next, we analyze second-best optimal regulation in this setting. To capture the two motivations for policy discussed above, we pay special attention to contrasting the role of climate-related risks when regulation has a narrow/financial mandate versus a broad/environmental mandate. Our results, which directly leverage the formulae from the general model, yield new insights into the differences between these two cases, and into the way in which climate-conscious regulation should be adjusted for imperfections. Finally, we characterize

[22]The Bank for International Settlements has recently summarized these two concerns as follows: "Given the impact of climate change on traditional risk categories, the speech makes the case that prudential policy needs to be adjusted to account for the impact of climate-related risks on the safety and soundness of financial institutions as part of the core mandate of supervisory authorities (what we could call the financial motivation for regulatory action). Moreover, this adjustment has often been presented as a contribution by prudential regulation to facilitate the transition to a more sustainable economy by providing incentives for a more climate-friendly allocation of financial resources (that would be the economic motivation)."

[23]For example, Andrew Bailey, the Governor of the Bank of England, has recently commented that "any incorporation of climate change into regulatory capital requirements would need to be grounded in robust data and be designed to support safety and soundness while avoiding unintended consequences or compromising our other objectives". See: https://www.bankofengland.co.uk/speech/2021/june/andrew-bailey-reuters-events-global-responsible-business-2021.

the value of extending the set of policy tools in the face of environmental externalities, which relates to the Le Chatelier/reverse leakage adjustments that we have characterized in the general case.

Environment. We assume that there is a single type of investor, in unit measure and indexed by i, and a unit measure of creditors, indexed by C. Both investors and creditors have risk-neutral preferences given by

$$c_0^i + \beta^i \int c_1^i(s)\, dF(s) \quad \text{and} \quad c_0^C + \beta^C \int c_1^C(s)\, dF(s) - \Psi\left(\theta^i\right) k^i,$$

where the term $\Psi(\theta^i)\, k^i$ introduces an environmental externality, as described below.[24] The budget constraints of investors at date 0 and date 1 are given by

$$\begin{aligned} c_0^i &= n_0^i + Q^i\left(b^i, \theta^i\right) k^i - \Upsilon\left(k^i\right) - \Omega\left(\theta^i\right) k^i, \\ c_1^i(s) &= k^i \max\left\{d_1(s)\,\theta^i + d_2(s)\left(1-\theta^i\right) + t\left(b^i, \theta^i, s\right) - b^i, 0\right\}. \quad \forall s. \end{aligned}$$

At date 0, investors, endowed with n_0^i dollars, make capital investments k^i in two sectors of the economy. A fraction θ^i is invested in sector 1, and the remaining $1-\theta^i$ in in sector 2. Investors issue debt with face value $b^i k^i$ to creditors, so that b^i measures investors' leverage. We conjecture and verify that the equilibrium price of debt can be written as $Q^i(b^i, \theta^i)\, k^i$, where $Q^i(b^i, k^i)$ denotes the market value per unit of capital. Capital investments are subject to an adjustment cost $\Upsilon(k^i)$ and an additional cost $\Omega(\theta^i)\, k^i$ of adjusting the sectoral composition of investors' portfolios.[25] At date 1, once a state s is realized, investor i receives $d_j(s)$ dollars for each unit of investment in sector $j \in \{1, 2\}$ and a bailout transfer $t^i(b^i, \theta^i, s)$ per unit of capital that potentially depends on the amount of debt issued by the investor and portfolio weights. If the sum of these revenues exceeds the face value of debt, then investors repay their debt and consume the residual claim. Otherwise, as discussed below, they optimally choose to default and consume zero.[26]

In this application, motivated by the existing regulatory instruments, we assume that investors

[24] The assumption that this distortion only impacts creditors and is linear in capital simplifies the exposition, but does not affect the qualitative insights of our analysis.

[25] Alternatively, the investors' problem can be formulated in terms of the total capital investments, namely, $k_1^i = \theta^i k^i$ and $k_2^i = \left(1-\theta^i\right) k^i$. Our formulation holds as long as portfolio adjustment costs are homogeneous of degree 1 in capital investments.

[26] This specification of bailouts corresponds to a model where the government has limited commitment, which connects our work to the treatment of bailouts in Farhi and Tirole (2012), Bianchi (2016), Chari and Kehoe (2016), Keister (2016), Gourinchas and Martin (2017), Cordella, Dell'Ariccia and Marquez (2018), Dávila and Walther (2020*a*), and Dovis and Kirpalani (2020), among others.

are subject to a *risk-weighted capital requirement*:[27]

$$b^i + \varphi\theta^i \le \bar{b}. \tag{32}$$

As we show below, imposing this constraint on investors is equivalent to imposing corrective taxes. Therefore, this application also serves to illustrate how our approach to imperfect regulation can be applied to quantity-based instruments that are often used in practice. Intuitively, the requirement in Equation (32) places an upper bound $\bar{b}$ on investors' leverage, which is adjusted in proportion to the share θ^i invested in sector 1. In the case with $\varphi > 0$, on which we will focus without loss of generality, the relative risk weight φ on sector 1 is positive, and the leverage cap becomes tighter when investors increase θ^i.

The budget constraints of creditors at date 0 and date 1 are given by

$$\begin{aligned} c_0^C &= n_0^C - h^i Q^i\left(b^i, \theta^i\right) k^i, \\ c_1^C(s) &= n_1^C(s) - (1+\kappa)\, t\left(b^i, \theta^i, s\right) k^i + h^i \mathcal{P}^i\left(b^i, \theta^i, s\right) k^i. \end{aligned}$$

At date 1, creditors are taxed $(1+\kappa)$ times the government bailout, where $\kappa > 0$ denotes the deadweight cost of fiscal intervention. Moreover, creditors who buy a fraction h^i of investors' debt pay the market price at date 0, and receive a payment $\mathcal{P}^i(b^i, \theta^i, s)\, k^i$ at date 1. This payment, which preemptively incorporates investors' optimal default decision, is defined as follows:

$$\mathcal{P}^i\left(b^i, \theta^i, s\right) = \begin{cases} b^i, & d_1(s)\,\theta^i + d_2(s)\left(1-\theta^i\right) + t\left(b^i, \theta^i, s\right) \ge b^i \\ \phi\left[d_1(s)\,\theta^i + d_2(s)\left(1-\theta^i\right)\right], & \text{otherwise.} \end{cases}$$

Investors default when their assets are worth less than the promised repayment b^i per unit of capital, and repay b^i in full otherwise. In default, creditors recover a fraction $\phi < 1$ of their assets, so that $1-\phi$ can be interpreted as the deadweight cost of default. For simplicity, we assume that primitives are such that there exists a default threshold $s^\star(b^i, \theta^i)$, so that investors default when $s < s^\star(b^i, \theta^i)$ and repay otherwise.[28]

Finally, recall that creditors' preferences include a utility loss of $\Psi(\theta^i)\, k^i$ as a result of investors' choices. This term reflects an environmental externality. Investors' portfolio choices θ^i can affect

[27] Risk-weighted capital requirements under the Basel accords ensure that the ratio of equity to risk-weighted assets in leveraged institutions (e.g., banks) is at least equal to a constant fraction C. In our context, equity is $\left(1-b^i\right) k^i$ and risk-weighted assets can be represented as $\left[w_1\theta^i + w_2\left(1-\theta^i\right)\right] k^i$, where w_j is the risk weight on sector j investments. Thus, we can express a risk-weighted capital requirement as

$$1 - b^i \ge C\left[w_1\theta^i + w_2\left(1-\theta^i\right)\right] \iff b^i + \underbrace{(w_1 - w_2)}_{\equiv\varphi}\theta^i \le \underbrace{1 - Cw_2}_{\equiv\bar{b}},$$

which is equivalent to our formulation in (32), with φ denoting the relative risk weight on sector 1 investments.

[28] The uniqueness of this threshold $s^\star\left(b^i, \theta^i\right)$ is guaranteed under the standard assumptions that i) $d_j(s)$, $j \in \{1,2\}$, is increasing in s (i.e., higher asset returns in good states), and ii) the bailout transfer $t\left(b^i, \theta^i, s\right)$ is decreasing in s and increasing in b^i (i.e., larger bailouts in bad states/for more levered investors).

this loss. For example, if $\frac{\partial \Psi'}{\partial \theta^i} > 0$, then the environmental externality is increasing in the investment share in sector 1, meaning that sector 1 is associated with more pollution than sector 2.

Equilibrium. For given regulatory parameters $\left\{\bar{b}, \varphi\right\}$ defining the constraint (32) and a given bailout policy $t\left(b^i, \theta^i, s\right)$, an *equilibrium* is defined by leverage, portfolio, and investment decisions $\left\{b^i, \theta^i, k^i\right\}$, a default decision rule, and a pricing schedule $Q\left(b^i, \theta^i\right)$ such that investors and creditors maximize their utility and the market for debt clears, i.e., $h^i = 1$.

We rely on the following characterization of the equilibrium.

Lemma 1. [Equilibrium characterization] *Equilibrium choices $\left\{b^i, \theta^i, k^i\right\}$ are given by the solution to the following reformulation of the problem faced by investors:*

$$\max_{\left\{b^i, \theta^i, k^i\right\}} \left[M\left(b^i, \theta^i\right) - \Omega\left(\theta^i\right) - 1\right] k^i - \Upsilon\left(k^i\right) \text{ subject to } k^i\left(b^i + \varphi\theta^i\right) \leq k^i \bar{b}, \tag{33}$$

where $M\left(b^i, \theta^i\right)$ is given by

$$\begin{aligned} M\left(b^i, \theta^i\right) = & \underbrace{\beta^i \int_{s^\star\left(b^i, \theta^i\right)}^{\bar{s}} \left(d_1(s)\,\theta^i + d_2(s)\left(1-\theta^i\right) + t\left(b^i, \theta^i, s\right) - b^i\right) dF(s)}_{equity} \\ & + \underbrace{\beta^C \left(\int_{s^\star\left(b^i, \theta^i\right)}^{\bar{s}} b^i dF(s) + \phi \int_{\underline{s}}^{s^\star\left(b^i, \theta^i\right)} \left[d_1(s)\,\theta^i + d_2(s)\left(1-\theta^i\right) + t\left(b^i, \theta^i, s\right)\right] dF(s)\right)}_{debt\ = Q\left(b^i, \theta^i\right)}, \end{aligned} \tag{34}$$

and $s^\star\left(b^i, \theta^i\right)$ solves the equation

$$d_1(s^\star)\,\theta^i + d_2(s^\star)\left(1-\theta^i\right) + t\left(b^i, \theta^i, s^\star\right) = b^i.$$

Intuitively, we characterize the equilibrium by incorporating the pricing of debt into the investors' problem at date 1. The function $M\left(b^i, \theta^i\right)$ can be interpreted as the sum of the market values of equity (owned by investors) and debt (owned by creditors) per unit of investment. Notice that the second term in Equation (34) corresponds to the equilibrium price of debt $Q\left(b^i, \theta^i\right)$, which incorporates the fact that investors default in states $s < s^\star\left(b^i, \theta^i\right)$ in which the value of their assets is less than the promised repayment b^i. In problem (33), investors maximize the market value of investment net of costs. For convenience, and without loss of generality, we have scaled the regulatory constraint in this problem by total investment $k^i \geq 0$.

An important aspect of this application is that the planner's instruments are imperfect. This can be seen by writing investors' first-order conditions as

$$\frac{\partial M\left(b^i,\theta^i\right)}{\partial b^i} = \mu \equiv \tau_b \tag{35}$$

$$\frac{\partial M\left(b^i,\theta^i\right)}{\partial \theta^i} - \Omega'\left(\theta^i\right) = \mu\varphi \equiv \tau_\theta \tag{36}$$

$$M\left(b^i,\theta^i\right) - \Omega\left(\theta^i\right) - 1 - \Upsilon'\left(k^i\right) = 0, \tag{37}$$

where $\mu \geq 0$ is the Lagrange multiplier on the regulatory constraint. The first two conditions, which define optimal leverage and portfolio weights, show that the constraint in Equation (32) implies effective corrective taxes τ_b on leverage b^i and τ_θ on portfolios θ^i. The third condition, which defines optimal total investment k^i, does not contain a corrective tax. Intuitively, the capital requirement (32) constrains ratios but leaves the overall scale k^i of investors' balance sheet as a free, unregulated variable. By contrast, in a world with perfect instruments, the planner would be able to set a corrective tax τ_k on k^i in addition to τ_b and τ_θ. We return to the value of introducing such a tax below.

Optimal Corrective Policy. In this environment, we can express the marginal externalities $\{\delta_k, \delta_b, \delta_\theta\}$ associated with investors' choices and decompose them into a financial (i.e., bailout-related) and an environmental component as follows:

$$\delta_b = \underbrace{(1+\kappa)\,\beta^C \int_{\underline{s}}^{\bar{s}} \frac{\partial t\left(b^i,\theta^i,s\right)}{\partial b^i} dF(s)}_{\equiv \chi_b} \tag{38}$$

$$\delta_\theta = \underbrace{(1+\kappa)\,\beta^C \int_{\underline{s}}^{\bar{s}} \frac{\partial t\left(b^i,\theta^i,s\right)}{\partial \theta^i} dF(s)}_{\equiv \chi_\theta} + \underbrace{\frac{\partial \Psi\left(\theta^i\right)}{\partial \theta^i}}_{\equiv \psi_\theta} \tag{39}$$

$$\delta_k = \underbrace{(1+\kappa)\,\beta^C \int_{\underline{s}}^{\bar{s}} t\left(b^i,\theta^i,s\right) dF(s)}_{\equiv \chi_k} + \underbrace{\Psi\left(\theta^i\right)}_{\equiv \psi_k}. \tag{40}$$

For instance, χ_k in Equation (40) measures the marginal distortion in capital choices due to bailouts, while ψ_k is the distortion due to environmental externalities. Equations (38) and (39) define the distortions associated with leverage and portfolio choices per unit of capital. An important point is that leverage induces only a financial distortion, since environmental damage is determined by the technologies that are operated in this economy, and is independent of how these technologies are financed.

In Proposition 8, we characterize the form of the second-best policy.

Proposition 8. [Financial Regulation with Environmental Externalities]
a) The marginal welfare effects of varying the leverage cap $\bar{b}$ and the risk weight φ, respectively, are

given by

$$\frac{dW}{d\bar{b}} = \frac{db^i}{d\bar{b}} (\tau_b - \delta_b) k^i + \frac{d\theta^i}{d\bar{b}} (\tau_\theta - \delta_\theta) k^i - \frac{dk^i}{d\bar{b}} \delta_k, \tag{41}$$

$$\frac{dW}{d\varphi} = \frac{db^i}{d\varphi} (\tau_b - \delta_b) k^i + \frac{d\theta^i}{d\varphi} (\tau_\theta - \delta_\theta) k^i - \frac{dk^i}{d\varphi} \delta_k. \tag{42}$$

b) The optimal regulation satisfies

$$\begin{pmatrix} \tau_b \\ \tau_\theta \end{pmatrix} = \begin{pmatrix} \delta_b \\ \delta_\theta \end{pmatrix} + \begin{pmatrix} \frac{db^i}{d\bar{b}} & \frac{d\theta^i}{d\bar{b}} \\ \frac{db^i}{d\varphi} & \frac{d\theta^i}{d\varphi} \end{pmatrix}^{-1} \begin{pmatrix} \frac{d\log k^i}{d\bar{b}} \\ \frac{d\log k^i}{d\varphi} \end{pmatrix} \delta_k. \tag{43}$$

Proposition 8, which is an instance of Propositions 1 and 3, characterizes the marginal welfare effects of adjusting the two instruments available to the planner and the optimal regulation in terms of the parameters of the risk-weighted capital constraint, which are the leverage cap $\bar{b}$ and the relative risk weight φ. Notice that, even though we are working in terms of a quantity constraint, our general characterization of welfare effects from Proposition 1 applies, after suitably adjusting for k^i. This feature highlights the usefulness of our approach for analyzing quantity-based regulation.

Specifically, Equations (41) and (42) show that marginal welfare effects depend on Pigouvian wedges — defined in terms of the equivalent taxes $\{\tau_b, \tau_\theta\}$ in Equations (35) and (36) — as well as policy elasticities. First-best regulation is prevented by the fact that the unregulated scale decision k^i introduces an additional distortion δ_k. The optimal regulation, which we discuss in more detail below, takes into account this distortion along with the appropriate leakage elasticities $\frac{d\log k^i}{d\bar{b}}$ and $\frac{d\log k^i}{d\varphi}$.[29]

In the remainder of this section, we use this characterization to derive several concrete insights into optimal regulation with environmental externalities. First, we analyze the distinction between optimal policy motivated by narrow financial stability mandates and broader mandates that take environmental externalities into account. Importantly, we provide a novel treatment of these questions taking into account imperfections in policy instruments. Finally, we characterize the value of relaxing regulatory constraints by imposing corrective regulation on the total scale of investment.

Imperfect Regulation with Narrow/Financial Mandates. We first consider a financial regulator who has a narrow mandate and is only concerned with financial externalities. In terms of our decomposition of distortions, we interpret a narrow mandate as meaning that the regulator acts as if the climate-related distortions $\{\psi_\theta, \psi_k\}$ are both equal to zero. In the background, one can interpret that the distribution of states, $F(s)$, and the payoffs of the different investments, $d_1(s)$ and $d_2(s)$, account for climate risks. Applying Proposition 8 and substituting Equations (38) through (40) yields the optimal policy in this case:

[29]The appropriate leakage elasticities in this application are semi-elasticities, i.e., responses of log investment to policy reforms. Compared to our general model, this formulation arises because we have expressed leverage and portfolio choices per unit of capital.

Corollary 1. [Imperfect Regulation with Narrow/Financial Mandates] *The optimal policy of a regulator with a narrow/financial mandate is given by*

$$\begin{pmatrix} \tau_b \\ \tau_\theta \end{pmatrix} = \begin{pmatrix} \chi_b \\ \chi_\theta \end{pmatrix} + \begin{pmatrix} \frac{db^i}{d\bar{b}} & \frac{d\theta^i}{d\bar{b}} \\ \frac{db^i}{d\varphi} & \frac{d\theta^i}{d\varphi} \end{pmatrix}^{-1} \begin{pmatrix} \frac{d\log k^i}{d\bar{b}} \\ \frac{d\log k^i}{d\varphi} \end{pmatrix} \chi_k, \tag{44}$$

where χ_b, χ_θ, and χ_k denote the financial component of the respective externalities, defined in Equations (38) through (40).

Equation (44) shows that the optimal leverage cap — represented by τ_b — and the optimal risk weight — represented by τ_θ — are set in response to two terms. The first term captures the marginal externality associated with a change in b or θ, which in this case corresponds to the marginal response of expected bailouts to more leverage or more investment in sector 1.

The second term, which arises only with imperfect instruments, is proportional to the leakage elasticities $\frac{d\log k^i}{d\bar{b}}$ and $\frac{d\log k^i}{d\varphi}$, and scales with the total expected bailout, via χ_k. In the Appendix, we prove that both these elasticities fall into the "complements" case: Stricter leverage regulation ($\downarrow \bar{b}$) or a stricter relative risk weight ($\uparrow \varphi$) both lead to increases in k^i in equilibrium. Therefore, Equation (44) generally calls for *overregulation* of leverage and risk. Finally, notice that the relevant leakage elasticities are modulated by an inverse matrix of policy elasticities between b^i and θ^i.

The implication for financial regulation with environmental externalities is that any adjustment for climate-related risk should be determined only by its impact on financial externalities (in this particular case, this emerges from the presence of bailouts). For instance, the risk weight equivalent tax τ_θ should be increased if sector 1 is associated with climate-related tail risk that makes large bailouts more likely (i.e., if $\mathbb{E}_s\left[\frac{\partial t(b^i,\theta^i,s)}{\partial\theta^i}\right] > 0$). In addition, the setting with imperfect instruments implies that taxes on *both* leverage and portfolio weights should increase if climate-related risk increases the magnitude of the total expected bailout. This prediction is unique to our analysis and directly leverages our general tools.

Imperfect Regulation with Broad/Environmental Mandates. We now consider a financial regulator with a broad mandate who cares directly about mitigating environmental distortions. We will focus now on the case where the environmental distortions satisfy $\psi_\theta > 0$ and $\psi_k > 0$. In this case, increases in overall scale as well as concentrated investments in sector 1 are associated with greater environmental damage.

Corollary 2. [Imperfect Regulation with Broad/Environmental Mandates]

a) When policy has been set optimally according to a narrow/financial mandate, the welfare benefits of marginal policy changes are given by

$$\frac{dW}{d\bar{b}} = -\frac{d\theta^i}{d\bar{b}}\psi_\theta - \frac{dk^i}{d\bar{b}}\psi_k \tag{45}$$

$$\frac{dW}{d\varphi} = -\frac{d\theta^i}{d\varphi}\psi_\theta - \frac{dk^i}{d\varphi}\psi_k, \tag{46}$$

where ψ_θ and ψ_k denote the environmental component of the respective externalities, defined in Equations (39) and (40).

b) The optimal policy of a regulator with a broad/environmental mandate is given by

$$\begin{pmatrix} \tau_b \\ \tau_\theta \end{pmatrix} = \begin{pmatrix} \chi_b \\ \chi_\theta + \psi_\theta \end{pmatrix} + \begin{pmatrix} \frac{db^i}{d\bar{b}} & \frac{d\theta^i}{d\bar{b}} \\ \frac{db^i}{d\varphi} & \frac{d\theta^i}{d\varphi} \end{pmatrix}^{-1} \begin{pmatrix} \frac{d\log k^i}{d\bar{b}} \\ \frac{d\log k^i}{d\varphi} \end{pmatrix} (\chi_k + \psi_k) \,, \tag{47}$$

where χ_b, χ_θ, and χ_k denote the financial component and ψ_θ and ψ_k denote the environmental component of the respective externalities, defined in Equations (38) through (40).

Corollary 2 develops two insights into the distinction between narrow/financial and broad/environmental mandates. First, Equations (45) and (46) highlight the additional welfare effects, from the perspective of a broad mandate, of adjusting either the leverage cap and risk weights, when policy has previously been optimized according a narrow mandate. These equations are useful for deciding whether policy should be adjusted at the margin once a regulator decides to take environmental outcomes into account. The relevant marginal welfare effects are determined by the environmental distortions ψ_θ and ψ_k and the associated leakage elasticities. It is interesting to note that the leakage elasticities to leverage (i.e., $\frac{db^i}{d\bar{b}}$ and $\frac{db^i}{d\varphi}$) are irrelevant here, because the mode of financing has no marginal impact on environmental concerns.

Equation (45) shows that a regulator who adjusts the leverage cap $\bar{b}$ in response to environmental concerns faces a potential conflict of interest. Indeed, while it is natural that scale and leverage are generally complements, implying $\frac{dk^i}{d\bar{b}} > 0$,[30] the response of optimal portfolio choices $\frac{d\theta^i}{d\bar{b}}$ is ambiguous in theory, and depends on the functional form of returns to investment in each sector. Since the environmental distortions ψ_θ, ψ_b are assumed positive, the two terms in Equation (45) may have opposite signs. As a result, it is unclear whether leverage requirements should be relaxed or tightened in response to environmental concerns, and their impact on welfare may be offset by portfolio adjustments.

By contrast, Equation (46) demonstrates that risk weights are a natural tool for addressing environmental concerns. Both the portfolio share θ^i and total capital k^i are generally complements to the risk weight, implying that $\frac{d\theta^i}{d\varphi} < 0$ and $\frac{dk^i}{d\varphi} < 0$. Therefore, it is clear that risk weights ought to be tightened when regulators account for environmental externalities.

The second insight emerging from Corollary 2 is the characterization of optimal policy in Equation (47). There are two differences to the equivalent characterization with a narrow mandate in Equation (44). First, the marginal distortion on portfolio choices is augmented, which calls for greater relative risk weights on the polluting sector (sector 1). Second, the scale distortion is augmented by ψ_k. The latter point is particularly important for our analysis. The scale distortion matters purely due to imperfect regulation and leakage elasticities. Equation (47) demonstrates

[30] Recall that $d\bar{b} > 0$ stands for a *looser* leverage cap, that is, a lower effective tax on leverage. Hence, $\frac{dk^i}{d\bar{b}} > 0$ is equivalent to $\frac{dk^i}{d\tau_b} < 0$, which corresponds to the complements case in our general, tax-based notation.

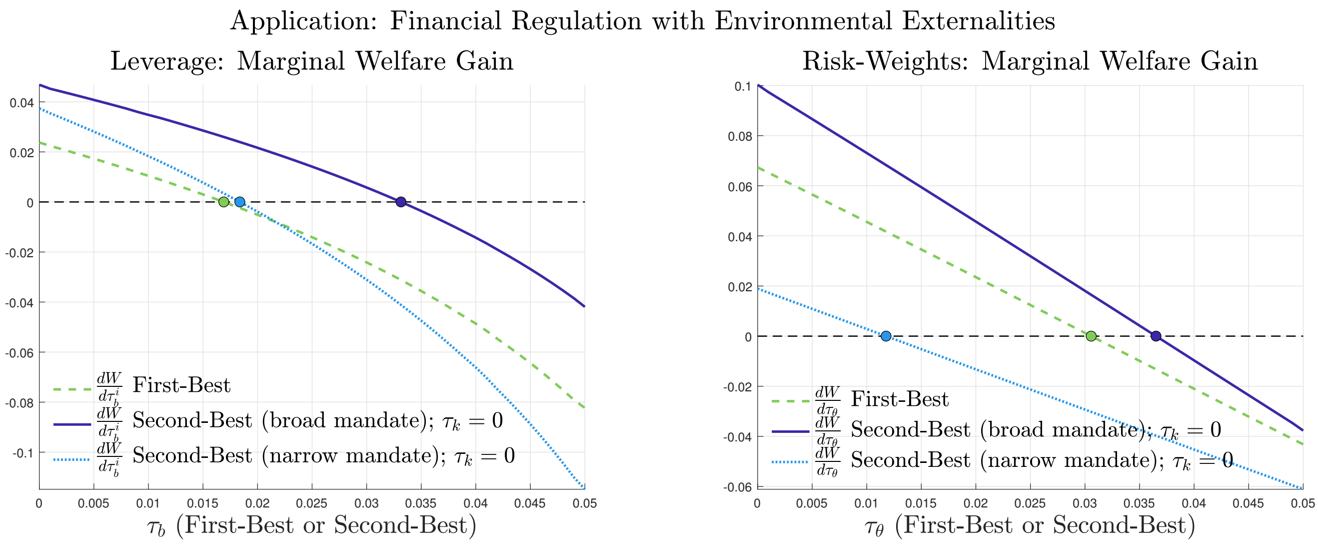

Figure 1: Financial Regulation with Environmental Externalities

Note: The left panel of Figure 1 compares the marginal welfare effects of varying corrective leverage regulation (τ_b) in three different scenarios. The green dashed line corresponds to the first-best scenario, in which τ_θ and τ_k are held fixed at their first-best levels (previously computed). The solid dark blue line corresponds to a second-best scenario in which the regulator has a broad mandate and cares about financial and environmental distortions. In this case, we compute welfare gains setting $\tau_k = 0$ and holding τ_θ fixed at the optimal second-best level for a broad mandate (previously computed). The light blue dotted line corresponds to a second-best scenario in which the regulator has a narrow mandate and cares exclusively about financial distortions. In this case, we compute welfare gains setting $\tau_k = 0$ and holding τ_θ fixed at the optimal second-best level for a narrow mandate (previously computed). The right panel of Figure 1 compares the analogous marginal welfare effects of varying corrective risk-weights regulation (τ_θ) in the same three scenarios.
To generate this figure, we assume that the bailout policy is linearly separable, $t^i\left(b^i, s\right) = \alpha_0^i + \alpha_b^i b^i + \alpha_\theta^i \theta^i - \alpha_s^i s$, that the adjustment cost is quadratic, $\Upsilon\left(k^i\right) = \frac{a}{2}\left(k^i\right)^2$, and that the functions $\Omega\left(\theta^i\right)$ and $\Psi\left(\theta^i\right)$ are of the CES (constant elasticity of substitution) form in terms of k_1^i and k_2^i, so $\Omega\left(\theta\right) = z_\Omega\left(a_\Omega\left(\theta^i\right)^{\eta_\Omega} + \left(1-a_\Omega\right)\left(1-\theta^i\right)^{\eta_\Omega}\right)^{\frac{1}{\eta_\Omega}}$ and $\Psi\left(\theta\right) = z_\Psi\left(a_\Psi\left(\theta^i\right)^{\eta_\Psi} + \left(1-a_\Psi\right)\left(1-\theta^i\right)^{\eta_\Psi}\right)^{\frac{1}{\eta_\Psi}}$. The parameters used to generate this figure are $\beta^i = 0.9$, $\beta^C = 0.98$ $\phi^i = 0.7$, $a = 1$, $\alpha_0^i = \alpha_s^i = 0$, $\alpha_b^i = 0.015$, $\alpha_\theta^i = 0.01$, $\kappa = 0.15$, $d_1\left(s\right) = d_1 s$ with $d_1 = 1.01$, $d_2\left(s\right) = d_2 s$ with $d_2 = 1$, $z_\Omega = 0.25$, $a_\Omega = 1.5$, $\eta_\Omega = 1.5$, $z_\Psi = 0.25$, $a_\Psi = 0.55$, $\eta_\Psi = 1.5$, $n_0^C = 50$, and $n_1^C\left(s\right) = 50 + 0.1s$, where s is normally distributed with mean 1.3 and standard deviation 0.8, truncated to the interval $[0, 3]$. For reference, the optimal first-best regulation is $\tau_b = 1.69\%$, $\tau_\theta = 3.05\%$, and $\tau_k = 14.22\%$, the optimal second-best regulation with a broad mandate is $\tau_b = 3.33\%$, $\tau_\theta = 3.65\%$, and $\tau_k = 0$, while the optimal second-best regulation with a narrow mandate is $\tau_b = 1.83\%$, $\tau_\theta = 1.11\%$, and $\tau_k = 0$.

that adjustments for leakage elasticities become *more* important once the regulator cares about environmental effects.

Figure 1 illustrates the relation between the first-best and second-best solutions in both the narrow and the broad mandate cases. In particular, the left panel shows the marginal welfare effect of varying leverage regulation (in terms of τ_b), while the right panel shows the marginal welfare effect of varying risk-weights (in terms of τ_θ). As we have formally shown above, Figure 1 illustrates that the optimal second-best policy under a broad mandate overregulates both leverage and portfolio weights relative to the first-best. However, consistent with the insights discussed above, the relation between the first-best regulation and the second-best regulation for a regulator with a narrow mandate is more nuanced. In the case we illustrate, it turns out that a narrow regulator overregulates leverage relative to the first-best, but not portfolio weights. This is mainly due to the fact that the leakage elasticity with respect to capital is greater in magnitude for leverage, as Figures OA-1 and OA-2 illustrate. By contrast, a broad regulator overregulates both leverage and portfolio weights relative to first best, because she places a greater weight on all leakage elasticities to capital.

The Value of Regulating Scale To close the analysis of this application, we consider a regulator who is able to impose a corrective tax $\tau_k k^i$ on investors in order to correct for the (previously unregulated) externalities associated with the scale decision k^i. The key economic insights can be obtained by considering the marginal welfare effect of increasing τ_k.

Corollary 3. [Environmental Externalities/Regulating Unregulated Decision]

When the planner can impose a corrective tax τ_k on the total scale of investment k^i, the marginal welfare effect of varying τ_k is given by

$$\begin{aligned}\frac{dW}{d\tau_k} &= \underbrace{\frac{db^i}{d\tau_k}}_{=0}\left(\tau_b - \delta_b\right)k^i + \underbrace{\frac{d\theta^i}{d\tau_k}}_{=0}\left(\tau_\theta - \delta_\theta\right)k^i - \frac{dk^i}{d\tau_k}\left(\tau_k - \delta_k\right) \\ &= -\frac{dk^i}{d\tau_k}\omega_k.\end{aligned} \tag{48}$$

An interesting property of this environment is that there are no reverse leakage effects from regulating scale onto leverage and portfolio decisions. Intuitively, the investors' problem (see Lemma 1) can be broken down into a two-step procedure. First, investors choose leverage and portfolios to maximize market values $M\left(b^i, \theta^i\right)$ per unit of total capital. Second, they set the marginal cost of capital equal to its maximized market value. Since the first step does not depend on the cost/tax of capital, b^i and θ^i are independent of τ_k in equilibrium.

This fact has two novel economic implications. First, we note that the case for regulating scale here is much stronger than in other applications. In particular, the capital-specific elements of the Le Chatelier/reverse leakage adjustment matrix $\boldsymbol{L}$, which usually dampens the welfare impact of regulating unregulated decisions, are zero. Moreover, the case for regulating scale is clearly *stronger* when the regulator has a *broad/environmental mandate*, other things equal, since this mandate takes into account the full marginal distortion $\delta_k = \chi_k + \psi_k$.

Second, we see from Equation (48) that the optimal level of the tax on capital is always given by $\tau_k = \delta_k$, which corresponds to the first best or Pigouvian correction. The absence of reverse leakage implies that there is no incentive to over- or underregulate scale, once the regulator is allowed to do so. This is true even when the regulation of leverage and portfolio decisions is imperfect (with $\tau_b \neq \delta_b$ and/or $\tau_\theta \neq \delta_\theta$).

In summary, this application highlights how the tools we have developed can yield tractable and novel insights into the issue of "green" capital regulation. Several of these insights cannot be obtained without taking seriously the imperfections that regulatory instruments exhibit in practice.

5 Further Applications

In this section, we present four additional applications of our general results. This section has several purposes. First, the study of these applications allows us to show how our results can be employed to determine the optimal second-best policy in several scenarios of practical relevance. Second, these applications, which are special cases of the general framework studied in Sections 2 and 3, illustrate how our results encompass widely studied rationales for regulation, including bailouts, pecuniary externalities, and internalities. Third, by studying specific applications, we can connect leakage (and policy) elasticities and Pigouvian wedges to model primitives. Finally, we discuss how our results can be used to interpret existing empirical findings and guide future measurement efforts in the context of each application.

Table 1 provides a schematic summary of our applications. Each application is designed to be the simplest one that illustrates the form of the optimal second-best policy in a particular second-best scenario. In the Online Appendix, we provide detailed derivations for each application.[31]

Table 1: Summary of Applications

Application		Restricted Instrument	$\lvert\mathcal{I}\rvert$	$\lvert\mathcal{X}\rvert$
#1	Shadow Banking	Unregulated Investors	2	1
#2	Behavioral Distortions	Unregulated Activities	1	2
#3	Asset Substitution	Uniform Activity Regulation	1	2
#4	Pecuniary Externalities	Uniform Investor Regulation	2	1

Note: Table 1 summarizes our applications. The column $\lvert\mathcal{I}\rvert$ denotes the number of investors and the column $\lvert\mathcal{X}\rvert$ denotes the number of balance-sheet decisions.

In Application 1, we study a model in which some investors are unregulated and regulation is motivated by the presence of implicit government subsidies. In Application 2, we study an environment where regulation constrains the ratio of investors' risky investments to borrowing. In

[31] These applications are not exhaustive. For instance, one could study the role of imperfect corrective regulation in models of strategic behavior and imperfect competition, as in Corbae and D'Erasmo (2010) and Corbae and Levine (2018, 2019), or in the context of regulation of asset markets, as in Dávila (2014) or Cai, He, Jiang and Xiong (2020).

this application, to illustrate how our model applies to a different rationale for intervention, we consider a behavioral distortion (distorted beliefs). In Application 3, regulation is constrained to be uniform across different investment activities, with intervention motivated by government bailouts, which yields new insights into asset substitution problems. In the final application, we analyze a model of fire-sale externalities, along the lines of Lorenzoni (2008), in which regulation is also constrained to be uniform across different types of investors.

5.1 Application 1: Shadow Banking/Unregulated Investors

The notion of shadow banking is typically used to describe the financial activities that take place outside of the regulated financial sector.[32] In this application, we consider an environment with two types of investors, in which only one type of investor can be regulated (the traditional sector), while the other is outside of the scope of the regulation (the shadow sector).

Environment. We assume that there are two types of investors $i \in \{1, 2\}$. In this application, investors should be broadly interpreted as financial intermediaries or banks. Investors have risk-neutral preferences of the form:

$$c_0^i + \beta^i \int c_1^i(s)\, dF(s),$$

with budget constraints given by

$$\begin{aligned} c_0^i &= n_0^i + Q^i\left(b^i\right) - \tau_b^i b^i + T_0^i \\ c_1^i(s) &= n_1^i(s) + \max\left\{v^i s + t^i\left(b^i, s\right) - b^i, 0\right\}, \quad \forall s. \end{aligned}$$

At date 0, an investor i endowed with n_0^i dollars chooses the face value of its debt, b^i, which determines the amount of financing obtained at date 0, $Q^i(b^i)$, determined in equilibrium by creditors, as described below. Investor i faces a corrective tax τ_b^i per unit of b^i due at date 0. At date 1 in state s, investor i receives $v^i s$ dollars, as well as a bailout transfer $t^i(b^i, s)$.

Creditors are risk-averse, with preferences of the form

$$u\left(c_0^C\right) + \beta^C \int u\left(c_1^C(s)\right) dF(s).$$

Their budget constraints are given by

$$\begin{aligned} c_0^C &= n_0^C - \sum_{i \in \mathcal{I}} h^i Q^i\left(b^i\right), \\ c_1^C(s) &= n_1^C(s) + \sum_{i \in \mathcal{I}} h^i \mathcal{P}^i\left(b^i, s\right) - (1+\kappa) \sum_{i \in \mathcal{I}} t^i\left(b^i, s\right), \quad \forall s, \end{aligned}$$

where h^i is the fraction of bonds purchased from investor i, and $\mathcal{P}^i(b^i, s)$ denotes the repayment

[32] Pozsar, Adrian, Ashcraft and Boesky (2010), Gorton, Metrick, Shleifer and Tarullo (2010), and Claessens, Pozsar, Ratnovski and Singh (2012) provide a detailed overview of shadow banking institutions, activities, and regulations.

received by creditors from investor i in state s, as we explicitly describe in the Online Appendix. At date 1, all bailout funds are raised from creditors, with a constant net marginal cost of public funds $\kappa \geq 0$. Note that investors only interact in this application through changes in the price of credit, i.e., through the stochastic discount factor of creditors: $m^C(s) = \frac{\beta^C u'\left(c_1^C(s)\right)}{u'\left(c_0^C\right)}$.

Equilibrium. In this application, for given corrective taxes/subsidies $\{\tau_b^1, \tau_b^2\}$, lump-sum transfers $\{T_0^1, T_0^2\}$, and bailout transfers $\{t^1(b^1, s), t^2(b^2, s)\}$, an *equilibrium* is fully determined by investors' borrowing decisions, $\{b^1, b^2\}$, and financing schedules, $\{Q^1(b^1), Q^2(b^2)\}$, such that investors maximize their utility, given the financing schedules, and creditors set the schedules optimally, so that $h^1 = h^2 = 1$.

In the first-best scenario, the planner is able to set τ_b^1 and τ_b^2 freely. However, we are interested in scenarios in which the planner cannot regulate type 2 investors, so

$$\tau_b^2 = 0,$$

which makes the problem of choosing the optimal τ_b^1 a second-best problem.

Optimal Corrective Policy/Simulation. First, in Proposition 9, we characterize the form of the optimal second-best policy. Next, we explore a numerical simulation of this application.

Proposition 9. [Shadow Banking/Unregulated Investors]
a) The marginal welfare effect of varying the corrective regulation of regulated investors, τ_b^1, is given by

$$\frac{dW}{d\tau_b^1} = \frac{db^1}{d\tau_b^1}\left(\tau_b^1 - \delta_b^1\right) - \frac{db^2}{d\tau_b^1}\delta_b^2,$$

where the marginal distortions in this application are defined by

$$\delta_b^i = (1+\kappa)\int m^C(s)\,\frac{\partial t^i(b^i, s)}{\partial b^i}\,dF(s), \tag{49}$$

where $m^C(s)$ denotes the stochastic discount factor of creditors.
b) The optimal corrective regulation satisfies

$$\tau_b^1 = \delta_b^1 - \left(-\frac{db^1}{d\tau_b^1}\right)^{-1}\frac{db^2}{d\tau_b^1}\delta_b^2.$$

Proposition 9 is an application of Propositions 3 and 5 and exploits the structure of this application to extract further insights. In this application, the marginal distortions associated with borrowing, δ_b^i, are determined by the expected marginal bailout $\frac{\partial t^i(b^i,s)}{\partial b^i}$, augmented by default deadweight losses κ if present, valued using the creditors' stochastic discount factor. The departure of the optimal regulation from the first-best critically depends on the leakage elasticity $\frac{db^2}{d\tau_b^1}$ and the unregulated distortion δ_b^2. A number of recent studies provide direct measurements of the relevant

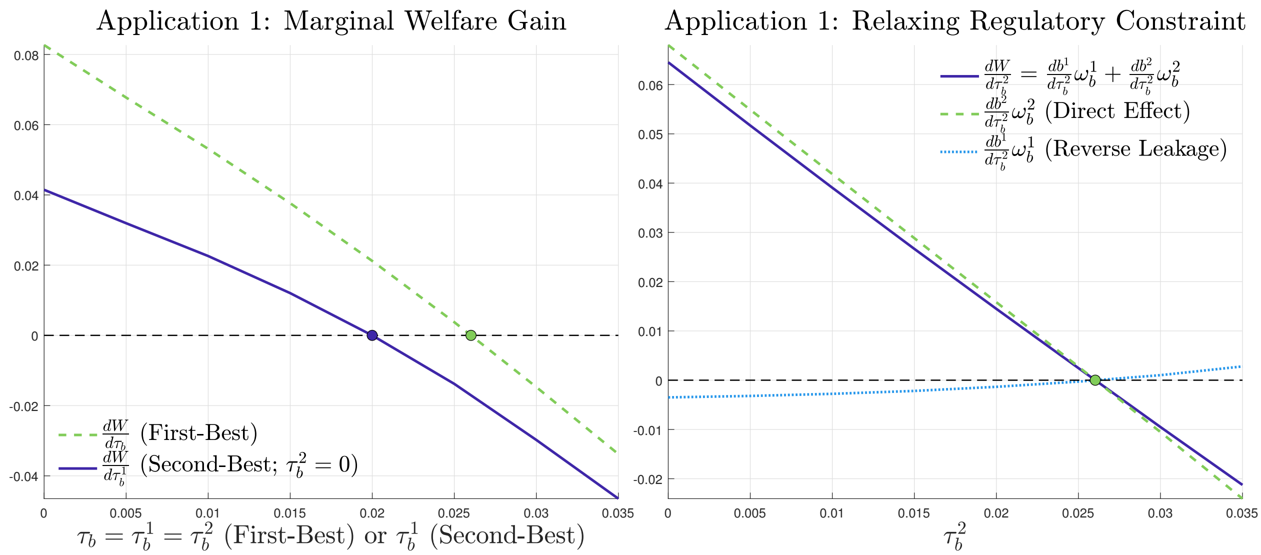

Figure 2: Shadow Banking/Unregulated Investors (Application 1)

Note: The left panel of Figure 2 compares the marginal welfare effects of varying corrective regulations in two different scenarios. The green dashed line corresponds to the first-best scenario in which the horizontal axis corresponds to $\tau_b = \tau_b^1 = \tau_b^2$. The solid blue line corresponds to a second-best scenario in which $\tau_b^2 = 0$ and the horizontal axis corresponds to τ_b^1. Since we assume that both types of investors are symmetric, the value of τ_b that makes the first-best marginal welfare effect zero defines the first-best regulation. The value of τ_b^1 that makes the second-best marginal welfare effect zero defines the second-best regulation.
The right panel of Figure 2 illustrates Proposition 4 by showing the marginal value of being able to regulate the shadow sector. The solid dark blue line corresponds to the total marginal welfare gain of increasing τ_b^2, while τ_b^1 is continually adjusted to be at the optimal second-best value given τ_b^2. The total gain can be decomposed into a direct effect, which corresponds to $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\boldsymbol{\omega}^U$ in Equation (24), and a reverse leakage effect, which corresponds to $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\boldsymbol{L}\boldsymbol{\omega}^U$ in Equation (24). The green dashed line corresponds to the direct effect of relaxing the regulatory constraint, while the light blue dotted line corresponds to the reverse leakage effect. Note that both the direct effect and the reverse leakage effect are zero at the first-best, when $\tau_b = \tau_b^1 = \tau_b^2 = 2.60\%$, but have opposite signs otherwise.
To generate this figure, we assume that the bailout policy is linearly separable: $t^i\left(b^i, s\right) = \alpha_0^i - \alpha_s^i s + \alpha_b^i b^i$, and that creditors' utility is isoelastic: $u\left(c\right) = \frac{c^{1-\gamma}}{1-\gamma}$. The parameters used to generate this figure are $\beta^i = 0.7$, $\phi^i = 0.25$, $v^i = 1$, $\alpha_0^i = \alpha_s^i = 0$, $\alpha_b^i = 0.01$, for $i \in \{1, 2\}$. Also $\kappa = 0.13$, $\gamma = 6$, $\beta^C = 0.98$, $n_0^C = 50$, and $n_1^C\left(s\right) = 50 + 0.1s$, where s is normally distributed with mean 1.3 and standard deviation 0.3, truncated to the interval $[0, 3]$. For reference, the optimal first-best regulation is $\tau_b^1 = \tau_b^2 = 2.60\%$, while the optimal second-best regulation, when the second type of investors cannot be regulated, is $\tau_b^1 = 1.99\%$. Since borrowing decisions are gross substitutes in this application, the optimal second-best policy is *sub-Pigouvian*.

leakage elasticity (e.g., Buchak, Matvos, Piskorski and Seru, 2018*a*; Irani, Iyer, Meisenzahl and Peydró, 2020).[33] As we show in the Online Appendix, in this application, consistent with the empirical literature, we find that tighter regulation on the regulated sector (higher τ_b^1) increases the activities carried out by the unregulated/shadow sector ($\frac{db^2}{d\tau_b^1} > 0$), so leverage choices are gross substitutes. Therefore, we expect the optimal second-best policy to be sub-Pigouvian.[34]

Moreover, the presence of unregulated investors may exacerbate the welfare distortion δ_b^1 associated with regulated investors. Concretely, when unregulated investors receive bailouts in state s, the marginal utility of creditors increases in this state due to taxation. In Equation (49), this increases the distortion associated with marginal increases in regulated investors' leverage. In this sense, our results reconcile two common narratives. On the one hand, leakage to the shadow banking system motivates sub-Pigouvian regulation. On the other hand, the optimal corrective policy must also adjust to increases in overall leverage, which raise marginal distortions δ_b^1 in general equilibrium.

An instructive special case, which we use to solve the model numerically, is obtained by using a linearly separable bailout policy: $t^i(b^i, s) = \alpha_0^i - \alpha_s^i s + \alpha_b^i b^i$, where $\alpha_s^i, \alpha_b^i \geq 0$. In this case, marginal distortions $\delta_b^i = \frac{1+\kappa}{R^f}\alpha_b^i$ are invariant to policy, and the optimal corrective regulation is

$$\tau_b^1 = \frac{1+\kappa}{R^f}\left[\alpha_b^1 - \left(-\frac{db^1}{d\tau_b^1}\right)^{-1}\frac{db^2}{d\tau_b^1}\alpha_b^2\right],$$

where $R^f = \left(\int m^C(s)\, dF(s)\right)^{-1}$ denotes the creditors' riskless discount rate.

The left panel of Figure 2 illustrates the comparison between the first-best and second-best policy when simulating this model.[35] To more clearly illustrate the insights that we present in this paper, in Figure 2 we assume that both types of investors are ex-ante identical, and that the only difference between the two is that investor 2 cannot be regulated. Given this symmetry assumption, it is possible to represent the marginal value of varying the regulation $\tau_b = \tau_b^1 = \tau_b^2$ for both investors, which yields the first-best regulation when $\frac{dW}{d\tau_b} = 0$. In contrast, the solid line in

[33] This work focuses on the elasticity of substitution between the market share of regulated and unregulated investments. Due to space constraints, we have held the scale of investment fixed in this application, but one could easily extend the framework to account for both leverage and investment choices, in which case the measured elasticities of substitution in those papers become directly relevant. In addition, our application highlights that the elasticity of substitution between regulated and unregulated leverage is a key statistic for second-best regulation.

[34] Note that one can also use this model to analyze quantity-based policies, such as capital requirements. For instance, suppose that regulated investors are subject to a binding quantity regulation $b^1 \leq \bar{b}^1$, where the regulator chooses the upper bound $\bar{b}^1$. In our model, a marginal change $d\bar{b}^1$ is equivalent to the local tax reform $d\tau_b^1 = \left(\frac{db^1}{d\tau_b^1}\right)^{-1} d\bar{b}^1$. The associated leakage elasticity is $\frac{db^2}{d\bar{b}^1} = \left(\frac{db^1}{d\tau_b^1}\right)^{-1}\frac{db^2}{d\tau_b^1}$, and the optimal corrective regulation in Proposition 9 can be alternatively expressed as

$$\tau_b^1 = \delta_b^1 + \frac{db^2}{d\bar{b}^1}\delta_b^2.$$

[35] Figure OA-3 in the Online Appendix illustrates comparative statistics of different endogenous outcomes of the model that are useful to better understand the form of the optimal second-best policy. Figures OA-4 through OA-6 in the Online Appendix do the same for the remaining applications.

Figure 2 shows the marginal value of varying the regulation that investor 1 faces (the traditional sector), when investor 2 (the shadow sector) is unregulated, that is, when $\tau_b^2 = 0$. As implied by our theoretical results, since $\frac{db^2}{d\tau_b^1} > 0$ and $\frac{db^1}{d\tau_b^1} < 0$, we find that the optimal second-best policy is sub-Pigouvian, so the optimal second-best regulation that investor 1 faces is lower than the first-best regulation. In this particular simulation, the optimal first-best regulation is $\tau_b^1 = \tau_b^2 = 2.60\%$, while the second-best regulation (when $\tau_b^2 = 0$) is $\tau_b^1 = 1.99\%$.

The right panel of Figure 2 illustrates Proposition 3 by showing the marginal value of being able to regulate the shadow sector. This panel provides a clear illustration of the Le Chatelier/reverse leakage adjustment discussed above. Regardless of whether the shadow sector is underregulated (when τ_b^2 is below first-best) or overregulated when (when τ_b^2 is above first-best), the reverse leakage effect has the opposite sign of the direct effect of adjusting the regulation of the shadow sector, attenuate welfare gains/losses. This illustrates how the presence of perfectly regulated decisions contributes to attenuating the welfare gains of relaxing regulatory constraints.

5.2 Application 2: Behavioral Distortions/Unregulated Activities

In this application, we characterize the form of the optimal scale-invariant policy in a model in which regulation is motivated by belief distortions.

Environment. We assume that there is a single type of investor, in unit measure and indexed by i, and a unit measure of creditors, indexed by C. Both investors and creditors have risk-neutral preferences given by

$$c_0^i + \beta^i \int c_1^i(s)\, dF^i(s) \quad \text{and} \quad c_0^C + \beta^C \int c_1^C(s)\, dF^C(s),$$

where $F^i(s)$ and $F^C(s)$ denote the beliefs (cumulative distribution functions) of investors and creditors over the possible states. Endowments and technologies are specified as in our canonical model in Section 4, with the simplification that investors do not choose the composition of their capital portfolio. Accordingly, the budget constraints of investors at date 0 and date 1 are given by

$$\begin{aligned} c_0^i &= n_0^i + Q^i\left(b^i\right) k^i - \Upsilon\left(k^i\right) \\ c_1^i(s) &= n_1^i(s) + \max\left\{s - b^i, 0\right\} k^i, \quad \forall s. \end{aligned}$$

Creditors' budget constraints are given by

$$\begin{aligned} c_0^C &= n_0^C - h^i Q^i\left(b^i\right) k^i \\ c_1^C(s) &= n_1^C(s) + h^i \mathcal{P}^i\left(b^i, s\right) k^i, \quad \forall s, \end{aligned}$$

where $\mathcal{P}^i\left(b^i, s\right)$ denotes the repayment received by creditors from investors in state s per unit of investment, explicitly described in the Online Appendix.

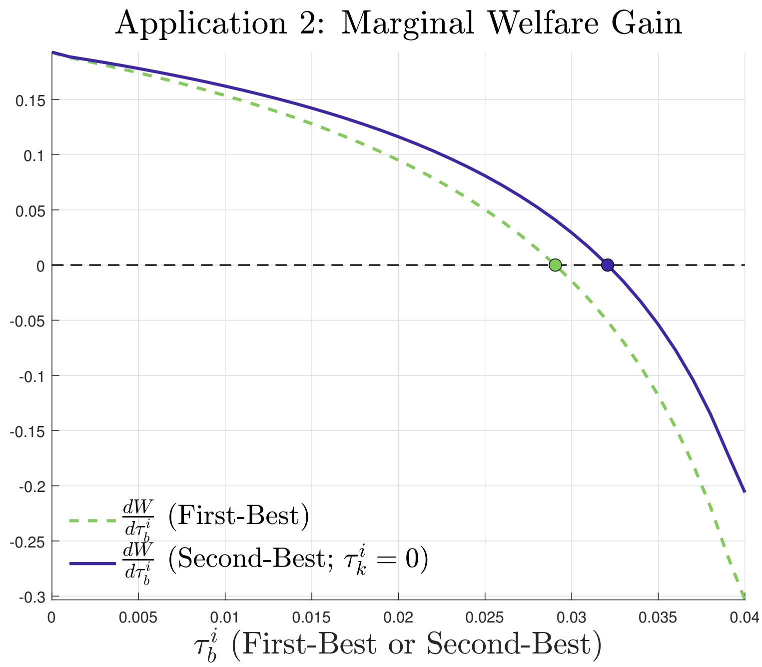

Figure 3: Behavioral Distortions/Unregulated Activities (Application 2)

Note: Figure 3 compares the marginal welfare effects of varying the corrective regulation in two different scenarios. The green dashed line corresponds to a scenario in which τ_k^i is set at the first-best level. The solid blue line corresponds to a second-best scenario in which $\tau_k^i = 0$. Therefore, the value of τ_b^i that makes the first-best marginal welfare effect zero defines the first-best leverage regulation, since τ_k^i is already set at the first-best level. The value of τ_b^i that makes the second-best marginal welfare effect zero defines the second-best regulation. To generate this figure, we assume that the adjustment cost is quadratic: $\Upsilon\left(k^i\right) = \frac{a}{2}\left(k^i\right)^2$. The parameters used to generate this figure are $\beta^i = 0.9$, $\beta^C = 0.95$, $\phi^i = 0.8$, and $a = 1$. We assume that investors and creditors perceive s to be normally distributed with mean 1.5 and standard deviation 0.4, and the planner perceives the mean to be 1.3 instead. For reference, the optimal first-best regulation is given by $\tau_b^i = 2.91\%$ and $\tau_k^i = 18.45\%$, while the second-best regulation, when investment cannot be regulated, is $\tau_b^i = 3.21\%$. Since leverage and investment decisions are gross complements in this application, the optimal second-best policy is *super-Pigouvian*.

As in Section 4, we consider regulation via a capital requirement

$$b^i \leq \bar{b}.$$

We show below that this is equivalent to a corrective tax on leverage choices b^i.

We assume that the planner computes welfare using different probability assessments than those used by investors and creditors to make decisions. This provides a corrective rationale for intervention. As highlighted in Dávila and Walther (2020*b*) and Proposition 10 below, the rationale for regulation is determined by the difference between private agents' and the planner's valuations per unit of risky investment, which represent a levered version of Tobin's q. These valuations are, respectively, given by

$$M\left(b^{i}\right)=\beta^{i}\int\max\left\{s-b^{i},0\right\}dF^{i}\left(s\right)+\beta^{C}\int\mathcal{P}^{i}\left(b^{i},s\right)dF^{C}\left(s\right)$$
$$M^{P}\left(b^{i}\right)=\beta^{i}\int\max\left\{s-b^{i},0\right\}dF^{P}\left(s\right)+\beta^{C}\int\mathcal{P}^{i}\left(b^{i},s\right)dF^{P}\left(s\right),$$

where $F^{P}(s)$ denotes the probability distribution used by the planner to calculate welfare.

Equilibrium. In this application, for a given leverage cap $\bar{b}$, an *equilibrium* is defined by an investment decision, k^i, a leverage decision, b^i, and a default decision rule such that i) investors maximize their utility given $Q^i(\cdot)$, and ii) creditors set the schedule $Q^i(\cdot)$ optimally, so that $h^i=1$.

In the first-best scenario, the planner is able to set corrective taxes on both leverage and investment. In this application, the planner's only instrument is the leverage cap $\bar{b}$, which is imperfect. This can be seen by writing investors' first-order conditions as

$$\frac{\partial M\left(b^{i}\right)}{\partial b^{i}}=\mu\equiv\tau_{b}$$
$$M\left(b^{i},\theta^{i}\right)-1-\Upsilon'\left(k^{i}\right)=0,$$

As in Section 4, the planner can therefore impose an effective tax on leverage via $\bar{b}$, but cannot affect investors' marginal incentive to create investment capital k^i.

Optimal Corrective Policy/Simulation. In Proposition 10, we characterize the form of the optimal second-best policy, which we discuss along with a numerical simulation.

Proposition 10. [**Behavioral Distortions/Unregulated Activities**]
a) The marginal welfare effect of varying the regulation of investors' leverage, τ_b^i, is given by

$$\frac{dW}{d\tau_{b}^{i}}=\frac{db^{i}}{d\tau_{b}^{i}}\left(\tau_{b}^{i}-\delta_{b}^{i}\right)-\frac{dk^{i}}{d\tau_{b}^{i}}\delta_{k}^{i},$$

where the marginal distortions in this application are defined by

$$\delta_{b}^{i}=\frac{dM\left(b^{i}\right)}{db^{i}}-\frac{dM^{P}\left(b^{i}\right)}{db^{i}}$$
$$\delta_{k}^{i}=M\left(b^{i}\right)-M^{P}\left(b^{i}\right).$$

b) The optimal corrective regulation satisfies

$$\tau_{b}^{i}=\delta_{b}^{i}-\left(-\frac{db^{i}}{d\tau_{b}^{i}}\right)^{-1}\frac{dk^{i}}{d\tau_{b}^{i}}\delta_{k}^{i}.$$

Proposition 10 is the counterpart of Propositions 3 and 5, and it identifies the distortions associated with leverage and investment the planner perceives. In this application, the welfare distortion associated with leverage, δ_b^i, is driven by the difference in marginal valuations, while the distortion associated with investment, δ_k^i, is driven by the difference in the level of valuations. In this application we have $\frac{db^i}{d\tau_b^i} < 0$ and, critically, the leakage elasticity from leverage to investment is negative, that is, $\frac{dk^i}{d\tau_b^i} < 0$, implying that leverage and investment are gross complements. As implied by our results in Section 3, the optimal second-best regulation on leverage is super-Pigouvian.

Importantly, a comparison between this application with the previous one (shadow banking) highlights that both leakage elasticities featuring substitutes and those featuring complements are important in common regulatory scenarios. A number of recent empirical studies confirm that the leakage elasticity from leverage to risky investments is negative, in the sense that banks with lower capital ratios originate a larger volume of risky loans (e.g., Jiménez, Ongena, Peydró and Saurina, 2014; Dell'Ariccia, Laeven and Suarez, 2017; Acharya, Eisert, Eufinger and Hirsch, 2018).

Figure 3 compares the marginal welfare effects of varying regulation in the first-best and second-best scenarios when simulating this model. To illustrate the first-best solution for leverage, we fix τ_k^i to its first-best value when showing the marginal welfare associated with varying τ_b^i. The second-best marginal welfare gain simply sets $\tau_k^i = 0$. As implied by our theoretical results, the optimal second-best policy is super-Pigouvian, so it is optimal for the planner to overregulate leverage relative to the first-best scenario. In this particular simulation, the optimal first-best regulation is $\tau_b^i = 2.91\%$ and $\tau_k^i = 18.45\%$, while the second-best regulation (when $\tau_k^i = 0$) is $\tau_b^i = 3.21\%$.

5.3 Application 3: Asset Substitution/Uniform Activity Regulation

A common concern in financial regulation is that corrective policy instruments are somewhat coarse in practice. For example, when imposing capital requirements on banks, financial regulators tend to set risk weights for wide classes of risky investments (e.g., mortgage loans), but within the class, banks can freely optimize their portfolios (e.g., among loans to borrowers with different credit scores) without any change in the associated capital charge. In our model, this situation corresponds to a uniform regulation across different capital investments. In this application, we consider uniform corrective policy in a model where investors enjoy government guarantees. We use the properties of uniform regulation to derive new insights into the classical asset substitution problem (e.g., Jensen and Meckling, 1976), and characterize the optimal second-best policy.

Environment. We assume that there is a single type of investor, in unit measure and indexed by i, and a unit measure of creditors, indexed by C. Both investors and creditors have risk-neutral preferences given by

$$c_0^i + \beta^i \int c_1^i(s)\, dF(s) \quad \text{and} \quad c_0^C + \beta^C \int c_1^C(s)\, dF(s)\,.$$

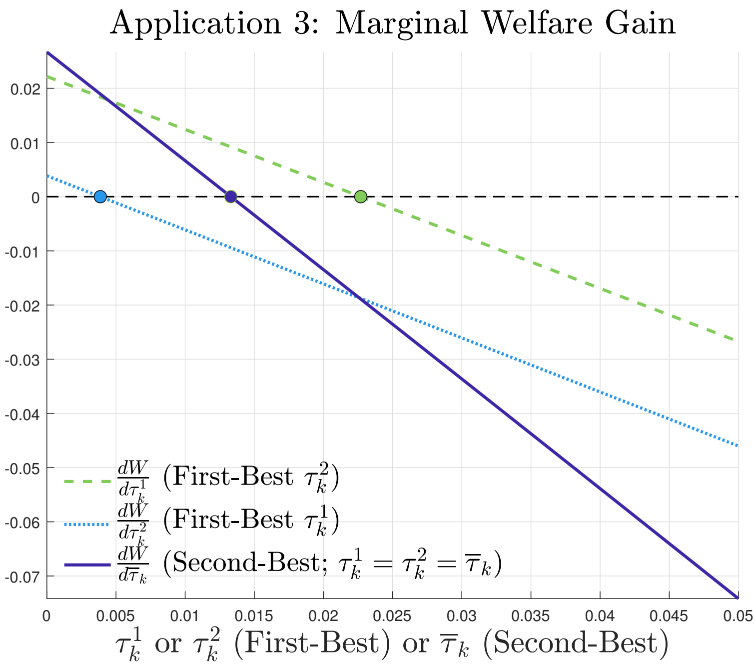

Figure 4: Asset Substitution/Uniform Activity Regulation (Application 3)

Note: Figure 4 compares the marginal welfare effects of varying the corrective regulation in two different scenarios. The green dashed line and the light blue dotted line illustrate the first-best regulation. The green dashed line corresponds to a scenario in which τ_k^2 is set at the first-best level (previously computed), while the light blue dotted line corresponds to a scenario in which τ_k^1 is set at the first-best level (previously computed). Therefore, the values of τ_k^1 and τ_k^2 that respectively make each line zero define the first-best regulation. The solid dark blue line corresponds to a second-best scenario in which $\overline{\tau}_k = \tau_k^1 = \tau_k^2$, so its zero defines the second-best regulation. To generate this figure, we assume that the adjustment cost is quadratic: $\Upsilon\left(k_1^i, k_2^i\right) = \frac{z_1}{2}\left(k_1^i\right)^2 + \frac{z_2}{2}\left(k_2^i\right)^2$. We also assume that $d_1(s) = \mu_1 + \sigma_1 s$ and $d_2(s) = \mu_2 + \sigma_2 s$ when s is distributed as a standard normal. The parameters used to generate this figure are $\beta^i = 0.8$, $\beta^C = 1$, $\kappa = 0.1$, $z_1 = z_2 = 1$, $b^i = 1.4$, $\mu_1 = 1.5$, $\mu_2 = 1.3$, $\sigma_1 = 0.3$, and $\sigma_2 = 0.5$. For reference, the optimal first-best regulation is given by $\tau_k^1 = 2.27\%$ and $\tau_k^2 = 0.39\%$, while the second-best regulation, when the regulation is uniform, is $\overline{\tau}_k = 1.33\%$.

The budget constraints of investors at date 0 and date 1 are given by

$$c_0^i = n_0^i - \Upsilon\left(k_1^i, k_2^i\right) - \tau_k^1 k_1^i - \tau_k^2 k_2^i + T_0^i$$
$$c_1^i(s) = \max\left\{d_1(s)\, k_1^i + d_2(s)\, k_2^i + t\left(k_1^i, k_2^i, b^i, s\right) - b^i, 0\right\}, \quad \forall s.$$

At date 0, investors, endowed with n_0^i dollars, choose the scale of two risky capital investments k_1^i and k_2^i, which are subject to an adjustment cost of $\Upsilon\left(k_1^i, k_2^i\right)$. Hence, investors make $|\mathcal{X}| = 2$ free choices regarding their balance-sheet.

At date 1, investors earn the realized returns on capital investments k_1^i and k_2^i, which are given by $d_1(s)$ and $d_2(s)$ and are increasing in s. In addition, they receive a bailout transfer $t\left(k_1^i, k_2^i, b^i, s\right)$ from the government. We further assume that investors have legacy debt (i.e., debt issued before the start of the model) with face value b^i. Hence, investors owe a predetermined repayment of b^i to creditors at date 1. We make this simplifying assumption in order to sharpen our focus on *asset substitution*, which describes investors' choice between different risky investments, as opposed to leverage choices. At date 1, investors consume the difference between i) the cash flow from investments augmented by the bailout transfer and ii) the debt owed, if this difference is positive. Otherwise, they default and consume zero.

For simplicity, we focus on a particular form of bailout that fully prevents default — this may correspond to an investor that is "too big to fail". Concretely, we assume that the government bailout is equal to the minimum amount required to avoid default

$$t\left(k_1^i, k_2^i, b^i, s\right) = \max\left\{b^i - d_1(s)\,k_1^i - d_2(s)\,k_2^i, 0\right\}. \tag{50}$$

Given this form of bailout policy, creditors are guaranteed a repayment of b^i at date 1. We write $s^\star\left(k_1^i, k_2^i\right)$ for the threshold state below which bailouts are positive.[36]

Hence, the budget constraints of creditors at date 0 and date 1 are given by

$$\begin{aligned} c_0^C &= n_0^C \\ c_1^C(s) &= n_1^C(s) + b^i - (1+\kappa)\,t\left(k_1^i, k_2^i, b^i, s\right), \ \forall s. \end{aligned}$$

Even though creditors are always repaid b^i in every state, we assume that in order to finance the bailout, the government imposes a tax of $(1+\kappa)$ per dollar of bailout on creditors, where $\kappa > 0$ measures the deadweight fiscal cost of bailout transfers. The rationale for regulation in this environment is a classical "moral hazard" argument. Investors, whose debt is implicitly guaranteed by the government, do not internalize the impact of their risky capital investments on fiscal costs, which ultimately reduces the consumption of creditors.

Equilibrium. In this application, for given corrective taxes/subsidies $\{\tau_k^1, \tau_k^2\}$, lump-sum transfers $T_0^i = \tau_k^1 k_1^i + \tau_k^2 k_2^i$, bailout policy $t\left(k_1^i, k_2^i, b^i, s\right)$, and legacy debt b^i, an *equilibrium* is defined by investment decisions such that investors maximize their utility. In the first-best scenario, the planner is able to set τ_k^1 and τ_k^2 freely. However, we are interested in a scenario in which the planner is unable to treat investments differentially for regulation purposes. Thus, the planner chooses $\tau_k^1 \geq 0$ and $\tau_k^2 \geq 0$ subject to the uniform regulation constraint:

$$\overline{\tau}_k = \tau_k^1 = \tau_k^2.$$

Optimal Corrective Policy/Simulation. In Proposition 11 we characterize the form of the second-best policy, which we discuss along with a numerical simulation.

Proposition 11. [Asset Substitution/Uniform Activity Regulation]
a) The marginal welfare effect of varying the uniform corrective regulation of capital investments, $\overline{\tau}_k = \tau_k^1 = \tau_k^2$, is given by

$$\frac{dW}{d\overline{\tau}_k} = \frac{dk_1^i}{d\overline{\tau}_k}\left(\overline{\tau}_k - \delta_1\right) + \frac{dk_2^i}{d\overline{\tau}_k}\left(\overline{\tau}_k - \delta_2\right),$$

[36] Formally, for a fixed value b^i of legacy debt, this threshold is the unique solution to $b^i - d_1(s)\,k_1^i - d_2(s)\,k_2^i = 0$.

where the marginal distortions in this application are defined by

$$\delta_j = (1+\kappa)\,\beta^C \int_{\underline{s}}^{s^\star\left(k_1^i, k_2^i\right)} d_j(s)\, dF(s)\,.$$

b) The optimal corrective regulation satisfies

$$\overline{\tau}_k = \frac{\frac{dk_1^i}{d\overline{\tau}_k}}{\frac{dk_1^i}{d\overline{\tau}_k} + \frac{dk_2^i}{d\overline{\tau}_k}}\delta_1 + \frac{\frac{dk_2^i}{d\overline{\tau}_k}}{\frac{dk_1^i}{d\overline{\tau}_k} + \frac{dk_2^i}{d\overline{\tau}_k}}\delta_2.$$

Proposition 11 identifies the distortions associated with the different types of investment decisions in this application. The shape of the distortions δ_j highlights the nature of the asset substitution problem: investors' private incentives are driven by the returns to investment in "upside" states $s \geq s^\star\left(k_1^i, k_2^i\right)$, while the planner's concern about bailouts focuses on "downside" states $s < s^\star\left(k_1^i, k_2^i\right)$. The optimal uniform regulation is a weighted average of the downside distortions imposed by both types of capital. As implied by our general results in Section 3, the appropriate weight assigned by the planner to each of the distortions in the optimal second-best policy is given by how sensitive each capital decision is to changes in the regulation, $\frac{\frac{dk_1^i}{d\overline{\tau}_k}}{\frac{dk_1^i}{d\overline{\tau}_k} + \frac{dk_2^i}{d\overline{\tau}_k}}$ and $\frac{\frac{dk_2^i}{d\overline{\tau}_k}}{\frac{dk_1^i}{d\overline{\tau}_k} + \frac{dk_2^i}{d\overline{\tau}_k}}$. Figure 4 illustrates this intuition by comparing the marginal welfare effects of varying regulation in the first-best and second-best scenarios.

In the Online Appendix, assuming that investment costs are quadratic, we provide further intuition on how the weights $\frac{dk_1^i}{d\overline{\tau}_k}$ and $\frac{dk_2^i}{d\overline{\tau}_k}$ are determined. We show that the sufficient statistics for the optimal weights are i) the sensitivity of the probability of receiving a bailout to the uniform regulation, and ii) the marginal contribution $d_n(s^\star)$ of each asset class at the bailout boundary. Intuitively, a large ratio $\frac{d_2(s^\star)}{d_1(s^\star)}$ means that changes in the default boundary affect mostly returns to k_2^i, which makes investors' optimal investment in k_2^i more sensitive to the uniform regulation.

5.4 Application 4: Pecuniary Externalities/Uniform Investor Regulation

Pecuniary/fire-sale externalities coupled with incomplete markets and/or collateral constraints provide a well-studied rationale for corrective macro-prudential regulation. The natural notion of efficiency in those environments, constrained efficiency, typically requires individual-specific regulations, which can be mapped to our first-best benchmark. In this application, we study the form of the second-best policy in an environment in which it would be optimal to set investor-specific regulations, but the planner is constrained to set the same corrective regulation for all investors.

Environment. We assume that there are two types of investors/entrepreneurs, indexed by $i \in \{1, 2\}$, and households, indexed by H — who in a richer model would also play the role of creditors. There are three dates, $t \in \{0, 1, 2\}$ and no uncertainty.[37] Investors, who for simplicity do not discount the future, have preferences of the form:

$$u^i = c_0^i + c_1^i + c_2^i,$$

subject to non-negativity constraints, $c_0^i \geq 0$, $c_1^i \geq 0$, $c_2^i \geq 0$, where their budget constraints are given by

$$\begin{aligned} c_0^i &= n_0^i - \Upsilon^i\left(k_0^i\right) - \tau_k^i k_0^i + T_0^i \\ c_1^i &= q\left(k_0^i - k_1^i\right) - \xi^i k_0^i \\ c_2^i &= z^i k_1^i. \end{aligned}$$

At date 0, an investor i endowed with n_0^i dollars chooses how much to produce, k_0^i, given a technology $\Upsilon^i\left(k_0^i\right)$. Investor i also faces a corrective tax τ_k^i per unit invested at date 0. At date 1, an investor i must reinvest $\xi^i > 0$ per unit of invested capital at date 0, which needs to be satisfied by selling $k_0^i - k_1^i$ units of capital at a market price q — this is a simple way to generate a fire-sale. At the final date, whatever capital is left yields an output $z^i k_1^i$. For simplicity, we assume that, in equilibrium, $T_0^i = \tau_k^i k_0^i$, $\forall i$.

Households, who exclusively consume at date 1, have access to a decreasing returns to scale technology to transform capital into output at date 1. Formally, the utility of households is given by

$$u^H = c_1^H = F\left(k_1^H\right) - qk_1^H,$$

where $F(\cdot)$ is a well-behaved concave function and k_1^H denotes the amount of capital purchased by households at date 1. The solution to the households' problem will define a downward sloping demand curve for sold capital at date 1.

Equilibrium. In this application, for given corrective taxes/subsidies $\{\tau_k^1, \tau_k^2\}$ and lump-sum transfers $\{T_0^1, T_0^2\} = \{\tau_k^1 k_0^1, \tau_k^2 k_0^2\}$, an *equilibrium* is fully determined by investors/entrepreneurs' investment decisions $\{k_0^i, k_1^i\}$ at dates 0 and 1, households' capital allocation k_1^H at date 1, and an equilibrium price q, such that investors' and households' utilities are maximized, subject to constraints, and the capital market clears, that is, $\sum_i \left(k_0^i - k_1^i\right) = k_1^H$.

In the first-best scenario, the planner is able to set τ_k^1 and τ_k^2 freely. However, we are interested in scenarios in which the planner must regulate both investors equally, so

$$\overline{\tau}_k = \tau_k^1 = \tau_k^2,$$

which makes the problem of choosing the optimal $\overline{\tau}_k$ a second-best problem.

In Section D of the Online Appendix, we provide a detailed characterization of the equilibrium.

[37] It is well known that for pecuniary externalities to matter there must be more than one trading stage. The two final dates in this application can be mapped to the second date in Section 2.

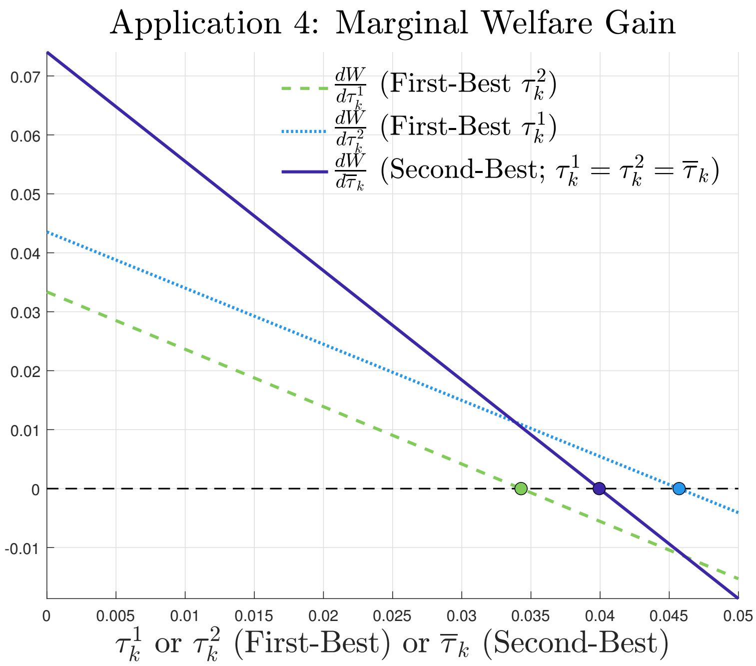

Figure 5: Pecuniary Externalities/Uniform Investor Regulation (Application 4)

Note: Figure 5 compares the marginal welfare effects of varying the corrective regulation in two different scenarios. The green dashed line and the light blue dotted line illustrate the first-best regulation. The green dashed line corresponds to a scenario in which τ_k^2 is set at the first-best level (previously computed), while the light blue dotted line corresponds to a scenario in which τ_k^1 is set at the first-best level (previously computed). Therefore, the values of τ_k^1 and τ_k^2 that respectively make each line zero define the first-best regulation. The solid dark blue line corresponds to a second-best scenario in which $\tau_k^1 = \tau_k^2 = \overline{\tau}_k$, so its zero defines the second-best regulation. To generate this figure, we assume that the adjustment cost of investment is quadratic: $\Upsilon^i\left(k_0^i\right) = \frac{a^i}{2}\left(k_0^i\right)^2$, and that the technology of households is isoelastic: $F\left(k_1^H\right) = \frac{\left(k_1^H\right)^\alpha}{\alpha}$. The parameters used to generate this figure are $\alpha = 0.5$, $a^1 = a^2 = 1$, $z^1 = z^2 = 1.5$, $\xi^1 = 0.3$, and $\xi^2 = 0.4$. For reference, the optimal first-best regulation is given by $\tau_k^1 = 3.43\%$ and $\tau_k^2 = 4.57\%$, while the second-best regulation, when the regulation is uniform, is $\overline{\tau}_k = 3.99\%$.

At date 1, the non-negativity constraint of investors' consumption will necessarily bind, so the amount sold by investor i at date 1 will be proportional to date 0 investment: $k_0^i - k_1^i = \frac{\xi^i}{q}k_0^i$. The households' optimality condition is given by $q = F'\left(k_1^H\right)$. When combined with market clearing and the characterization of optimal investment at date 0 that we present in the Online Appendix, we show that the equilibrium price can be characterized in terms of primitives as the solution to

$$q = \left(\sum_i \frac{\xi^i}{a^i}\left(z^i\left(1 - \frac{\xi^i}{q}\right) - \tau_k^i\right)\right)^{\frac{\alpha-1}{\alpha}},$$

where we have assumed quadratic adjustment costs $\Upsilon^i\left(k_0^i\right) = \frac{a^i}{2}\left(k_0^i\right)^2$ and the isoelastic production function $F\left(k_1^H\right) = \frac{\left(k_1^H\right)^\alpha}{\alpha}$.

Optimal Corrective Policy/Simulation. In Proposition 12 we characterize the form of the second-best policy, which we discuss along with a numerical simulation.

Proposition 12. [Pecuniary Externalities/Uniform Investor Regulation]
a) The marginal welfare effect of varying the uniform corrective regulation of investments, $\overline{\tau}_k = \tau_k^1 = \tau_k^2$*, is given by*

$$\frac{dW}{d\overline{\tau}_k} = \frac{dk_0^1}{d\overline{\tau}_k}\left(\overline{\tau}_k - \delta_k^1\right) + \frac{dk_0^2}{d\overline{\tau}_k}\left(\overline{\tau}_k - \delta_k^2\right),$$

where

$$\delta_k^i = -\frac{\partial q}{\partial k_0^i}\sum_{\ell=1}^{2}\left(\frac{z^\ell}{q} - 1\right)\left(k_0^\ell - k_1^\ell\right).$$

b) The optimal corrective regulation satisfies

$$\overline{\tau}_k = \frac{\frac{dk_0^1}{d\overline{\tau}_k}}{\frac{dk_0^1}{d\overline{\tau}_k} + \frac{dk_0^2}{d\overline{\tau}_k}}\delta_k^1 + \frac{\frac{dk_0^2}{d\overline{\tau}_k}}{\frac{dk_0^1}{d\overline{\tau}_k} + \frac{dk_0^2}{d\overline{\tau}_k}}\delta_k^2.$$

Proposition 12 identifies the distortions associated with the investment choices of investors/entrepreneurs. In this application, the distortion is generated by a distributive pecuniary externality, using the terminology of Dávila and Korinek (2018). Consistent with the results in that paper, this type of externality is determined by price sensitivities, differences in marginal valuations, and net trade positions. In this case, these three statistics are given by $\frac{\partial q}{\partial k_0^i}$, $\frac{z^\ell}{q} - 1$, and $k_0^\ell - k_1^\ell$. Note that δ_k^i includes the sum of the latter two terms across both types of investors, since a given investor does not internalize how his individual investment decision affects prices and consequently the welfare of other investors of the same and different types.

As implied once again by our general results in Section 3, the appropriate weight assigned by the planner to each of the distortions in the optimal second-best policy is given by how sensitive each capital decision is to changes in the regulation. Figure 5 illustrates this intuition by comparing the marginal welfare effects of varying regulation in the first-best and second-best scenarios. By comparing Application 3 with Application 4, it becomes evident that the principles that guide the second-best regulation when it is forced to be uniform across choices for a given agent or across agents for a given choice are identical.

6 Conclusion

This paper provides a systematic study of optimal corrective regulation with imperfect instruments. We have shown that leakage elasticities and Pigouvian wedges are sufficient statistics to account for the marginal welfare impact of imperfect regulatory policies in a large class of environments. The same statistics can also serve to characterize the social value of relaxing regulatory constraints. We have explicitly characterized the optimal regulatory policy with unregulated investors, unregulated activities, with uniform regulation across heterogeneous investors and activities, and with costly regulation.

A central insight is that leakage elasticities from perfectly regulated to imperfectly regulated activities play a crucial role in determining second-best policy. In particular, we show that the optimal second-best policy depends crucially on whether perfectly and imperfectly regulated decisions are gross substitutes or gross complements. Our work provides concrete examples of the relevant elasticities. We have leveraged the general methodology to highlight the common fundamental economic principles in a number of practical scenarios, such as financial regulation with environmental externalities, shadow banking, behavioral distortions, asset substitution, and fire-sale externalities with heterogeneous investors. We hope that our general results spur the development of future measurement efforts and new applications of practical interest.

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A Proofs and derivations: Section 2

Investors' problem: The problem solved by investor i in Lagrangian form is

$$\max_{c_0^i, \{c_1^i(s)\}, \boldsymbol{x}^i} \mathcal{L}^i,$$

where $\mathcal{L}^i$ is given by

$$\begin{aligned}
\mathcal{L}^i &= u^i\left(c_0^i, \{c_1^i(s)\}_{s\in S}, \{\overline{\boldsymbol{x}}^j\}_{j\in\mathcal{I}}\right) \\
&\quad - \lambda_0^i\left(c_0^i - n_0^i - Q^i\left(\boldsymbol{x}^i\right) + \Upsilon^i\left(\boldsymbol{x}^i\right) + \boldsymbol{\tau}^i\cdot\boldsymbol{x}^i - T_0^i\right) \\
&\quad - \int \lambda_1^i(s)\left(c_1^i(s) - n_1^i(s) - \rho_i\left(\boldsymbol{x}^i, s\right)\right) dF(s),
\end{aligned}$$

where λ_0^i and $\lambda_1^i(s)$ denote the Lagrange multipliers that correspond to investor i's budget constraints.[38] The consumption optimality conditions imply that $\lambda_0^i = \frac{\partial u^i}{\partial c_0^i}$ and $\lambda_1^i(s)\,dF(s) = \frac{\partial u^i}{\partial c_1^i(s)}$. The balance-sheet optimality conditions for investor i are given by

$$-\lambda_0^i\left(-\frac{\partial Q^i\left(\boldsymbol{x}^i\right)}{\partial \boldsymbol{x}^i} + \frac{\partial \Upsilon^i\left(\boldsymbol{x}^i\right)}{\partial \boldsymbol{x}^i} + \boldsymbol{\tau}^i\right) + \int \lambda_1^i(s)\frac{\partial \rho_i\left(\boldsymbol{x}^i, s\right)}{\partial \boldsymbol{x}^i} dF(s) = 0,\ \forall i, \tag{51}$$

where Equation (51) corresponds to Equation (12) in the text, and where $m^i(s) = \frac{\lambda_1^i(s)}{\lambda_0^i}$.[39] Formally, the $|\mathcal{X}|\times 1$ vectors $\frac{\partial Q^i\left(\boldsymbol{x}^i\right)}{\partial \boldsymbol{x}^i}$, $\frac{\partial \Upsilon^i\left(\boldsymbol{x}^i\right)}{\partial \boldsymbol{x}^i}$, and $\boldsymbol{\tau}^i$ are given by:

$$\frac{\partial Q^i}{\partial \boldsymbol{x}^i} = \begin{pmatrix} \frac{\partial Q^i}{\partial b_1^i} \\ \vdots \\ \frac{\partial Q^i}{\partial b_{|\mathcal{B}|}^i} \\ \frac{\partial Q^i}{\partial k_1^i} \\ \vdots \\ \frac{\partial Q^i}{\partial k_{|\mathcal{K}|}^i} \end{pmatrix}, \quad \frac{\partial \Upsilon^i}{\partial \boldsymbol{x}^i} = \begin{pmatrix} \frac{\partial \Upsilon^i}{\partial b_1^i} \\ \vdots \\ \frac{\partial \Upsilon^i}{\partial b_{|\mathcal{B}|}^i} \\ \frac{\partial \Upsilon^i}{\partial k_1^i} \\ \vdots \\ \frac{\partial \Upsilon^i}{\partial k_{|\mathcal{K}|}^i} \end{pmatrix}, \quad \text{and} \quad \boldsymbol{\tau}^i = \begin{pmatrix} \tau_{b,1}^i \\ \vdots \\ \tau_{b,|\mathcal{B}|}^i \\ \tau_{k,1}^i \\ \vdots \\ \tau_{k,|\mathcal{K}|}^i \end{pmatrix}.$$

Similarly, we define the $|\mathcal{X}|\times 1$ vector $\int \lambda_1^i(s)\frac{\partial \rho_i\left(\boldsymbol{x}^i, s\right)}{\partial \boldsymbol{x}^i} dF(s)$ as follows:

$$\int \lambda_1^i(s)\frac{\partial \rho_i\left(\boldsymbol{x}^i, s\right)}{\partial \boldsymbol{x}^i} dF(s) = \begin{pmatrix} \int \lambda_1^i(s)\frac{\partial \rho_i\left(\boldsymbol{x}^i, s\right)}{\partial b_1^i} dF(s) \\ \vdots \\ \int \lambda_1^i(s)\frac{\partial \rho_i\left(\boldsymbol{x}^i, s\right)}{\partial b_{|\mathcal{B}|}^i} dF(s) \\ \int \lambda_1^i(s)\frac{\partial \rho_i\left(\boldsymbol{x}^i, s\right)}{\partial k_1^i} dF(s) \\ \vdots \\ \int \lambda_1^i(s)\frac{\partial \rho_i\left(\boldsymbol{x}^i, s\right)}{\partial k_{|\mathcal{K}|}^i} dF(s) \end{pmatrix}.$$

[38] Without loss of generality, we define the state s multipliers $\lambda_1^i(s)$ inside the expectation.

[39] Note that a sufficient regularity condition for the second term of Equation (51) to be valid is that $\rho_i\left(\boldsymbol{x}^i, s\right)$ is continuous. Otherwise, all results follow when the second term is $\frac{\partial}{\partial \boldsymbol{x}^i}\left[\int \lambda_1^i(s)\rho_i\left(\boldsymbol{x}^i, s\right) dF(s)\right]$.

Creditors' problem: The problem solved by creditors in Lagrangian form is

$$\max_{c_0^C, \left\{c_1^C(s)\right\}, \left\{h_i^C\right\}} \mathcal{L}^C,$$

where $\mathcal{L}^C$ is given by

$$\mathcal{L}^C = u^C\left(c_0^C, \left\{c_1^C(s)\right\}_{s\in S}, \left\{\overline{\boldsymbol{x}}^j\right\}_{j\in\mathcal{I}}\right) - \lambda_0^C\left(c_0^C - n_0^C + \sum_{i\in\mathcal{I}} h_i^C Q^i\left(\overline{\boldsymbol{x}}^i\right)\right)$$
$$-\int \lambda_1^C(s)\left(c_1^C(s) - n_1^C(s) - \sum_{i\in\mathcal{I}} h_i^C \rho_i^C\left(\overline{\boldsymbol{x}}^i, s\right)\right) dF(s),$$

where λ_0^C and $\lambda_1^C(s)$ denote the Lagrange multipliers that correspond to the creditors' budget constraints. The consumption optimality conditions imply that $\lambda_0^C = \frac{\partial u^C}{\partial c_0^C}$ and $\lambda_1^C(s)\,dF(s) = \frac{\partial u^C}{\partial c_1^C(s)}$. The optimality conditions for creditors regarding $\left\{h_i^C\right\}$ are

$$-\lambda_0^C Q^i\left(\boldsymbol{x}^i\right) + \int \lambda_1^C(s)\,\rho_i^C\left(\boldsymbol{x}^i, s\right) dF(s) = 0,\ \forall i, \tag{52}$$

where we use the fact that $\boldsymbol{x}^i = \overline{\boldsymbol{x}}^i$ in equilibrium. Equation (52), which exactly corresponds to Equation (11) in the text once we define $m^C(s) = \frac{\lambda_1^C(s)}{\lambda_0^C}$, characterizes the financing schedules $Q^i\left(\boldsymbol{x}^i\right)$ that investors face.

B Proofs and derivations: Section 3

Proof of Proposition 1 [Marginal Welfare Effects of Corrective Regulation: Policy Elasticities and Pigouvian Wedges]:

Proof. We initially characterize the $|\mathcal{X}| \times 1$ vectors $\frac{\frac{dV^i}{d\tau^j}}{\lambda_0^j}$ and $\frac{\frac{dV^C}{d\tau^j}}{\lambda_0^C}$, which correspond to the money-metric welfare changes of type i investors and creditors when $\boldsymbol{\tau}^j$ changes. In vector form, these are given by

$$\frac{dV_m^i}{d\boldsymbol{\tau}^j} = \frac{\frac{dV^i}{d\boldsymbol{\tau}^j}}{\lambda_0^i} = \begin{pmatrix} \frac{\frac{dV^i}{d\tau_1^j}}{\lambda_0^i} \\ \vdots \\ \frac{\frac{dV^i}{d\tau_{|\mathcal{X}|}^j}}{\lambda_0^i} \end{pmatrix} \quad \text{and} \quad \frac{dV_m^C}{d\boldsymbol{\tau}^j} = \frac{\frac{dV^C}{d\boldsymbol{\tau}^j}}{\lambda_0^C} = \begin{pmatrix} \frac{\frac{dV^C}{d\tau_1^j}}{\lambda_0^C} \\ \vdots \\ \frac{\frac{dV^C}{d\tau_{|\mathcal{X}|}^j}}{\lambda_0^C} \end{pmatrix}.$$

Investors. We express the financing schedules faced by investors as a function of the stochastic discount factor of creditors $m^C(s)$, which is in turn in equilibrium a function of the consumption of creditors in all dates and states. This allows us to separately account for any general equilibrium pecuniary effects. Formally, we represent the equilibrium financing schedules in Equation (11) for an investor i as follows:

$$Q^i\left(\boldsymbol{x}^i; m^C(s)\right) = \int m^C(s)\, \rho_i^C\left(\boldsymbol{x}^i, s\right) dF(s),$$

where we make explicit the dependence on $m^C(s)$. The money-metric change in indirect utility for investor i when varying the regulation that investor j faces is given by the following $|\mathcal{X}| \times 1$ vector:

$$\begin{aligned}\frac{\frac{dV^i}{d\boldsymbol{\tau}^j}}{\lambda_0^i} = {} & \frac{dc_0^i}{d\boldsymbol{\tau}^j} \overbrace{\left(\frac{\frac{\partial u^i}{\partial c_0^i} - \lambda_0^i}{\lambda_0^i}\right)}^{=0} + \int \frac{dc_1^i(s)}{d\boldsymbol{\tau}^j} \overbrace{\left(\frac{\frac{\frac{\partial u^i}{\partial c_1^i(s)}}{dF(s)} - \lambda_1^i(s)}{\lambda_0^i}\right)}^{=0} dF(s) \\ & + \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} \overbrace{\left(\frac{\partial Q^i\left(\boldsymbol{x}^i; m^C(s)\right)}{\partial \boldsymbol{x}^i} - \frac{\partial \Upsilon^i\left(\boldsymbol{x}^i\right)}{\partial \boldsymbol{x}^i} - \boldsymbol{\tau}^i + \int m^i(s) \frac{\partial \rho_i\left(\boldsymbol{x}^i, s\right)}{\partial \boldsymbol{x}^i} dF(s)\right)}^{=0} \\ & + \frac{dT_0^i}{d\boldsymbol{\tau}^j} - \frac{d\boldsymbol{\tau}^i}{d\boldsymbol{\tau}^j}\boldsymbol{x}^i + \frac{\partial Q^i\left(\boldsymbol{x}^i; m^C(s)\right)}{\partial m^C(s)} \frac{dm^C(s)}{d\boldsymbol{\tau}^j} + \sum_{\ell \in \mathcal{I}} \frac{d\boldsymbol{x}^\ell}{d\boldsymbol{\tau}^j} \frac{1}{\lambda_0^i} \frac{\partial u^i}{\partial \overline{\boldsymbol{x}}^\ell},\end{aligned} \tag{53}$$

where the $|\mathcal{X}| \times 1$ vectors $\frac{dT_0^i}{d\boldsymbol{\tau}^j}$ and $\boldsymbol{x}^j$ are given by

$$\frac{dT_0^i}{d\boldsymbol{\tau}^j} = \begin{pmatrix} \frac{dT_0^i}{d\tau_1^j} \\ \vdots \\ \frac{dT_0^i}{d\tau_{|\mathcal{X}|}^j} \end{pmatrix} \quad \text{and} \quad \boldsymbol{x}^i = \begin{pmatrix} b_1^i \\ \vdots \\ b_{|\mathcal{B}|}^i \\ k_1^i \\ \vdots \\ k_{|\mathcal{K}|}^i \end{pmatrix} = \begin{pmatrix} x_1^i \\ \vdots \\ x_n^i \\ \vdots \\ x_{|\mathcal{X}|}^i \end{pmatrix},$$

and where the matrix $\frac{d\boldsymbol{\tau}^i}{d\boldsymbol{\tau}^j}$, of dimension $|\mathcal{X}| \times |\mathcal{X}|$, is given by

$$\frac{d\boldsymbol{\tau}^i}{d\boldsymbol{\tau}^j} = \begin{cases} I_{|\mathcal{X}|}, & \text{if } i = j \\ 0_{|\mathcal{X}|}, & \text{if } i \neq j, \end{cases}$$

which is either a $|\mathcal{X}|$-dimensional identity matrix, $I_{|\mathcal{X}|}$, when $i = j$, or a $|\mathcal{X}| \times |\mathcal{X}|$ matrix of zeros, $0_{|\mathcal{X}|}$, when $i \neq j$. We also define the $|\mathcal{X}| \times 1$ vector $\frac{\partial Q^i(\boldsymbol{x}^i; m^C(s))}{\partial m^C(s)} \frac{dm^C(s)}{d\boldsymbol{\tau}^j}$ as

$$\frac{\partial Q^i\left(\boldsymbol{x}^i; m^C(s)\right)}{\partial m^C(s)} \frac{dm^C(s)}{d\boldsymbol{\tau}^j} = \begin{pmatrix} \int \frac{dm^C(s)}{d\tau_1^j} \rho_i^C\left(\boldsymbol{x}^i, s\right) dF(s) \\ \vdots \\ \int \frac{dm^C(s)}{d\tau_{|\mathcal{X}|}^j} \rho_i^C\left(\boldsymbol{x}^i, s\right) dF(s) \end{pmatrix}.$$

Note that we use the fact that

$$\frac{dQ^i\left(\boldsymbol{x}^i; m^C(s)\right)}{d\boldsymbol{\tau}^j} = \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} \frac{\partial Q^i}{\partial \boldsymbol{x}^i} + \frac{\partial Q^i}{\partial m^C(s)} \frac{dm^C(s)}{d\boldsymbol{\tau}^j},$$

as well as

$$\frac{d\left(\boldsymbol{\tau}^i \cdot \boldsymbol{x}^i - T_0^i\right)}{d\boldsymbol{\tau}^j} = \frac{d\boldsymbol{\tau}^i}{d\boldsymbol{\tau}^j} \boldsymbol{x}^i + \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} \boldsymbol{\tau}^i - \frac{dT_0^i}{d\boldsymbol{\tau}^j}.$$

Note that we define the $|\mathcal{X}| \times |\mathcal{X}|$ matrix $\frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}$ as follows:

$$\frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} = \begin{pmatrix} \frac{dx_1^i}{d\tau_1^j} & \cdots & \frac{dx_{|\mathcal{X}|}^i}{d\tau_1^j} \\ \vdots & \frac{dx_n^i}{d\tau_{n'}^j} & \vdots \\ \frac{dx_1^i}{d\tau_{|\mathcal{X}|}^j} & \cdots & \frac{dx_{|\mathcal{X}|}^i}{d\tau_{|\mathcal{X}|}^j} \end{pmatrix}. \tag{54}$$

This matrix is the Jacobian of the equilibrium vector of the balance-sheet decisions of investor i with respect to a change in the set of regulations that agent j faces. Note also that $\frac{\partial u^i}{\partial \overline{\boldsymbol{x}}^\ell}$ denotes a $|\mathcal{X}| \times 1$ gradient vector.

Creditors. In the case of creditors, we can express the $|\mathcal{X}| \times 1$ vector $\frac{\frac{dV^C}{d\boldsymbol{\tau}^j}}{\lambda_0^C}$ as follows:

$$\begin{aligned}
\frac{\frac{dV^C}{d\boldsymbol{\tau}^j}}{\lambda_0^C} &= \frac{dc_0^C}{d\boldsymbol{\tau}^j} \overbrace{\left(\frac{\frac{\partial u^C}{\partial c_0^C} - \lambda_0^C}{\lambda_0^C}\right)}^{=0} + \int \overbrace{\left(\frac{\frac{\frac{\partial u^C}{\partial c_1^C(s)}}{dF(s)} - \lambda_1^C(s)}{\lambda_0^C}\right)}^{=0} \frac{dc_1^C(s)}{d\boldsymbol{\tau}^j} dF(s) \\
&\quad - \sum_{i \in \mathcal{I}} \frac{dh_i^C}{d\boldsymbol{\tau}^j} \overbrace{\left(Q^i\left(\boldsymbol{x}^i; m^C(s)\right) - \int m^C(s) \rho_i^C\left(\boldsymbol{x}^i, s\right) dF(s)\right)}^{=0} \\
&\quad - \sum_{i \in \mathcal{I}} h_i^C \left(\frac{dQ^i\left(\boldsymbol{x}^i; m^C(s)\right)}{d\boldsymbol{\tau}^j} - \int m^C(s) \frac{d\rho_i^C\left(\boldsymbol{x}^i, s\right)}{d\boldsymbol{\tau}^j} dF(s)\right) + \sum_{\ell \in \mathcal{I}} \frac{d\boldsymbol{x}^\ell}{d\boldsymbol{\tau}^j} \frac{1}{\lambda_0^C} \frac{\partial u^C}{\partial \overline{\boldsymbol{x}}^\ell} \\
&= -\sum_{i \in \mathcal{I}} \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} \underbrace{\left(\frac{\partial Q^i}{\partial \boldsymbol{x}^i} - \int m^C(s) \frac{\partial \rho_i^C\left(\boldsymbol{x}^i, s\right)}{\partial \boldsymbol{x}^i} dF(s)\right)}_{=0} - \sum_{i \in \mathcal{I}} \frac{\partial Q^i}{\partial m^C(s)} \frac{dm^C(s)}{d\boldsymbol{\tau}^j} + \sum_{\ell \in \mathcal{I}} \frac{d\boldsymbol{x}^\ell}{d\boldsymbol{\tau}^j} \frac{1}{\lambda_0^C} \frac{\partial u^C}{\partial \overline{\boldsymbol{x}}^\ell},
\end{aligned} \tag{55}$$

where in the second equality we use the fact that $h_i^C = 1$ and the fact that $\frac{d\rho_i^C(\boldsymbol{x}^i,s)}{d\boldsymbol{\tau}^j} = \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\frac{\partial \rho_i^C}{\partial \boldsymbol{x}^i}$, and where the $|\mathcal{X}| \times 1$ vector $\frac{\partial \rho_i^C}{\partial \boldsymbol{x}^i}$ is given by

$$\frac{\partial \rho_i^C}{\partial \boldsymbol{x}^i} = \begin{pmatrix} \frac{\partial \rho_i^C}{\partial x_1^i} \\ \vdots \\ \frac{\partial \rho_i^C}{\partial x_{|\mathcal{X}|}^i} \end{pmatrix}.$$

Note that $\frac{d\boldsymbol{x}^\ell}{d\boldsymbol{\tau}^j}$ is defined as in Equation (54), and that $\frac{\partial u^C}{\partial \overline{\boldsymbol{x}}^\ell}$ denotes a $|\mathcal{X}| \times 1$ gradient vector.

Social Welfare: First, we can express the sum among investors of the change in money-metric indirect utilities as follows:

$$\begin{aligned}
\sum_{i\in\mathcal{I}} \frac{dV_m^i}{d\boldsymbol{\tau}^j} &= \sum_{i\in\mathcal{I}} \left(\frac{dT_0^i}{d\boldsymbol{\tau}^j} - \frac{d\boldsymbol{\tau}^i}{d\boldsymbol{\tau}^j}\boldsymbol{x}^i\right) + \sum_{i\in\mathcal{I}} \frac{\partial Q^i}{\partial m^C(s)} \frac{dm^C(s)}{d\boldsymbol{\tau}^j} + \sum_{i\in\mathcal{I}}\sum_{\ell\in\mathcal{I}} \frac{d\boldsymbol{x}^\ell}{d\boldsymbol{\tau}^j}\frac{1}{\lambda_0^i}\frac{\partial u^i}{\partial \overline{\boldsymbol{x}}^\ell} \\
&= \sum_{i\in\mathcal{I}} \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\left(\boldsymbol{\tau}^i + \sum_{\ell\in\mathcal{I}} \frac{1}{\lambda_0^\ell}\frac{\partial u^\ell}{\partial \overline{\boldsymbol{x}}^i}\right) + \sum_{i\in\mathcal{I}} \frac{\partial Q^i}{\partial m^C(s)} \frac{dm^C(s)}{d\boldsymbol{\tau}^j},
\end{aligned}$$

where we use the fact that Equation (8) implies that

$$\sum_{i\in\mathcal{I}} \left(\frac{dT_0^i}{d\boldsymbol{\tau}^j} - \frac{d\boldsymbol{\tau}^i}{d\boldsymbol{\tau}^j}\boldsymbol{x}^i\right) = \sum_{i\in\mathcal{I}} \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\boldsymbol{\tau}^i,$$

as well as the following identity:

$$\sum_{i\in\mathcal{I}}\sum_{\ell\in\mathcal{I}} \frac{d\boldsymbol{x}^\ell}{d\boldsymbol{\tau}^j}\frac{1}{\lambda_0^i}\frac{\partial u^i}{\partial \overline{\boldsymbol{x}}^\ell} = \sum_{i\in\mathcal{I}}\sum_{\ell\in\mathcal{I}} \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\frac{1}{\lambda_0^\ell}\frac{\partial u^\ell}{\partial \overline{\boldsymbol{x}}^i}.$$

Therefore, we can express $\frac{dW}{d\boldsymbol{\tau}^j}$ as follows:

$$\begin{aligned}
\frac{dW}{d\boldsymbol{\tau}^j} &= \sum_{i\in\mathcal{I}} \frac{dV_m^i}{d\boldsymbol{\tau}^j} + \frac{dV_m^C}{d\boldsymbol{\tau}^j} \\
&= \sum_{i\in\mathcal{I}} \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\left(\boldsymbol{\tau}^i + \sum_{\ell\in\mathcal{I}} \frac{1}{\lambda_0^\ell}\frac{\partial u^\ell}{\partial \overline{\boldsymbol{x}}^i}\right) + \sum_{i\in\mathcal{I}} \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\frac{1}{\lambda_0^C}\frac{\partial u^C}{\partial \overline{\boldsymbol{x}}^i} \\
&= \sum_{i\in\mathcal{I}} \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\left(\boldsymbol{\tau}^i - \underbrace{\left(-\left(\sum_{\ell\in\mathcal{I}} \frac{1}{\lambda_0^\ell}\frac{\partial u^\ell}{\partial \overline{\boldsymbol{x}}^i} + \frac{1}{\lambda_0^C}\frac{\partial u^C}{\partial \overline{\boldsymbol{x}}^i}\right)\right)}_{=\boldsymbol{\delta}^i}\right),
\end{aligned}$$

which, after being stacked, corresponds to Equation (19) in the text — see also Footnote 10. Note that $\boldsymbol{\delta}^i$ is a $|\mathcal{X}| \times 1$ vector. □

Redistributional concerns: Given the money-metric marginal welfare effects of varying $\boldsymbol{\tau}^j$, defined in Equations (53) and (55), we can express the marginal welfare effects of varying $\boldsymbol{\tau}^j$, for any set of generalized social marginal welfare weights (Saez and Stantcheva, 2016), ω^i for $i \in \mathcal{I}$ and ω^j, as follows:

$$\frac{dW}{d\tau^j} = \sum_i \omega^i \frac{\frac{dV^i}{d\boldsymbol{\tau}^j}}{\lambda_0^i} + \omega^j \frac{\frac{dV^C}{d\boldsymbol{\tau}^j}}{\lambda_0^C} = \mathbb{E}_{iC}\left[\frac{\frac{dV^{iC}}{d\boldsymbol{\tau}^j}}{\lambda_0^x}\right] + \underbrace{\mathbb{C}ov_{iC}\left[\omega^{iC}, \frac{\frac{dV^{iC}}{d\boldsymbol{\tau}^j}}{\lambda_0^x}\right]}_{\text{Redistribution}}, \tag{56}$$

where we assume, without loss of generality that the weights add up to one, that is, $\sum_i \omega^i + \omega^j = 1$, and where we use the index iC to refer to the set of investors and creditors, that is, $\{\mathcal{I}, C\}$.[40]

When $\omega^i = \omega^C = 1$, then the redistribution term in Equation (56) is zero. This case is the one studied in the body of the paper. When $\omega^i \neq \omega^C \neq 1$, Equation (56) clearly shows that redistributional concerns enter additively to the marginal welfare effects of varying τ^j. A utilitarian planner simple corresponds to setting marginal welfare weights of the form $\omega^i = \lambda_0^i$, where λ_0^i typically equals marginal utility of consumption. Note that a utilitarian planner with access to lump-sum taxes/transfers finds it optimal to endogenously set $\omega^i = \omega^C = 1$.

Proof of Proposition 2 [First-Best Regulation/Pigouvian Principle]:

Proof. The optimal first-best regulation is characterized by

$$\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}\boldsymbol{\omega} = 0,$$

which defines a system of homogeneous linear equations in $\boldsymbol{\omega}$. If the matrix of policy elasticities $\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}$ is invertible (i.e., has full rank), the only solution to this system is the trivial solution $\boldsymbol{\omega} = 0$. □

Proof of Proposition 3 [Second-Best Regulation: General Case]:

Proof. Note that the Jacobian matrix $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}$, of dimensions $|\mathcal{U}| \times |\mathcal{R}|$, can be written as

$$\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U} = \begin{pmatrix} & \cdots & \\ \vdots & \frac{dx_n^i}{d\tau_{n'}^j} & \vdots \\ & \cdots & \end{pmatrix},$$

where the balance-sheet activities are such that $(i, n) \in \mathcal{R}$ and $(j, n') \in \mathcal{U}$. One can similarly define $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}$, $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}$, and $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$, with dimensions $|\mathcal{U}| \times |\mathcal{U}|$, $|\mathcal{R}| \times |\mathcal{U}|$, and $|\mathcal{R}| \times |\mathcal{R}|$ respectively, by switching the sets of coefficients.

Consider the marginal welfare effects of increasing the perfectly regulated taxes/subsidies $\boldsymbol{\tau}^R$. By definition of the perfectly regulated activities, we have $\boldsymbol{\eta}^R = 0$, so that Equation (20) yields $\frac{dW}{d\boldsymbol{\tau}^R} = 0$ at the second-best optimum. Using Equation (19) (or, more directly, its expanded version in Footnote 10) we obtain:

$$0 = \frac{dW}{d\boldsymbol{\tau}^R} = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}^R}(\boldsymbol{\tau} - \boldsymbol{\delta}) = \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) + \frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\left(\boldsymbol{\tau}^R - \boldsymbol{\delta}^R\right).$$

[40] In Equation (56), the expectation and covariance operators, indexed by iC, correspond to cross-sectional moments including investors and creditors.

Assuming that the matrix $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$ is invertible, we rearrange this equation as follows to complete the proof:

$$\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\left(\boldsymbol{\tau}^R-\boldsymbol{\delta}^R\right)=-\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\left(\boldsymbol{\tau}^U-\boldsymbol{\delta}^U\right)\iff\boldsymbol{\tau}^R=\boldsymbol{\delta}^R-\underbrace{\left(\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}}_{|\mathcal{R}|\times|\mathcal{R}|}\underbrace{\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}}_{|\mathcal{R}|\times|\mathcal{U}|}\underbrace{\left(\boldsymbol{\tau}^U-\boldsymbol{\delta}^U\right)}_{|\mathcal{U}|\times 1}.$$

$\square$

Derivations with diagonal policy elasticities: When the own-regulatory policy elasticity matrix $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$ is diagonal, we have

$$\begin{aligned}
\left(\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\boldsymbol{\omega}^U &= \begin{pmatrix} \left(\frac{dx_1^R}{d\tau_1^R}\right)^{-1} & 0 & \\ 0 & \ddots & \\ & & \left(\frac{dx_{|\mathcal{R}|}^R}{d\tau_{|\mathcal{R}|}^R}\right)^{-1}\end{pmatrix}\begin{pmatrix} \frac{dx_1^U}{d\tau_1^R} & \frac{dx_2^U}{d\tau_1^R} & & \\ \frac{dx_1^U}{d\tau_2^R} & \frac{dx_2^U}{d\tau_2^R} & & \\ & & \ddots & \\ & & & \frac{dx_{|\mathcal{U}|}^U}{d\tau_{|\mathcal{R}|}^R}\end{pmatrix}\begin{pmatrix}\omega_1^U\\ \vdots \\ \omega_{|\mathcal{U}|}^U\end{pmatrix}\\
&= \begin{pmatrix} \left(\frac{dx_1^R}{d\tau_1^R}\right)^{-1}\frac{dx_1^U}{d\tau_1^R} & \left(\frac{dx_1^R}{d\tau_1^R}\right)^{-1}\frac{dx_2^U}{d\tau_1^R} & & \\ \left(\frac{dx_2^R}{d\tau_2^R}\right)^{-1}\frac{dx_1^U}{d\tau_2^R} & \left(\frac{dx_2^R}{d\tau_2^R}\right)^{-1}\frac{dx_2^U}{d\tau_2^R} & & \\ & & \ddots & \\ & & & \left(\frac{dx_{|\mathcal{R}|}^R}{d\tau_{|\mathcal{R}|}^R}\right)^{-1}\frac{dx_{|\mathcal{U}|}^U}{d\tau_{|\mathcal{R}|}^R}\end{pmatrix}\begin{pmatrix}\omega_1^U\\ \vdots \\ \omega_{|\mathcal{U}|}^U\end{pmatrix}\\
&= \begin{pmatrix} \left(\frac{dx_1^R}{d\tau_1^R}\right)^{-1}\left(\frac{dx_1^U}{d\tau_1^R}\omega_1^U+\frac{dx_2^U}{d\tau_1^R}\omega_2^U+\cdots\right)\\ \left(\frac{dx_2^R}{d\tau_2^R}\right)^{-1}\left(\frac{dx_1^U}{d\tau_2^R}\omega_1^U+\frac{dx_2^U}{d\tau_2^R}\omega_2^U+\cdots\right)\\ \vdots \\ \left(\frac{dx_{|\mathcal{R}|}^R}{d\tau_{|\mathcal{R}|}^R}\right)^{-1}\left(\frac{dx_1^U}{d\tau_{|\mathcal{R}|}^R}\omega_1^U+\frac{dx_2^U}{d\tau_{|\mathcal{R}|}^R}\omega_2^U+\cdots\right)\end{pmatrix}.
\end{aligned}$$

It follows that the second-best regulation on choice $(j,n)\in\mathcal{R}$ is given by

$$\tau_n^j=\delta_n^j-\left(\frac{dx_n^j}{d\tau_n^j}\right)^{-1}\sum_{(j',n')\in\mathcal{U}}\frac{dx_{n'}^{j'}}{d\tau_n^j}\omega_{n'}^{j'}.$$

Proof of Proposition 4 [Welfare effects of relaxing regulatory constraints]:

Proof. Using Equation (19) (or, more directly, its expanded version in Footnote 10) we obtain the welfare effects of changes in the imperfectly regulated taxes/subsidies $\boldsymbol{\tau}^U$:

$$\frac{dW}{d\boldsymbol{\tau}^U}=\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{\tau}^U-\boldsymbol{\delta}^U\right)+\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}\left(\boldsymbol{\tau}^R-\boldsymbol{\delta}^R\right).$$

From the characterization of $\boldsymbol{\tau}^R$ at the second-best optimum from Proposition 3, we have that

$$\boldsymbol{\tau}^R-\boldsymbol{\delta}^R=\left(-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\left(\boldsymbol{\tau}^U-\boldsymbol{\delta}^U\right).$$

Combining, we obtain the required expression as follows:

$$\begin{aligned}
\frac{dW}{d\boldsymbol{\tau}^U} &= \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) - \frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}\left(\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) \\
&= \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I} - \underbrace{\left(\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\right)^{-1}\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}\left(\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}}_{\equiv \boldsymbol{L}}\right)\underbrace{\left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right)}_{\equiv \boldsymbol{\omega}^U} \\
&= \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I} - \boldsymbol{L}\right)\boldsymbol{\omega}^U.
\end{aligned}$$

□

Proof of Proposition 5 [Second-Best Regulation: Unregulated Investors/Activities]:

Proof. This proposition follows directly from Proposition 3 and the observation that, at the second-best optimum, the constraints are binding with $\boldsymbol{\tau}^U = 0$. Concretely, we have

$$\boldsymbol{\tau}^R = \boldsymbol{\delta}^R + \left(-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\left(\underbrace{\boldsymbol{\tau}^U}_{=0} - \boldsymbol{\delta}^U\right) = \boldsymbol{\delta}^R - \left(-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\boldsymbol{\delta}^U,$$

as required. □

Proof of Proposition 6 [Second-Best Regulation: Imperfect Targeting]:

Proof. Note that the case of uniform regulation is a particular case of linear constraints — see Equation (9) — in which

$$\boldsymbol{A} = \begin{pmatrix} 1 & -1 & & \cdots & 0 \\ & 1 & -1 & & \vdots \\ \vdots & & \ddots & \ddots & \\ 0 & \cdots & & 1 & -1 \end{pmatrix},$$

where $\boldsymbol{A}$ has dimensions $(|\mathcal{U}| - 1) \times |\mathcal{U}|$. In this case, $\frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}} = \boldsymbol{A}'$, so Equation (20) implies that

$$\frac{dW}{d\boldsymbol{\tau}^U} = \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I} - \boldsymbol{L}\right)\boldsymbol{\omega}^U = \boldsymbol{A}'\mu.$$

Adding up across all $|\mathcal{U}|$, where $\boldsymbol{\iota}$ denotes a column vector of ones with dimension $|\mathcal{U}|$, we obtain that

$$\boldsymbol{\iota}'\frac{dW}{d\boldsymbol{\tau}^U} = \boldsymbol{\iota}'\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I} - \boldsymbol{L}\right)\boldsymbol{\omega}^U = \boldsymbol{\iota}'\boldsymbol{A}'\mu = 0,$$

since $\boldsymbol{A}\boldsymbol{\iota} = 0$ — this corresponds to adding up the rows of $\boldsymbol{A}$ — which in turn implies that $\boldsymbol{\iota}'\boldsymbol{A}' = 0$ — this correspond to adding up the columns of $\boldsymbol{A}'$). It then immediately follows that

$$\begin{aligned} 0 &= \boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} (\boldsymbol{I} - \boldsymbol{L})\, \boldsymbol{\omega}^U \\ &= \boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} (\boldsymbol{I} - \boldsymbol{L}) \left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) \\ &= \boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} (\boldsymbol{I} - \boldsymbol{L}) \left(\overline{\tau}^U \boldsymbol{\iota} - \boldsymbol{\delta}^U\right), \end{aligned}$$

where the last line uses the fact that all elements of $\boldsymbol{\tau}^U$ must be equal to the same scalar, denoted $\overline{\tau}^U$, at the constrained solution; that is, $\boldsymbol{\tau}^U = \iota\overline{\tau}^U$. We solve as follows for the scalar $\overline{\tau}^U$ to complete the proof:

$$\underbrace{\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} (\boldsymbol{I} - \boldsymbol{L})\, \boldsymbol{\iota}}_{\text{scalar}} \overline{\tau}^U = \underbrace{\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} (\boldsymbol{I} - \boldsymbol{L})\, \boldsymbol{\delta}^U}_{\text{scalar}} \iff \overline{\tau}^U = \frac{\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} (\boldsymbol{I} - \boldsymbol{L})\, \boldsymbol{\delta}^U}{\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} (\boldsymbol{I} - \boldsymbol{L})\, \boldsymbol{\iota}}.$$

□

Proof of Proposition 7 [Second-Best Regulation: Attenuation under Quadratic Costs of Regulation]:

Proof. Note that we can express Equation (29) as follows:

$$\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} (\boldsymbol{I} - \boldsymbol{L}) \left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) = \boldsymbol{B}\boldsymbol{\tau}^U. \tag{57}$$

Solving for $\boldsymbol{\tau}^U$ simply yields

$$\boldsymbol{\tau}^U = \left(\boldsymbol{B} + \left(-\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\right) (\boldsymbol{I} - \boldsymbol{L})\right)^{-1} \left(\left(-\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\right) (\boldsymbol{I} - \boldsymbol{L})\, \boldsymbol{\delta}^U\right),$$

which corresponds to Equation (30) in the paper. With a single agent and activity, $\boldsymbol{L} = 0$, and Equation (57) collapses to

$$\frac{dx^U}{d\tau^U} \left(\tau^U - \delta^U\right) = b\tau^U,$$

which in turn implies Equation (31) in the text. □

Online Appendix

Sections C and D of this Online Appendix include detailed proofs and derivations associated with the applications described in Sections 4 and 5 of the paper. Section E of this Online Appendix shows that the results of the paper apply unchanged to a classical consumer theory scenario.

C Proofs and derivations: Section 4

Proof of Lemma 1 [Welfare effects of relaxing regulatory constraints]: Since creditors are risk-neutral, they must be indifferent between all quantities of debt purchased in equilibrium. Hence, the valuation of debt *per unit of capital* in equilibrium is

$$Q^i\left(b^i,\theta^i\right)=\int_{s^\star\left(b^i,\theta^i\right)}^{\bar{s}} b^i dF\left(s\right)+\phi\int_{\underline{s}}^{s^\star\left(b^i,\theta^i\right)}\left[d_1\left(s\right)\theta^i+d_2\left(s\right)\left(1-\theta^i\right)\right]dF\left(s\right).$$

Substituting the valuation of debt and the budget constraints into investors' objective function, and ignoring exogenous endowments, we obtain the simplified version of their maximization problem in Lemma 1.

Proof of Proposition 8 [Financial Regulation With Environmental Externalities]: Adding the utilities of investors and creditors, imposing market clearing, and ignoring exogenous environments, we find that maximizing welfare is equivalent to maximizing

$$\begin{aligned}W\left(b^i,\theta^i,k^i\right)&=\left[M\left(b^i,\theta^i\right)-\Omega\left(\theta^i\right)-1\right]k^i-\Upsilon\left(k^i\right)\\&\quad-\left(1+\kappa\right)\int_{\underline{s}}^{\bar{s}} t\left(b^i,\theta^i,s\right)k^i dF\left(s\right)-\Psi\left(\theta^i\right)k^i.\end{aligned}$$

We can now write $b^i\left(\bar{b},\varphi\right)$, $\theta^i\left(\bar{b},\varphi\right)$ and $k^i\left(\bar{b},\varphi\right)$ for optimal choices as a function of regulatory parameters, and totally differentiate the welfare function with respect to $\bar{b}$ to obtain

$$\begin{aligned}\frac{dW}{d\bar{b}}&=k^i\left(\frac{\partial M}{\partial b^i}-\delta_b\right)\frac{db^i}{d\bar{b}}+k^i\left(\frac{\partial M}{\partial\theta^i}-\Omega'\left(\theta^i\right)-\delta_\theta\right)\frac{d\theta^i}{d\bar{b}}-\delta_k\frac{dk^i}{d\bar{b}}\\&=k^i\left(\tau_b-\delta_b\right)\frac{db^i}{d\bar{b}}+k^i\left(\tau_\theta-\delta_\theta\right)\frac{d\theta^i}{d\bar{b}}-\delta_k\frac{dk^i}{d\bar{b}},\end{aligned}$$

where we have substituted the definitions of $\{\delta_b,\delta_\theta,\delta_k\}$ from Equations (38) through (40), as well as the first-order conditions (35), (36) and (37). Similarly, we differentiate with respect to φ to obtain

$$\begin{aligned}\frac{dW}{d\varphi}&=k^i\left(\frac{\partial M}{\partial b^i}-\delta_b\right)\frac{db^i}{d\varphi}+k^i\left(\frac{\partial M}{\partial\theta^i}-\Omega'\left(\theta^i\right)-\delta_\theta\right)\frac{d\theta^i}{d\varphi}-\delta_k\frac{dk^i}{d\varphi}\\&=k^i\left(\tau_b-\delta_b\right)\frac{db^i}{d\varphi}+k^i\left(\tau_\theta-\delta_\theta\right)\frac{d\theta^i}{d\varphi}-\delta\frac{dk^i}{d\varphi},\end{aligned}$$

as stated in Proposition 8, part a). Part b) of the proposition follows by solving for τ_b and τ_θ in the system $\frac{dW}{d\bar{b}}=0$, $\frac{dW}{d\varphi}=0$.

Characterization of complementarities: Investors' first-order condition for k^i can be written as

$$J\left(\bar{b},\varphi\right)=1+\Upsilon'\left(k^i\right),\tag{58}$$

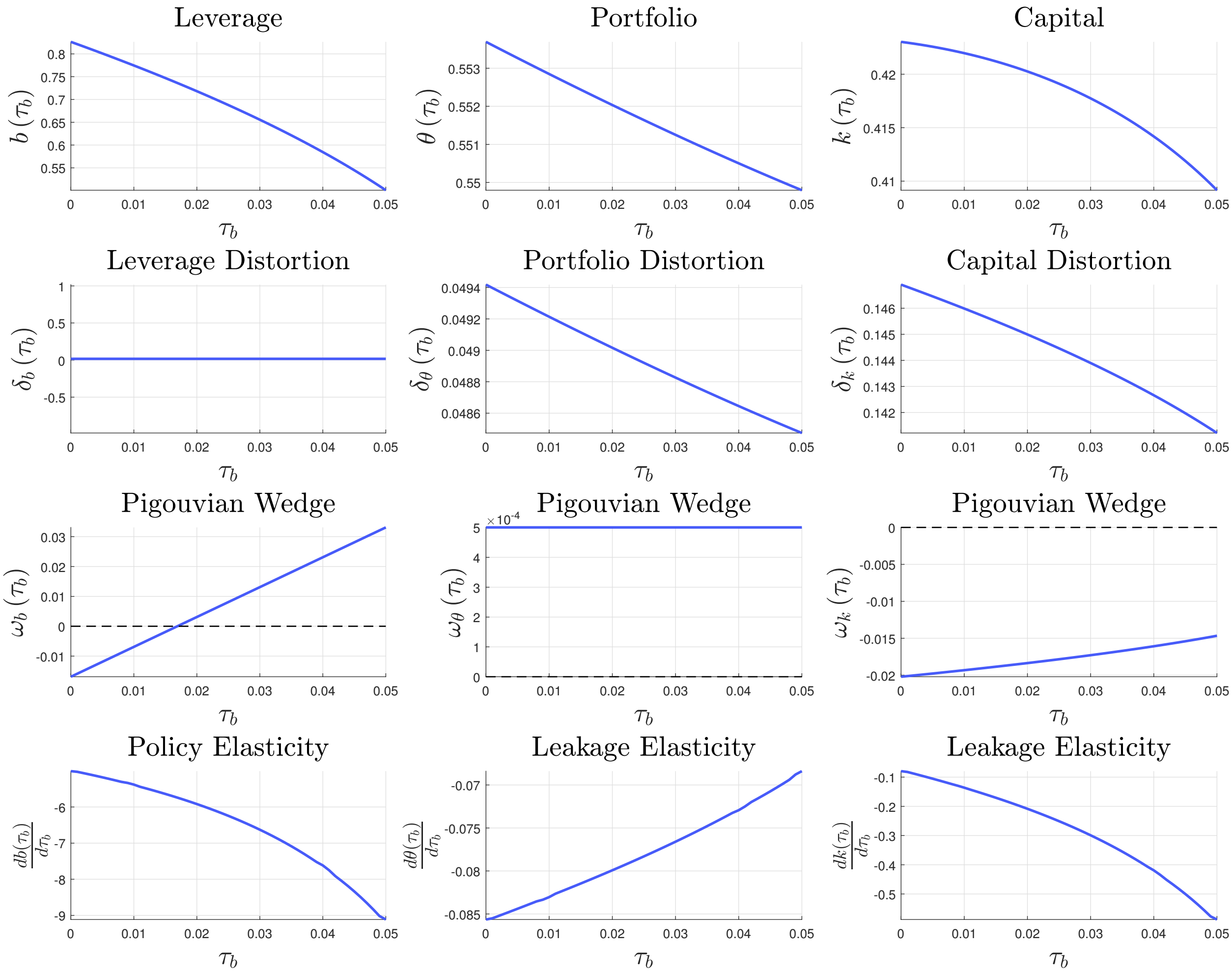

Figure OA-1: Financial Regulation with Environmental Externalities: Second-Best Comparative Statics, Leverage (τ_b)

Note: Figure OA-1 illustrates relevant comparative statics of our application on financial regulation with environmental externalities. In particular, we show how different variables vary with different values of τ_b, when $\tau_k = 0$ and when τ_θ is set at the second-best level (previously computed). The top row show equilibrium leverage b^i, portfolio allocations θ^i, and capital k^i. The second row shows leverage, portfolio, and capital distortions, defined in Equations (38) through (40), while the third row shows the associated Pigouvian wedges. The bottom row shows the policy elasticity $\frac{db}{d\tau_b}$ and the two leakage elasticities, $\frac{d\theta}{d\tau_b}$ and $\frac{dk}{d\tau_b}$ (since in this figure we are keeping τ_θ predetermined). The parameters used are described in Figure 1.

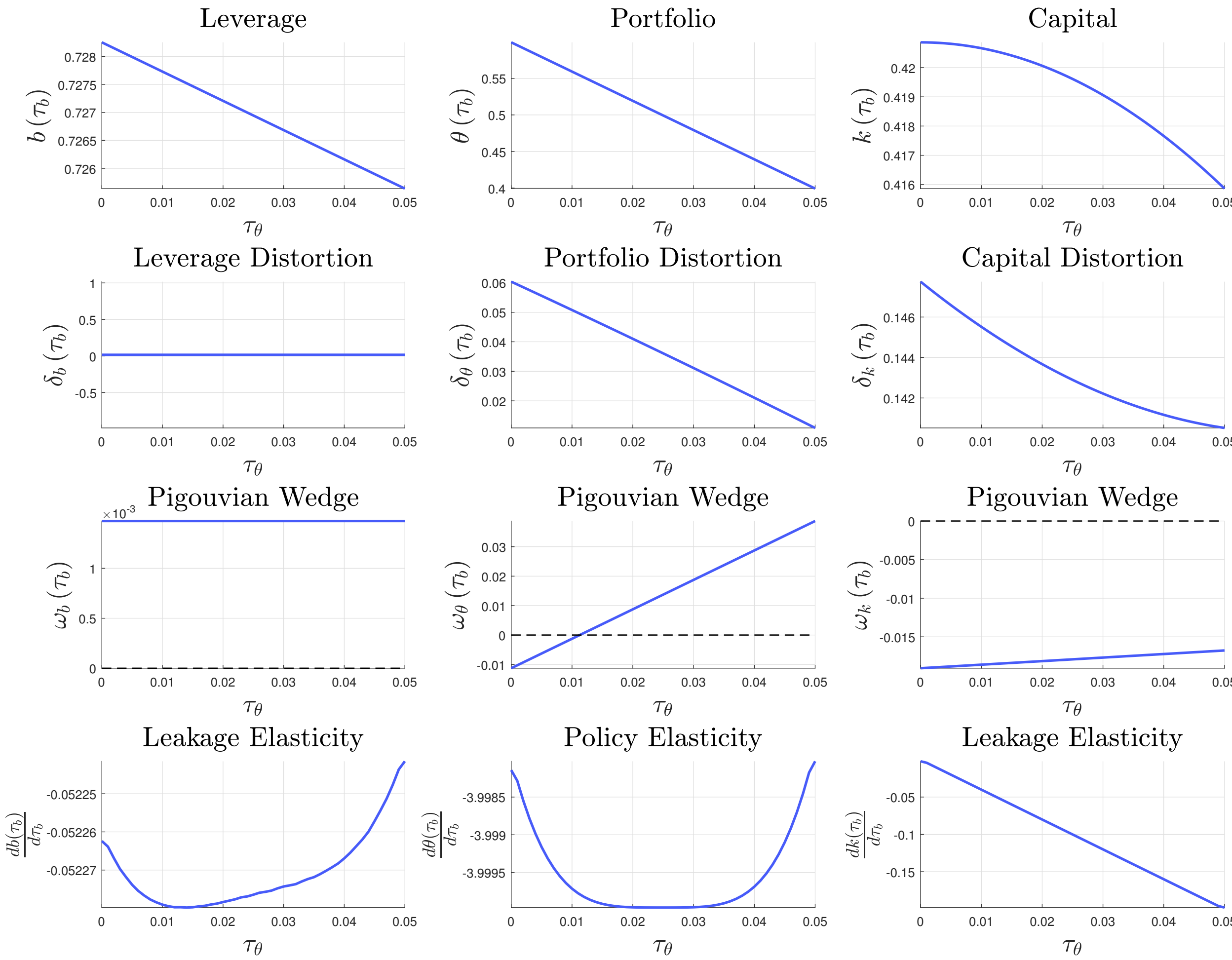

Figure OA-2: Financial Regulation with Environmental Externalities: Second-Best Comparative Statics, Risk Weights (τ_θ)

Note: Figure OA-2 illustrates relevant comparative statics of our application on regulation with environmental externalities. In particular, we show how different variables vary with different values of τ_b, when $\tau_k = 0$ and when τ_θ is set at the second-best level (previously computed). The top row show equilibrium leverage b^i, portfolio allocations θ^i, and capital k^i. The second row shows leverage, portfolio, and capital distortions, defined in Equations (38) through (40), while the third row shows the associated Pigouvian wedges. The bottom row shows the policy elasticity $\frac{db}{d\tau_b}$ and the two leakage elasticities, $\frac{d\theta}{d\tau_b}$ and $\frac{dk}{d\tau_b}$ (since in this figure we are keeping τ_θ predetermined). The parameters used are described in Figure 1.

where

$$J\left(\bar{b},\varphi\right)=\max_{b^i,\theta^i}\left\{M\left(b^i,\theta^i\right)-\Omega\left(\theta^i\right)\text{ subject to }b^i+\varphi\theta^i\leq\bar{b}\right\}. \tag{59}$$

By the envelope theorem, we have

$$\begin{aligned}\frac{\partial J}{\partial\bar{b}}&=\upsilon\geq 0\\ \frac{\partial J}{\partial\varphi}&=-\upsilon\theta^i\leq 0,\end{aligned}$$

where υ denotes the Lagrange multiplier on the constraint in (59). Totally differentiating (58), we now obtain

$$\begin{aligned}\frac{\partial J}{\partial\bar{b}}&=\frac{dk^i}{d\bar{b}}\Upsilon''\left(k^\star\right)\\ \frac{\partial J}{\partial\varphi}&=\frac{dk^i}{d\varphi}\Upsilon''\left(k^\star\right),\end{aligned}$$

so that the convexity of $\Upsilon\left(.\right)$ immediately implies

$$\frac{dk^i}{d\bar{b}}\geq 0,\frac{dk^i}{d\varphi}\leq 0,$$

as claimed in the text.

Further simulation results: Given our functional form assumptions for the simulation, note that we can express the default threshold $s^\star\left(b^i,\theta^i\right)$ as

$$s^\star\left(b^i,\theta^i\right)=\frac{b^i-\left(\alpha_0^i+\alpha_b^ib^i+\alpha_\theta^i\theta^i\right)}{d_1\theta^i+d_2\left(1-\theta^i\right)-\alpha_s^i}.$$

Relatedly, note that the marginal distortions in Equations (38) through (40) correspond then to

$$\begin{aligned}\delta_b&=\underbrace{\beta^C\left(1+\kappa\right)\alpha_b}_{\equiv\chi_b}\\ \delta_\theta&=\underbrace{\beta^C\left(1+\kappa\right)\alpha_\theta}_{\equiv\chi_\theta}+\underbrace{\frac{\partial\Psi\left(\theta^i\right)}{\partial\theta^i}}_{\equiv\psi_\theta}\\ \delta_k&=\underbrace{\beta^C\left(1+\kappa\right)\left(\alpha_0^i+\alpha_b^ib^i+\alpha_\theta^i\theta^i-\alpha_s^i\right)}_{\equiv\chi_k}+\underbrace{\Psi\left(\theta^i\right)}_{\equiv\psi_k}.\end{aligned}$$

Also, note that

$$\Omega'\left(\theta^i\right)=z^\eta\left(\Omega\left(\theta\right)\right)^{1-\eta}\left(a\left(\theta^i\right)^{\eta-1}-\left(1-a\right)\left(1-\theta^i\right)^{\eta-1}\right),$$

and similarly for $\Psi'\left(\theta^i\right)=\frac{\partial\Psi\left(\theta^i\right)}{\partial\theta^i}$.

D Proofs and derivations: Section 5

D.1 Application 1

Default and repayments: Investor i optimally defaults at date 1 if $v^i s + t^i \left(b^i, s\right) - b^i < 0$.[41] Assuming that $v^i + \frac{\partial t^i\left(b^i, s\right)}{\partial s} > 0$, there exists a unique threshold $s^{i\star}\left(b^i\right)$ such that default occurs if and only if $s < s^{i\star}\left(b^i\right)$. Therefore, the definition of the repayment eventually received by creditors, $\mathcal{P}^i\left(b^i, s\right)$, is

$$\mathcal{P}^i\left(b^i, s\right) = \begin{cases} \phi^i v^i s + t^i\left(b^i, s\right) & s \in \left[\underline{s}, s^{i*}\left(b^i\right)\right) \\ b^i & s \in \left[s^{i*}\left(b^i\right), \overline{s}\right]. \end{cases}$$

In our simulation, we let $t^i\left(b^i, s\right) = \alpha_0^i - \alpha_s^i s + \alpha_b^i b^i$, with $\alpha_s^i < v^i$, so that we can solve explicitly for the default threshold

$$s^{i\star}\left(b^i\right) = \left(\frac{1-\alpha_b^i}{v^i - \alpha_s^i}\right) b^i - \frac{1}{v^i - \alpha_s^i}\alpha_0^i.$$

We further assume that creditors have constant relative risk aversion with coefficient γ.

Creditors' optimal choices and asset pricing: We conjecture and verify that the price $Q^i\left(b^i; m^C(s)\right)$ of investors' debt is a function of b^i and creditors' stochastic discount factor $m^C(s) = \beta^C \frac{u'\left(c_1^C(s)\right)}{u'\left(c_0^C\right)}$. Substituting creditors' budget constraints into their objective, we obtain the simplified version of their maximization problem:

$$\begin{aligned} V^C\left(b^i, m^C(s)\right) = \max_{\{h^i\}_{i\in\mathcal{I}}} u\left(n_0^C - \sum_{i\in\mathcal{I}} h^i Q^i\left(b^i; m^C(s)\right)\right) \\ + \beta^C \int u\left(n_1^C(s) + \sum_{i\in\mathcal{I}} h^i \mathcal{P}^i\left(b^i, s\right) - (1+\kappa)\sum_{i\in\mathcal{I}} t^i\left(b^i, s\right)\right) dF(s), \end{aligned}$$

where $V^C(\cdot)$ denotes creditors' indirect utility as a function of investors' debt choice and market prices. The first-order conditions for this problem, combined with market clearing ($h^i = 1$), yield the following debt-pricing equation:

$$Q^i\left(b^i; m^C(s)\right) = \int_{\underline{s}}^{s^{i\star}\left(b^i\right)} m^C(s)\left(\phi^i v^i s + t^i\left(b^i, s\right)\right) dF(s) + \int_{s^{i\star}\left(b^i\right)}^{\overline{s}} m^C(s)\, b^i dF(s).$$

Note that the stochastic discount factor in equilibrium must satisfy the fixed-point equation

$$m^C(s) = \beta^C \frac{u'\left(n_1^C(s) + \sum_{i\in\mathcal{I}} \mathcal{P}^i\left(b^i, s\right) - (1+\kappa)\sum_{i\in\mathcal{I}} t^i\left(b^i, s\right)\right)}{u'\left(n_0^C - \sum_{i\in\mathcal{I}} Q^i\left(b^i; m^C(s)\right)\right)}.$$

Investors' optimal choices: Substituting investors' budget constraints into their objective, and ignoring exogenous endowments, we obtain the simplified version of their maximization problem:

[41] Note that it is straightforward to make bailouts depend on the decisions of all investors, as in, e.g., Farhi and Tirole (2012).

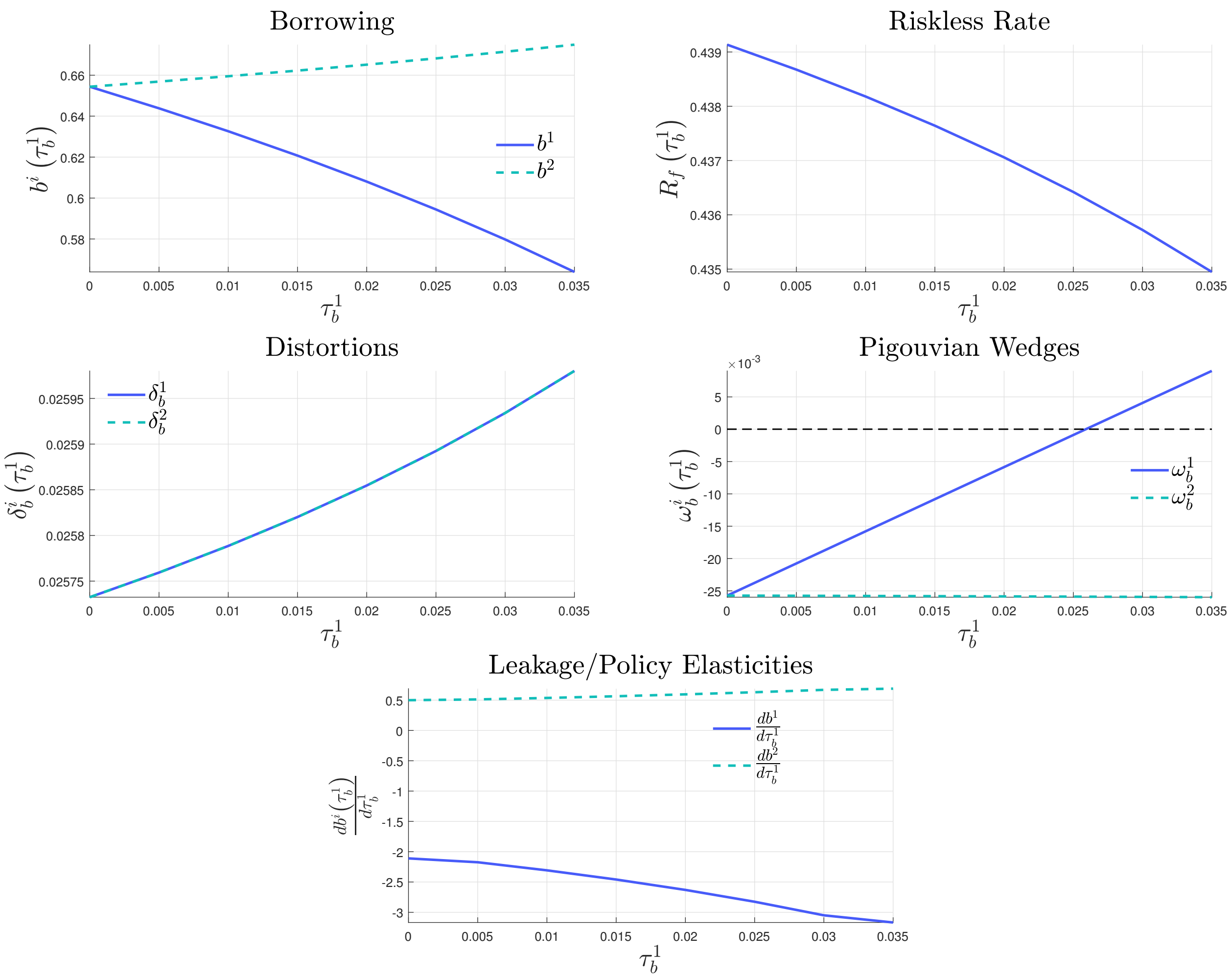

Figure OA-3: Application 1: Second-Best Comparative Statics

Note: Figure OA-3 illustrates relevant comparative statics of Application 1 for different values of τ_b^1, when $\tau_b^2 = 0$. The top left plot shows equilibrium borrowing b^i for both types of investors. The top right plot shows the equilibrium creditors' riskless rate, defined on page 42. The middle left plot shows the distortion associated with the borrowing choice of each investor, δ_b^1 and δ_b^2, defined in Equation (60) — note that the distortions move inversely with changes in the riskless rate R^f and quantitatively the changes are small. The middle right plot shows the Pigouvian wedge associated with the borrowing decision of each investor, ω_b^1 and ω_b^2. The bottom plot shows the policy elasticity $\frac{db^1}{d\tau_b^1}$ and the critical leakage elasticity $\frac{db^2}{d\tau_b^1} > 0$. The parameters used are described in Figure 2.

$$V^{i}\left(\tau_{b}^{i},T_{0}^{i},m^{C}\left(s\right)\right)=\max_{b^{i}}\beta^{i}\int_{s^{i\star}\left(b^{i}\right)}^{\overline{s}}\left(v^{i}s+t^{i}\left(b^{i},s\right)-b^{i}\right)dF\left(s\right)$$
$$+Q^{i}\left(b^{i};m^{C}\left(s\right)\right)-\tau_{b}^{i}b^{i}+T_{0}^{i},$$

where $V^{i}\left(\cdot\right)$ denotes investors' indirect utility as a function of regulation and market prices. The first-order condition determining the optimal b^{i} is

$$-\beta^{i}\int_{s^{i\star}\left(b^{i}\right)}^{\overline{s}}\left(1-\frac{\partial t^{i}}{\partial b}\left(b^{i},s\right)\right)dF\left(s\right)+\frac{\partial Q^{i}\left(b^{i};m^{C}\left(s\right)\right)}{\partial b^{i}}=\tau_{b}^{i},$$

where

$$\frac{\partial Q^{i}\left(b^{i};m^{C}\left(s\right)\right)}{\partial b^{i}}=\int_{s^{i\star}\left(b^{i}\right)}^{\overline{s}}m^{C}\left(s\right)dF\left(s\right)+\int_{\underline{s}}^{s^{i\star}\left(b^{i}\right)}\frac{\partial t^{i}}{\partial b}\left(b^{i},s\right)m^{C}\left(s\right)dF\left(s\right)$$
$$-\left(1-\phi\right)m^{C}\left(s^{i\star}\left(b^{i}\right)\right)v^{i}s^{i\star}\left(b^{i}\right)f\left(s^{i\star}\left(b^{i}\right)\right).$$

Marginal welfare effects: The money-metric marginal welfare effects of changing the regulation τ_{b}^{j} of investor type $j\in\{1,2\}$ are given by

$$\frac{dW}{d\tau_{b}^{j}}=\frac{1}{\lambda_{0}^{C}}\frac{dV^{C}}{d\tau_{b}^{j}}+\sum_{i\in\mathcal{I}}\frac{dV^{i}}{d\tau_{b}^{j}},$$

where $\lambda_{0}^{C}=u'\left(c_{0}^{C}\right)$, since $\lambda_{0}^{i}=1$. Using an envelope argument parallel to our general results in Proposition 1, we obtain, abstracting from pecuniary effects that cancel after aggregating,

$$\frac{dV^{C}}{d\tau_{b}^{j}}=-\left(1+\kappa\right)\beta^{C}\int u'\left(c_{1}\left(s\right)\right)\sum_{i\in\mathcal{I}}\frac{\partial t^{i}\left(b^{i},s\right)}{\partial b^{i}}\frac{db^{i}}{d\tau_{b}^{j}}dF\left(s\right),$$

and

$$\frac{dV^{i}}{d\tau_{b}^{j}}=\tau_{b}^{i}\frac{db^{i}}{d\tau_{b}^{j}},$$

where we have used the assumption that $T_{0}^{i}=\tau_{b}^{i}b^{i}$. Thus, we obtain

$$\frac{dW}{d\tau_{b}^{j}}=\sum_{i\in\mathcal{I}}\frac{db^{i}}{d\tau_{b}^{j}}\left(\tau_{b}^{i}-\underbrace{\left(1+\kappa\right)\int m^{C}\left(s\right)\frac{\partial t^{i}\left(b^{i},s\right)}{\partial b^{i}}dF\left(s\right)}_{=\delta_{b}^{i}}\right).\tag{60}$$

It follows that the first-best policy must satisfy $\tau_{b}^{i}=\delta_{b}^{i}$, $i\in\{1,2\}$.

Proof of Proposition 9 [Shadow Banking/Unregulated Investors]:

Proof. The proposition follows directly by evaluating Equation (60) in the case where the planner is forced to set $\tau_{b}^{2}\equiv0$. □

Further simulation results: Figure OA-3 illustrates comparative statics of the model in the context of the second-best policy, in which $\tau_b^2 = 0$.

D.2 Application 2

Default and repayments: At date 1, investors optimally decide to default when $s < b^i$, and to repay otherwise. Therefore, the definition of the repayment eventually received by creditors per unit of capital k^i, $\mathcal{P}^i\left(b^i, s\right)$, is

$$\mathcal{P}^i\left(b^i, s\right) = \begin{cases} \phi^i s & s \in \left[\underline{s}, b^i\right) \\ b^i & s \in \left[b^i, \overline{s}\right]. \end{cases}$$

Creditors' optimal choices and asset pricing: Since creditors are risk-neutral, they must be indifferent between all quantities of debt purchase in equilibrium. Hence, the valuation of debt *per unit of capital* in equilibrium satisfies

$$Q^i\left(b^i\right) = \beta^C \left(\int_{b^i}^{\overline{s}} b^i dF^C\left(s\right) + \phi \int_{\underline{s}}^{b^i} s dF^C\left(s\right)\right).$$

Investors' optimal choices: Substituting the valuation of debt and the budget constraints into investors' objective function, and ignoring exogenous endowments, we obtain the simplified version of their maximization problem:

$$\max_{b^i, k^i} M\left(b^i\right) k^i - \Upsilon\left(k^i\right) - \tau_b^i b^i k^i - \tau_k^i k^i + T_0^i,$$

where $M\left(b^i\right)$ is given by

$$M\left(b^i\right) = \beta^i \int_{b^i}^{\overline{s}} \left(s - b^i\right) dF^i\left(s\right) + Q^i\left(b^i\right).$$

We assume that all corrective taxes/subsidies are reimbursed to investors with $T_0^i = \tau_b^i b^i k^i + \tau_k^i k^i$. The first-order conditions in this problem, which yield demand functions for credit and investment, are given by the solution to

$$\frac{dM\left(b^i\right)}{db^i} - \tau_b^i = 0 \tag{61}$$

$$M\left(b^i\right) - \Upsilon'\left(k^i\right) - \tau_k^i = 0, \tag{62}$$

where

$$\frac{dM\left(b^i\right)}{db^i} = \beta^C \int_{b^i}^{\overline{s}} dF^C\left(s\right) - \beta^i \int_{b^i}^{\overline{s}} dF^i\left(s\right) - \left(1 - \phi\right) \beta^C b^i f^C\left(b^i\right).$$

As shown in Dávila and Walther (2020*b*), assuming that $0 < \beta^i < \beta^C \leq 1$ and that ϕ is not too small guarantees an interior solution for leverage. Note that the equilibrium value of b^i is independent of k^i, and consequently of τ_k^i. In our simulation, we assume that investment adjustment costs are quadratic, i.e., $\Upsilon\left(k^i\right) = \frac{a}{2}\left(k^i\right)^2$, in which case Equation (62) takes the form

$$k^i = \frac{1}{a}\left(M\left(b^i\right) - \tau_k^i\right).$$

Marginal welfare effects: As shown by Dávila and Walther (2020*b*), social welfare for a planner who computes welfare using beliefs $F^{i,P}$ and $F^{C,P}$ is given by

$$W = M^{P}\left(b^{i}\right)k^{i} - \Upsilon\left(k^{i}\right),$$

where $M^{P}\left(b^{i}\right)$ denotes the present value of payoffs under the planner's beliefs

$$M^{P}\left(b^{i}\right) = \beta^{i}\int_{b^{i}}^{\overline{s}}\left(s-b^{i}\right)dF^{i,P}\left(s\right) + \beta^{C}\left(\int_{b^{i}}^{\overline{s}} b^{i}dF^{C,P}\left(s\right) + \phi\int_{\underline{s}}^{b^{i}} sdF^{C,P}\left(s\right)\right).$$

The marginal welfare effects of varying τ_b^i, after differentiating and substituting investors' first-order conditions, can be written as

$$\begin{aligned}\frac{dW}{d\tau_b^i} &= \frac{dM^{P}\left(b^{i}\right)}{db^{i}}\frac{db^{i}}{d\tau_b^i} + \left(M^{P}\left(b^{i}\right) - \Upsilon'\left(k^{i}\right)\right)\frac{dk^{i}}{d\tau_b^i}\\ &= \left(\tau_b^i - \underbrace{\left(\frac{dM\left(b^{i}\right)}{db^{i}} - \frac{dM^{P}\left(b^{i}\right)}{db^{i}}\right)}_{\delta_b^i}\right)\frac{db^{i}}{d\tau_b^i} + \left(\tau_k^i - \underbrace{\left(M\left(b^{i}\right) - M^{P}\left(b^{i}\right)\right)}_{\delta_k^i}\right)\frac{dk^{i}}{d\tau_b^i}.\end{aligned} \tag{63}$$

Proof of Proposition 10 [Behavioral Distortions/Unregulated Activities]:

Proof. The proposition follows directly by evaluating Equation (63) in the case where the planner is forced to set $\tau_k^i \equiv 0$. □

Further simulation results: Figure OA-4 illustrates comparative statics of the model in the context of the second-best policy, in which $\tau_k^i = 0$.

D.3 Application 3

Default and repayments: The bailout policy specified in Equation (50) implies that investors always (weakly) prefer not to default. Creditors are therefore guaranteed a repayment equal to the face value of legacy debt, b^i. We treat b^i as an exogenous constant throughout this application. The threshold state below which bailouts are positive, denoted $s^{\star}\left(k_1^i, k_2^i\right)$, is implicitly defined by

$$b^{i} = d_1\left(s^{\star}\left(k_1^i, k_2^i\right)\right)k_1^i + d_2\left(s^{\star}\left(k_1^i, k_2^i\right)\right)k_2^i.$$

Notice that this equation has a unique solution because we have assumed that the returns to investment, $d_1(s)$ and $d_2(s)$, are increasing in s.

Creditors' optimal choices and asset pricing: In this application, we assume for simplicity that investors' debt b^i is legacy debt, i.e., issued before the start of the model. Therefore, there is no market for debt, and no market price, at date 0. Creditors are passive agents who simply consume their endowments and debt repayments, and pay the taxes raised for bailouts. Creditors' indirect utility, as a function of

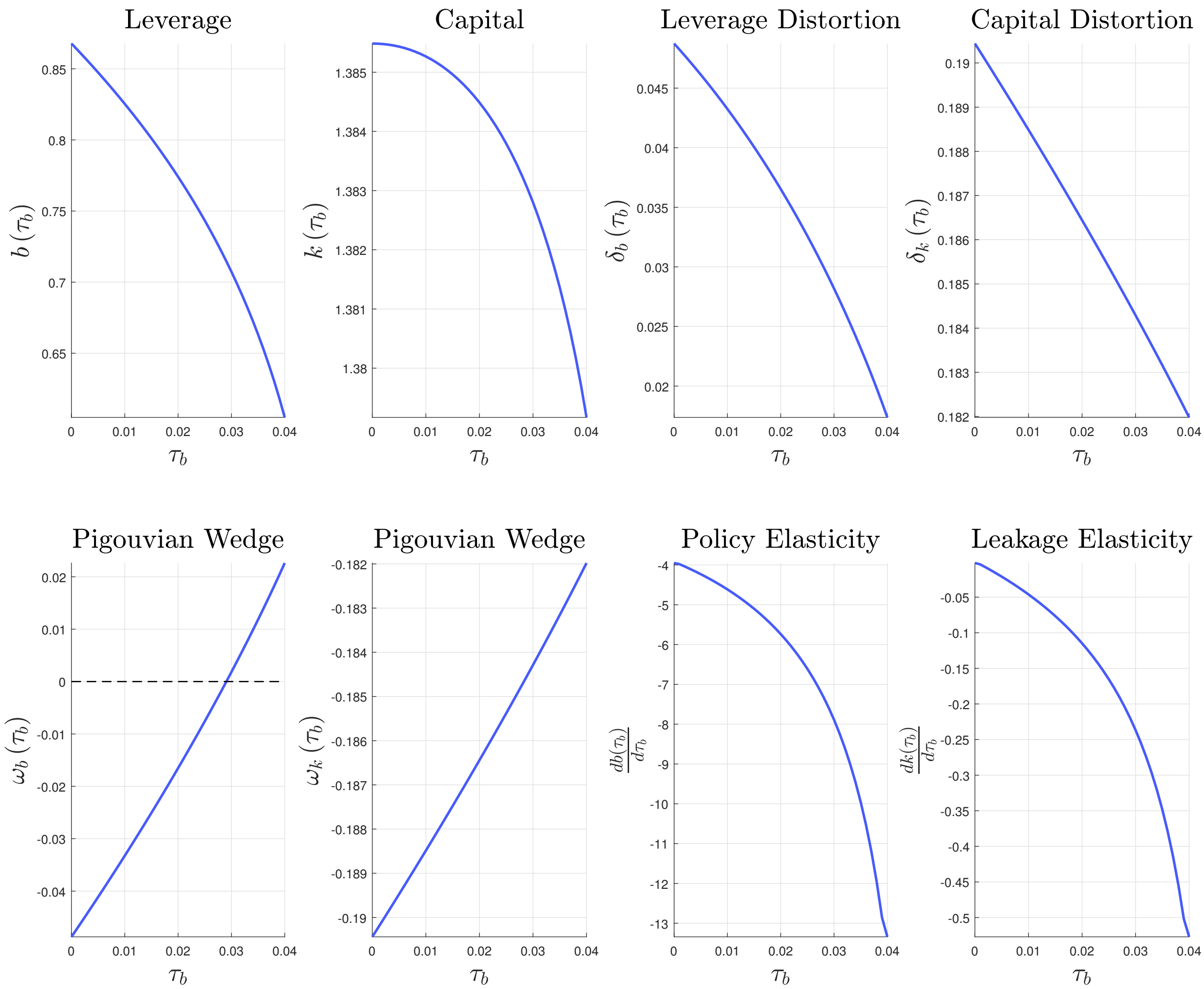

Figure OA-4: Application 2: Second-Best Comparative Statics

Note: Figure OA-4 illustrates relevant comparative statics of Application 2 for different values of τ_b, when $\tau_k = 0$. The top left plot and the top middle-left plot show equilibrium leverage b and investment k. The top middle-right and right plots show the leverage distortion δ_b and the capital distortion δ_k, respectively. The bottom left plot and the bottom middle-left plot show the associated Pigouvian wedges, ω_b and ω_k. The bottom middle-right plot and bottom right plot show the policy elasticity $\frac{db}{d\tau_b}$ and the leakage elasticity $\frac{dk}{d\tau_b}$. The parameters used are described in Figure 3.

investment choices, is then given by

$$V^C\left(k_1^i,k_2^i\right)=\beta^C\left(b^i-(1+\kappa)\int_{\underline{s}}^{\bar{s}}t\left(k_1^i,k_2^i,b^i,s\right)dF\left(s\right)\right)$$
$$=\beta^C\left(b^i-(1+\kappa)\int_{\underline{s}}^{s^\star\left(k_1^i,k_2^i\right)}\left(b^i-d_1\left(s\right)k_1^i-d_2\left(s\right)k_2^i\right)dF\left(s\right)\right).$$

Investors' optimal choices: Substituting investors' budget constraints into their objective, and ignoring exogenous endowments, we obtain the simplified version of their maximization problem:

$$V^i\left(\tau_k^1,\tau_k^2,T_0^i\right)=\max_{k_1^i,k_2^i}\beta^i\int_{s^\star\left(k_1^i,k_2^i\right)}^{\bar{s}}\left[d_1\left(s\right)k_1^i+d_2\left(s\right)k_2^i-b^i\right]dF\left(s\right)-\Upsilon\left(k_1^i,k_2^i\right)$$
$$-\tau_k^1k_1^i-\tau_k^2k_2^i+T_0^i,$$

where $V^i\left(\tau_k^1,\tau_k^2,T_0^i\right)$ denotes investors' indirect utility as a function of taxes/subsidies.

Investors' first-order conditions are given by

$$\beta^i\int_{s^\star\left(k_1^i,k_2^i\right)}^{\bar{s}}d_1\left(s\right)dF\left(s\right)-\frac{\partial\Upsilon\left(k_1^i,k_2^i\right)}{\partial k_1^i}-\tau_k^1=0$$
$$\beta^i\int_{s^\star\left(k_1^i,k_2^i\right)}^{\bar{s}}d_2\left(s\right)dF\left(s\right)-\frac{\partial\Upsilon\left(k_1^i,k_2^i\right)}{\partial k_2^i}-\tau_k^2=0.$$

Marginal welfare effects: The marginal welfare effect of changing the regulation τ_k^j of investment type $j\in\{1,2\}$ is given by

$$\frac{dW}{d\tau_k^j}=\frac{dV^C}{d\tau_k^j}+\frac{dV^i}{d\tau_k^j}.$$

Using the envelope theorem, parallel to our general results in Proposition 1, we obtain

$$\frac{dV^C}{d\tau_k^j}=-\left(1+\kappa\right)\beta^C\sum_{m\in\{1,2\}}\int_{\underline{s}}^{s^\star\left(k_1^i,k_2^i\right)}d_m\left(s\right)dF\left(s\right)\frac{dk_m^i}{d\tau_k^j},$$

and

$$\frac{dV^i}{d\tau_k^j}=\frac{\partial V^i}{\partial\tau_k^j}+\frac{\partial V^i}{\partial T_0^i}\frac{dT_0^i}{d\tau_k^j}$$
$$=\sum_{m\in\{1,2\}}\tau_m\frac{dk_m^i}{d\tau_k^j},$$

where we have used the assumption that $T_0^i=\tau_k^1k_1^i+\tau_k^2k_2^i$. Thus, we obtain

$$\frac{dW}{d\tau_k^j}=\sum_{m\in\{1,2\}}\frac{dk_m^i}{d\tau_k^j}\left(\tau_m-\underbrace{\left(1+\kappa\right)\beta^C\int_{\underline{s}}^{s^\star\left(k_1^i,k_2^i\right)}d_m\left(s\right)dF\left(s\right)}_{=\delta_m}\right).$$

Proof of Proposition 11 [Asset Substitution/Uniform Activity Regulation]:

Proof. To establish this proposition, we can use the general expression for optimal uniform regulation from Proposition 6

$$\overline{\tau}^U = \frac{\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} \left(\boldsymbol{I} - \boldsymbol{L}\right) \boldsymbol{\delta}^U}{\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} \left(\boldsymbol{I} - \boldsymbol{L}\right) \boldsymbol{\iota}}.$$

We have $\boldsymbol{L} = 0$ in this application, because there is no perfectly regulated choice. Hence, we obtain

$$\begin{aligned}\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} \boldsymbol{\delta}^U &= \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{dk_1^i}{d\tau_k^1} & \frac{dk_2^i}{d\tau_k^1} \\ \frac{dk_1^i}{d\tau_k^2} & \frac{dk_2^i}{d\tau_k^2} \end{pmatrix} \begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} \\ &= \left(\frac{dk_1^i}{d\tau_k^1} + \frac{dk_1^i}{d\tau_k^2}\right) \delta_1 + \left(\frac{dk_2^i}{d\tau_k^1} + \frac{dk_2^i}{d\tau_k^2}\right) \delta_2,\end{aligned}$$

and

$$\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} \boldsymbol{\iota} = \left(\frac{dk_1^i}{d\tau_k^1} + \frac{dk_1^i}{d\tau_k^2}\right) + \left(\frac{dk_2^i}{d\tau_k^1} + \frac{dk_2^i}{d\tau_k^2}\right).$$

Combining the last three expressions yields the required result, since

$$\begin{aligned}\overline{\tau}_k &= \frac{\left(\frac{dk_1^i}{d\tau_k^1} + \frac{dk_1^i}{d\tau_k^2}\right) \delta_1 + \left(\frac{dk_2^i}{d\tau_k^1} + \frac{dk_2^i}{d\tau_k^2}\right) \delta_2}{\left(\frac{dk_1^i}{d\tau_k^1} + \frac{dk_1^i}{d\tau_k^2}\right) + \left(\frac{dk_2^i}{d\tau_k^1} + \frac{dk_2^i}{d\tau_k^2}\right)} \\ &= \frac{\frac{dk_1^i}{d\overline{\tau}_k}}{\frac{dk_1^i}{d\overline{\tau}_k} + \frac{dk_2^i}{d\overline{\tau}_k}} \delta_1 + \frac{\frac{dk_2^i}{d\overline{\tau}_k}}{\frac{dk_1^i}{d\overline{\tau}_k} + \frac{dk_2^i}{d\overline{\tau}_k}} \delta_2,\end{aligned}$$

where we have defined the total response of k_m^i to a change in the uniform regulation as

$$\frac{dk_m^i}{d\overline{\tau}_k} = \frac{dk_m^i}{d\tau_k^1} + \frac{dk_m^i}{d\tau_k^2}.$$

□

Derivation of leakage elasticities with separable costs: Assume that the adjustment cost takes the form $\Upsilon\left(k_1^i, k_2^i\right) = \frac{z_1}{2}\left(k_1^i\right)^2 + \frac{z_2}{2}\left(k_2^i\right)^2$. Investors' first-order conditions now become

$$\begin{aligned}k_1^i &= \frac{1}{z_1}\left(\beta^i \int_{s^\star\left(k_1^i, k_2^i\right)}^{\overline{s}} d_1(s)\, dF(s) - \tau_k^1\right) \\ k_2^i &= \frac{1}{z_2}\left(\beta^i \int_{s^\star\left(k_1^i, k_2^i\right)}^{\overline{s}} d_2(s)\, dF(s) - \tau_k^2\right).\end{aligned}$$

Applying the implicit function theorem and Leibniz rule to investors' first-order conditions, and imposing uniform regulation $\tau_k^1 = \tau_k^2 = \overline{\tau}_k$, we have

$$\frac{dk_n^i}{d\overline{\tau}_k} = \frac{1}{z_n}\left(-\beta^i d_n\left(s^\star\left(k_1^i, k_2^i\right)\right) f\left(s^\star\left(k_1^i, k_2^i\right)\right) \frac{ds^\star\left(k_1^i, k_2^i\right)}{d\overline{\tau}_k} - 1\right).$$

Notice that the probability of bailout is

$$\mathcal{P}\left(k_1^i, k_2^i\right) = F\left(s^\star\left(k_1^i, k_2^i\right)\right),$$

and has the property that

$$\frac{d\mathcal{P}\left(k_1^i, k_2^i\right)}{d\overline{\tau}_k} = f\left(s^\star\left(k_1^i, k_2^i\right)\right) \frac{ds^\star\left(k_1^i, k_2^i\right)}{d\overline{\tau}_k}.$$

Hence, we can write

$$\frac{dk_n^i}{d\overline{\tau}_k} = \frac{1}{z_n}\left(-\beta^i d_n\left(s^\star\left(k_1^i, k_2^i\right)\right)\frac{d\mathcal{P}\left(k_1^i, k_2^i\right)}{d\overline{\tau}_k} - 1\right).$$

It follows that the sufficient statistics for leakage elasticities are i) the scaling factor z_n of the cost function, ii) the sensitivity of the probability of bailout to the regulation, and iii) the marginal contribution $d_n\left(s^\star\right)$ of each asset class at the bailout boundary. Notice that the weight on δ_1 in the optimal tax formula now becomes

$$\begin{aligned}
\frac{\frac{dk_1^i}{d\overline{\tau}_k}}{\frac{dk_1^i}{d\overline{\tau}_k} + \frac{dk_2^i}{d\overline{\tau}_k}} &= \frac{\frac{1}{z_1}\left(-\beta^i d_1\left(s^\star\left(k_1^i, k_2^i\right)\right)\frac{d\mathcal{P}\left(k_1^i, k_2^i\right)}{d\overline{\tau}_k} - 1\right)}{\frac{1}{z_1}\left(-\beta^i d_1\left(s^\star\left(k_1^i, k_2^i\right)\right)\frac{d\mathcal{P}\left(k_1^i, k_2^i\right)}{d\overline{\tau}_k} - 1\right) + \frac{1}{z_2}\left(-\beta^i d_2\left(s^\star\left(k_1^i, k_2^i\right)\right)\frac{d\mathcal{P}\left(k_1^i, k_2^i\right)}{d\overline{\tau}_k} - 1\right)} \\
&= \frac{1}{1+\xi_1},
\end{aligned}$$

where

$$\xi_1 = \frac{z_1}{z_2}\frac{1 + \beta^i d_2\left(s^\star\right)\frac{d\mathcal{P}\left(k_1^i, k_2^i\right)}{d\overline{\tau}_k}}{1 + \beta^i d_1\left(s^\star\right)\frac{d\mathcal{P}\left(k_1^i, k_2^i\right)}{d\overline{\tau}_k}}.$$

Further simulation results Figure OA-5 illustrates comparative statics of the model in the context of the second-best policy, in which $\overline{\tau}_k = \tau_k^1 = \tau_k^2$.

D.4 Application 4

Households' optimal choices and asset pricing: Households' optimization problem at date 1 can be expressed as

$$V^H(q) = \max_{k_1^H} F\left(k_1^H\right) - qk_1^H,$$

where $V^H(\cdot)$ denotes households' indirect utility as a function of market prices. The solution to the households' problem is characterized by $q = F'\left(k_1^H\right)$. When combined with market clearing, given by $\sum_i \left(k_0^i - k_1^i\right) = k_1^H$, we find the following equation, which the price q must satisfy:

$$q = F'\left(k_1^H\right) = F'\left(\sum_i \left(k_0^i - k_1^i\right)\right) = F'\left(\frac{1}{q}\sum_i \xi^i k_0^i\right).$$

Notice that this equation defines q as an implicit function of capital investments k_0^i. Below, we derive a solution for the equilibrium value of q in terms of primitives under standard functional forms.

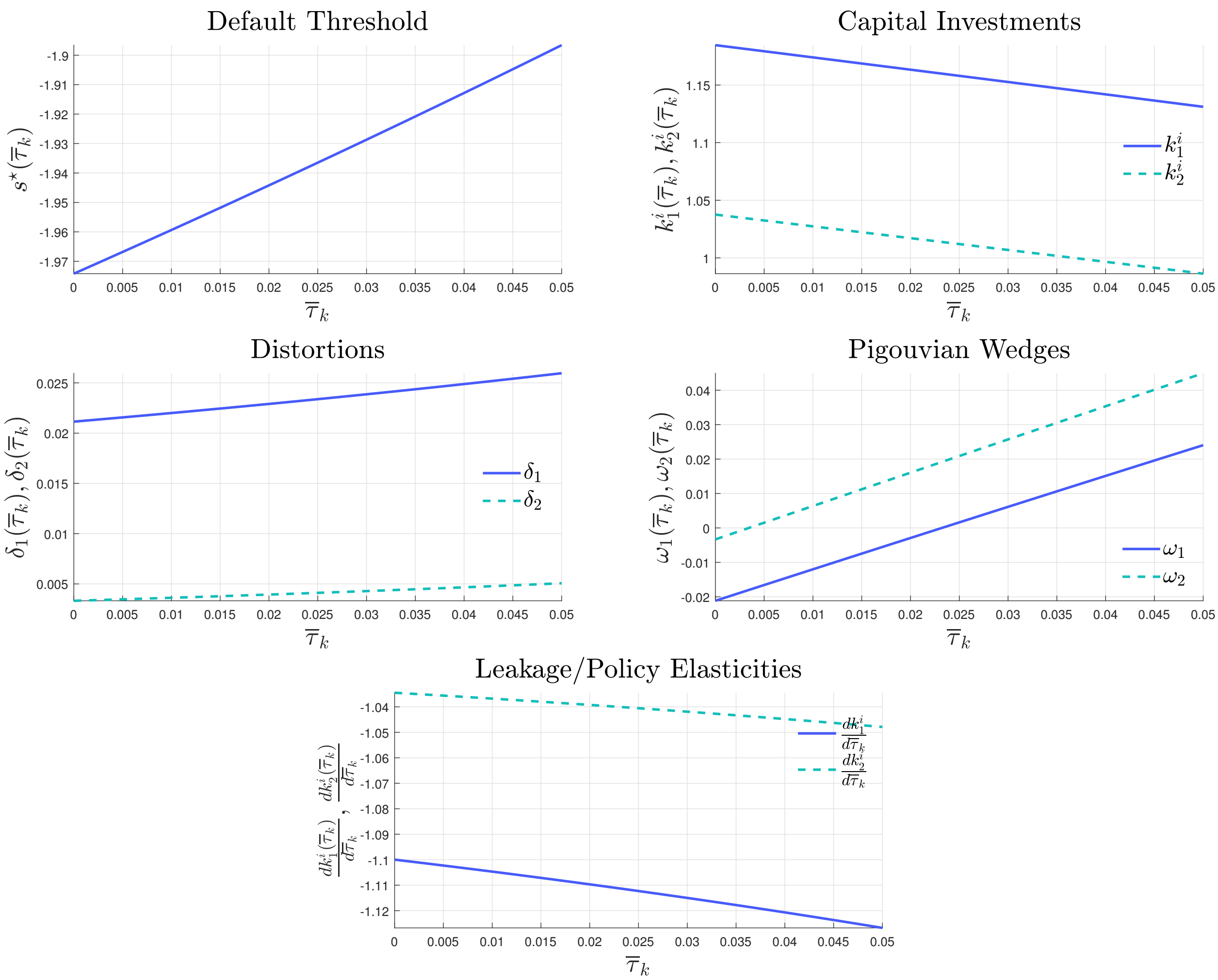

Figure OA-5: Application 3: Second-Best Comparative Statics

Note: Figure OA-5 illustrates relevant comparative statics of Application 3 for different values of $\overline{\tau}_k = \tau_k^1 = \tau_k^2$. The top left plot shows the default threshold $s^\star$. The top right plot shows risky capital investments k_1^i and k_2^i. The middle left plot shows the distortions associated with each investment decisions, δ_1 and δ_2, and the middle right plot shows the associated Pigouvian wedges, ω_1 and ω_2. The bottom plot shows the leakage/policy elasticities $\frac{dk_1^i}{d\overline{\tau}_k}$ and $\frac{dk_2^i}{d\overline{\tau}_k}$. The parameters used are described in Figure 4.

Investors' optimal choices: We solve the investors' problem recursively. At date 1, the non-negativity constraint on consumption is necessarily binding. It follows that the investor optimally chooses $c_1^i = 0$ and

$$k_1^i = \left(1 - \frac{\xi^i}{q}\right) k_0^i.$$

Thus, investor i's maximized utility (i.e., value function) from date 1 onwards is

$$v_1^i\left(q, k_0^i\right) = z^i \left(1 - \frac{\xi^i}{q}\right) k_0^i.$$

At date 0, ignoring exogenous endowments, we can express investors' optimization problem as

$$\begin{aligned} V^i\left(\tau_k^i, T_0^i, q\right) &= \max_{k_0^i} \left\{v_1^i\left(q, k_0^i\right) - \Upsilon^i\left(k_0^i\right) - \tau_k^i k_0^i + T_0^i\right\}, \\ &= \max_{k_0^i} \left\{z^i\left(1 - \frac{\xi^i}{q}\right) k_0^i - \Upsilon^i\left(k_0^i\right) - \tau_k^i k_0^i + T_0^i\right\}, \end{aligned}$$

where $V^i(\cdot)$ denotes investors' indirect lifetime utility as a function of taxes and market prices. The first-order condition determining optimal investment k_0^i is given by

$$z^i\left(1 - \frac{\xi^i}{q}\right) = \Upsilon^{i\prime}\left(k_0^i\right) + \tau_k^i.$$

Assuming quadratic adjustment costs, we obtain the closed form solution

$$k_0^i = \frac{1}{a^i}\left(z^i\left(1 - \frac{\xi^i}{q}\right) - \tau_k^i\right).$$

Marginal welfare effects: The marginal welfare effect of changing the regulation τ_k^j of investor type j is given by

$$\frac{dW}{d\tau_k^j} = \sum_{\ell \in \mathcal{I}} \frac{dV^\ell}{d\tau_k^j} + \frac{dV^H}{d\tau_k^j}.$$

Using the envelope theorem, parallel to our general results in Proposition 1, we obtain

$$\frac{dV^H}{d\tau_k^j} = \frac{\partial V^H}{\partial q}\frac{dq}{d\tau_k^j}.$$

Similarly, we have

$$\begin{aligned} \frac{dV^\ell}{d\tau_k^j} &= \frac{\partial V^\ell}{\partial \tau_k^j} + \frac{\partial V^\ell}{\partial T_0^\ell}\frac{dT_0^\ell}{d\tau_k^j} + \frac{\partial V^\ell}{\partial q}\frac{dq}{d\tau_k^j} \\ &= \tau_k^\ell \frac{dk_0^\ell}{d\tau_k^j} + \frac{\partial v_1^\ell}{\partial q}\frac{dq}{d\tau_k^j}, \end{aligned}$$

where we have used the assumption that $T_0^\ell = \tau_k^\ell k_0^\ell$. Combining, we obtain

$$\begin{aligned}\frac{dW}{d\tau_k^j} &= -k_1^H \frac{dq}{d\tau_k^j} + \sum_{\ell\in\mathcal{I}} \left(\tau_k^\ell \frac{dk_0^\ell}{d\tau_k^j} + \frac{\partial v_1^\ell}{\partial q}\frac{dq}{d\tau_k^j} \right) \\ &= \sum_{i\in\mathcal{I}} \tau_k^i \frac{dk_0^i}{d\tau_k^j} + \left(\sum_{\ell\in\mathcal{I}} \frac{\partial v_1^\ell}{\partial q} - k_1^H \right) \frac{dq}{d\tau_k^j}. \end{aligned} \tag{64}$$

Since q in equilibrium is an implicit function of initial capital investments k_0^i, $i \in \{1, 2\}$, we can write

$$\frac{dq}{d\tau_k^j} = \sum_{i\in\mathcal{I}} \frac{\partial q}{\partial k_0^i}\frac{dk_0^i}{d\tau_k^j}.$$

Moreover, notice that

$$\sum_{\ell\in\mathcal{I}} \frac{\partial v_1^\ell}{\partial q} - k_1^H = \sum_{\ell\in\mathcal{I}} \frac{z^\ell}{q}\frac{\xi^\ell}{q} k_0^\ell - k_1^H = \sum_{\ell\in\mathcal{I}} \left(\frac{z^\ell}{q} - 1 \right) \left(k_0^\ell - k_1^\ell \right),$$

where the last equality follows from the market clearing condition $k_1^H = \sum_{\ell\in\mathcal{I}} \left(k_0^\ell - k_1^\ell \right)$. Substituting into (64) yields

$$\begin{aligned}\frac{dW}{d\tau_k^j} &= \sum_{i\in\mathcal{I}} \tau_k^i \frac{dk_0^i}{d\tau_k^j} + \sum_{\ell\in\mathcal{I}} \left(\frac{z^\ell}{q} - 1 \right) \left(k_0^\ell - k_1^\ell \right) \sum_{i\in\mathcal{I}} \frac{\partial q}{\partial k_0^i}\frac{dk_0^i}{d\tau_k^j} \\ &= \sum_{i\in\mathcal{I}} \left(\tau_k^i - \underbrace{\left(-\frac{\partial q}{\partial k_0^i} \right) \sum_{\ell\in\mathcal{I}} \left(\frac{z^\ell}{q} - 1 \right) \left(k_0^\ell - k_1^\ell \right)}_{=\delta_k^i} \right) \frac{dk_0^i}{d\tau_k^j}. \end{aligned}$$

Proof of Proposition 12 [Fire-Sale Externalities/Uniform Investor Regulation]:

Proof. With uniform taxation, the planner is forced to set $\overline{\tau}_k = \tau_k^1 = \tau_k^2$. The marginal welfare effect of changing the uniform tax is

$$\begin{aligned}\frac{dW}{d\overline{\tau}_k} &= \sum_{j\in\mathcal{I}} \frac{dW}{d\tau_k^j} \\ &= \sum_{i\in\mathcal{I}} \left(\tau_k^i - \delta_k^i \right) \sum_{j\in\mathcal{I}} \frac{dk_0^i}{d\tau_k^j} \\ &= \sum_{i\in\mathcal{I}} \left(\overline{\tau}_k - \delta_k^i \right) \frac{dk_0^i}{d\overline{\tau}_k}, \end{aligned}$$

and solving for the optimal regulation $\frac{dW}{d\overline{\tau}_k} = 0$, we obtain the required second-best solution:

$$\overline{\tau}_k = \frac{\sum_{i\in\mathcal{I}} \frac{dk_0^i}{d\overline{\tau}_k} \delta_k^i}{\sum_{i\in\mathcal{I}} \frac{dk_0^i}{d\overline{\tau}_k}}.$$

□

Closed-form solutions: Under the assumption that $F\left(k_1^H\right) = \frac{\left(k_1^H\right)^\alpha}{\alpha}$, which implies that $F'\left(k_1^H\right) = \left(k_1^H\right)^{\alpha-1}$, we can express the equilibrium price in closed form as

$$q = \left(\sum_i \xi^i k_0^i\right)^{\frac{\alpha-1}{\alpha}}. \tag{65}$$

With quadratic adjustment costs $\Upsilon^i\left(k_0^i\right) = \frac{a^i}{2}\left(k_0^i\right)^2$, investors' optimal choices at date 0 satisfy

$$k_0^i = \frac{1}{a^i}\left(z^i\left(1-\frac{\xi^i}{q}\right) - \tau_k^i\right).$$

Note that $\frac{\partial k_0^i}{\partial q} = \frac{z^i}{a^i}\frac{\xi^i}{q^2} > 0$. Note also that $z^i\left(1-\frac{\xi^i}{q}\right) - \tau_k^i > 0$ is required for $k_0^i > 0$. Combining the optimal choice of k_0^i with the characterization of the price in Equation (65) yields a solution for q in terms of primitives:

$$q = \left(\sum_i \frac{\xi^i}{a^i}\left(z^i\left(1-\frac{\xi^i}{q}\right) - \tau_k^i\right)\right)^{\frac{\alpha-1}{\alpha}}.$$

As expected, the same change in k_0^i has a stronger impact on the price at date 1 for those investors with a higher ξ^i, who are forced to sell more at date 1. Note that we can write $\frac{\partial q}{\partial k_0^i} = \xi^i\frac{\alpha-1}{\alpha}q^{\frac{1}{1-\alpha}}$, so $\frac{\partial q}{\partial k_0^i}$ is higher in absolute, when q is higher.

Further simulation results Figure OA-6 illustrates comparative statics of the model in the context of the second-best policy, in which $\overline{\tau}_k = \tau_k^1 = \tau_k^2$.

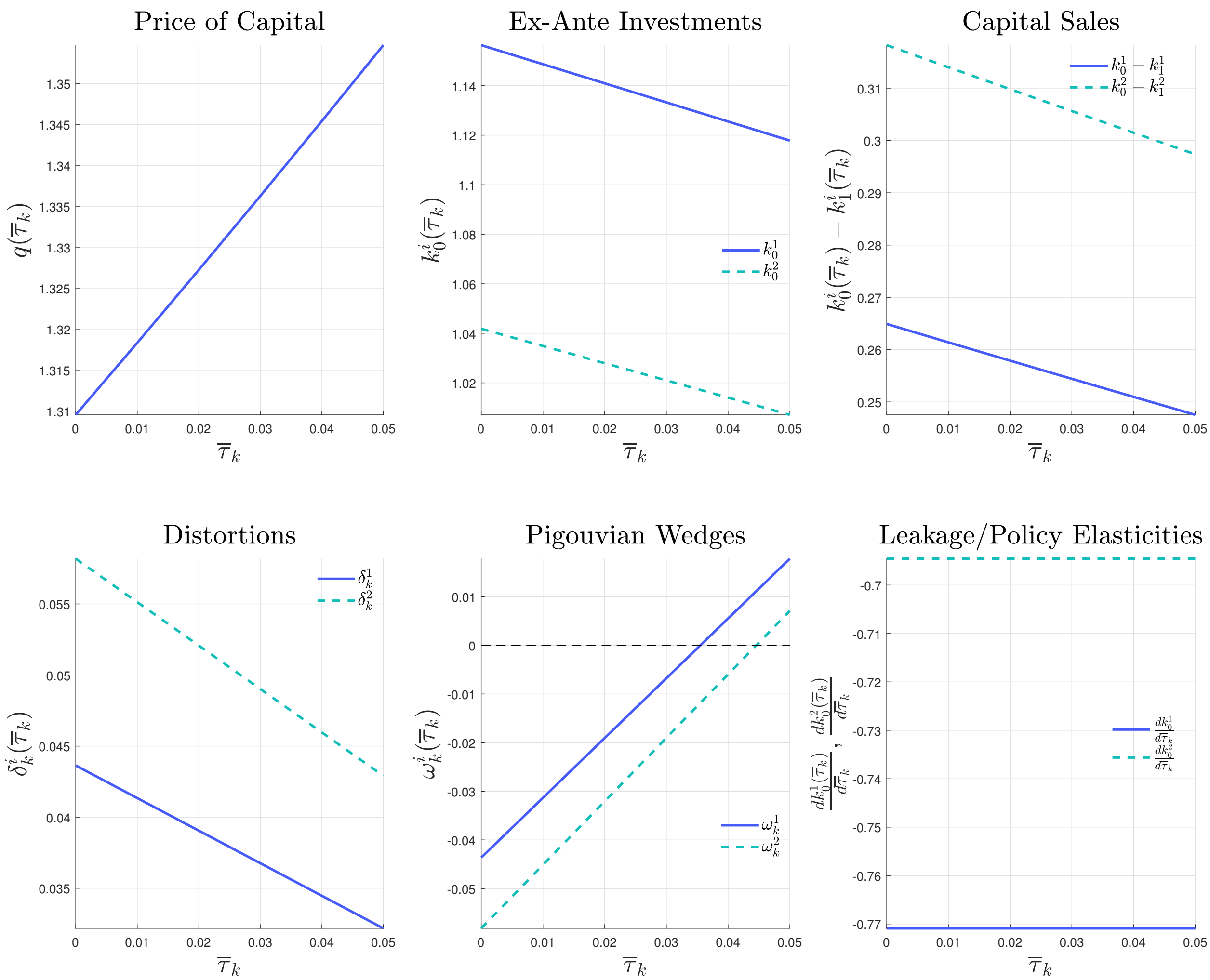

Figure OA-6: Application 4: Second-Best Comparative Statics

Note: Figure OA-6 illustrates relevant comparative statics of Application 4 for different values of $\tau_k^1 = \tau_k^2 = \overline{\tau}_k$. The top left plot shows the price of capital in equilibrium q. The top middle plot shows investment at date 0 for both investor types, k_0^1 and k_0^2. The top right plot shows the amount of capital sold at date 1 for both investor types, $k_0^1 - k_1^1$ and $k_0^2 - k_1^2$. The bottom left plot and the bottom middle plot show the distortions associated with the investment decisions of each investor, δ_k^1 and δ_k^2, and the associated Pigouvian wedges, ω_k^1 and ω_k^2. The bottom right plot shows the leakage/policy elasticities $\frac{dk_0^1}{d\overline{\tau}_k}$ and $\frac{dk_0^2}{d\overline{\tau}_k}$. The parameters used are described in Figure 5.

E Classical Consumer Theory Formulation

In this section, we show that Proposition 1 holds unchanged in the context of classical consumer theory after suitably reinterpreting some of the variables. Since the remaining propositions in the body of the paper follow from Proposition 1, showing that Proposition 1 holds unchanged in a classical consumer theory scenario is sufficient to establish that all other results are also applicable in that case. Here, we follow closely the notation in Mas-Colell, Whinston and Green (1995).

Environment There is a finite number of consumer types, indexed by $i, j, m \in \mathcal{I}$, where $\mathcal{I} = \{1, 2, \ldots, I\}$. There are L different commodities, indexed by ℓ. The preferences of a type i consumer are represented by the following utility function, which directly depends on the consumption of all consumers:

$$u^i \left(\boldsymbol{x}^i, \left\{ \overline{\boldsymbol{x}}^j \right\}_{j \in \mathcal{I}} \right), \tag{66}$$

where $\boldsymbol{x}^i \in \mathbb{R}^L$ denotes the consumption bundle of a type i consumer.[42] As in Section 2, we denote by $\overline{\boldsymbol{x}}^j \in \mathbb{R}^L$ the consumption bundles of type j consumers as a whole.

Thus a type i consumer maximizes Equation (66) subject to the budget constraint

$$\boldsymbol{p} \cdot \left(\boldsymbol{x}^i - \boldsymbol{e}^i \right) \leq w^i - \boldsymbol{\tau}^i \cdot \boldsymbol{x}^i + T_0^i, \tag{67}$$

where $\boldsymbol{p} \in \mathbb{R}^L$ is the vector of commodity prices, $\boldsymbol{e}^i \in \mathbb{R}^L$ is the endowment in terms of the different commodities of a type i consumer, w^i is the wealth of a type i consumer, and $\boldsymbol{\tau}^i \in \mathbb{R}^L$ and T_0^i are the (potentially consumer-type specific) taxes/subsidies and the transfer received by a type i consumer. As a whole, the transfers must satisfy

$$\sum_{i \in \mathcal{I}} \boldsymbol{\tau}^i \cdot \boldsymbol{x}^i = \sum_{i \in \mathcal{I}} T_0^i. \tag{68}$$

An *equilibrium,* given corrective taxes/subsidies $\left\{ \boldsymbol{\tau}^i \right\}_{i \in \mathcal{I}}$ and lump-sum transfers $\left\{ T_0^i \right\}_{i \in \mathcal{I}}$, consists of consumption bundles $\left\{ \boldsymbol{x}^i \right\}_{i \in \mathcal{I}}$ and prices $\boldsymbol{p}$, such that i) investors maximize utility, Equation (66), subject to budget constraint (67), ii) any revenue raised is returned back to investors, satisfying Equation (68), iii) markets clear, that is, $\sum_{i \in \mathcal{I}} \left(\boldsymbol{x}^i - \boldsymbol{e}^i \right) = 0$, and iv) consumption allocations are consistent in the aggregate, that is, $\boldsymbol{x}^i = \overline{\boldsymbol{x}}^i$, $\forall i$.

Result Here we provide the counterpart of Proposition 1 in the text. As in Section 3, welfare is computed in money-metric terms.

Proposition 13. *[Marginal Welfare Effects of Corrective Regulation: Classical Consumer Theory] The marginal welfare effects of varying the set of regulations $\boldsymbol{\tau}$, $\frac{dW}{d\boldsymbol{\tau}}$, are given by*

$$\frac{dW}{d\boldsymbol{\tau}} = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}} \left(\boldsymbol{\tau} - \boldsymbol{\delta} \right) = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}} \boldsymbol{\omega}, \tag{69}$$

[42] It should be evident that our results apply to even more general environments. For instance, it is straightforward to derive a counterpart of Propositions 1 and 13 in a game theoretic environment in which agent's utilities are given by

$$V^i = u^i \left(\boldsymbol{x}^i, \left\{ \overline{\boldsymbol{x}}^j \right\}_{j \in \mathcal{I}} \right) - \tau^i \cdot x^i + T_0^i.$$

where $\frac{dW}{d\boldsymbol{\tau}}$ is a vector of dimension $L \cdot I \times 1$, $\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}$ is the square Jacobian matrix of policy elasticities of dimension $L \cdot I \times L \cdot I$, and $\boldsymbol{\tau}$ and $\boldsymbol{\delta}$ are vectors of dimension $L \cdot I \times 1$, where

$$\boldsymbol{\tau} = \begin{pmatrix} \boldsymbol{\tau}^1 \\ \vdots \\ \boldsymbol{\tau}^i \\ \vdots \\ \boldsymbol{\tau}^{\mathcal{I}} \end{pmatrix} \quad \textit{and} \quad \boldsymbol{\delta} = \begin{pmatrix} \boldsymbol{\delta}^1 \\ \vdots \\ \boldsymbol{\delta}^i \\ \vdots \\ \boldsymbol{\delta}^{\mathcal{I}} \end{pmatrix}, \quad \textit{where} \quad \boldsymbol{\tau}^i = \begin{pmatrix} \tau_1^i \\ \vdots \\ \tau_\ell^i \\ \vdots \\ \tau_L^i \end{pmatrix} \quad \textit{and} \quad \boldsymbol{\delta}^i = \begin{pmatrix} \delta_1^i \\ \vdots \\ \delta_\ell^i \\ \vdots \\ \delta_L^i \end{pmatrix},$$

and where $\boldsymbol{\delta}^i = -\sum_{m \in \mathcal{I}} \frac{\nabla_{\overline{x}^i} u^m(\cdot)}{\lambda^m}$.

Proof. First, we characterize the change in indirect utility of consumer-type i when varying the vector of taxes/subsidies on consumer-type j:

$$\begin{aligned} \frac{dV^i}{d\boldsymbol{\tau}^j} &= \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} \nabla_{x^i} u^i(\cdot) + \sum_{m \in \mathcal{I}} \frac{d\overline{\boldsymbol{x}}^m}{d\boldsymbol{\tau}^j} \nabla_{\overline{x}^m} u^i(\cdot) - \lambda^i \left(\frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} \boldsymbol{p} + \frac{d\boldsymbol{p}}{d\boldsymbol{\tau}^j} \left(\boldsymbol{x}^i - \boldsymbol{e}^i \right) + \frac{d\boldsymbol{\tau}^i}{d\boldsymbol{\tau}^j} \boldsymbol{x}^i + \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} \boldsymbol{\tau}^i - \frac{dT_0^i}{d\boldsymbol{\tau}^j} \right) \\ &= \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} \underbrace{\left[\nabla_{x^i} u^i(\cdot) - \lambda^i \left(\boldsymbol{p} + \boldsymbol{\tau}^i \right) \right]}_{=0} + \sum_{m \in \mathcal{I}} \frac{d\overline{\boldsymbol{x}}^m}{d\boldsymbol{\tau}^j} \nabla_{\overline{x}^m} u^i(\cdot) - \lambda^i \left(\frac{d\boldsymbol{p}}{d\boldsymbol{\tau}^j} \left(\boldsymbol{x}^i - \boldsymbol{e}^i \right) + \frac{d\boldsymbol{\tau}^i}{d\boldsymbol{\tau}^j} \boldsymbol{x}^i - \frac{dT_0^i}{d\boldsymbol{\tau}^j} \right), \end{aligned}$$

which follows from the Envelope Theorem when we define $\frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}$, $\frac{d\overline{\boldsymbol{x}}^m}{d\boldsymbol{\tau}^j}$, $\frac{d\boldsymbol{p}}{d\boldsymbol{\tau}^j}$, and $\frac{d\boldsymbol{\tau}^i}{d\boldsymbol{\tau}^j}$ as $L \times L$ Jacobians and $\nabla_x u^i(\cdot)$, $\nabla_{\overline{x}} u^i(\cdot)$, and $\frac{dT_0^i}{d\boldsymbol{\tau}^j}$ as $L \times 1$ gradient vectors. Note that $\frac{dV^i}{d\boldsymbol{\tau}^j}$ is a $L \times 1$ vector and that we use $\frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}$ and $\frac{d\overline{\boldsymbol{x}}^i}{d\boldsymbol{\tau}^j}$ indistinctly going forward, since they are equal in equilibrium.

Normalizing by the marginal value of wealth, we can express this change in money-metric terms as follows:

$$\frac{\frac{dV^i}{d\boldsymbol{\tau}^j}}{\lambda^i} = \sum_{m \in \mathcal{I}} \frac{d\boldsymbol{x}^m}{d\boldsymbol{\tau}^j} \frac{\nabla_{\overline{x}^m} u^i(\cdot)}{\lambda^i} - \frac{d\boldsymbol{p}}{d\boldsymbol{\tau}^j} \left(\boldsymbol{x}^i - \boldsymbol{e}^i \right) - \frac{d\boldsymbol{\tau}^i}{d\boldsymbol{\tau}^j} \boldsymbol{x}^i + \frac{dT_0^i}{d\boldsymbol{\tau}^j}.$$

Now, adding up across consumer types, we have

$$\begin{aligned} \sum_{i \in \mathcal{I}} \frac{\frac{dV^i}{d\boldsymbol{\tau}^j}}{\lambda^i} &= \sum_{i \in \mathcal{I}} \sum_{m \in \mathcal{I}} \frac{d\boldsymbol{x}^m}{d\boldsymbol{\tau}^j} \frac{\nabla_{\overline{x}^m} u^i(\cdot)}{\lambda^i} - \frac{d\boldsymbol{p}}{d\boldsymbol{\tau}^j} \sum_{i \in \mathcal{I}} \left(\boldsymbol{x}^i - \boldsymbol{e}^i \right) - \sum_{i \in \mathcal{I}} \left(\frac{d\boldsymbol{\tau}^i}{d\boldsymbol{\tau}^j} \boldsymbol{x}^i - \frac{dT_0^i}{d\boldsymbol{\tau}^j} \right) \\ &= \sum_{i \in \mathcal{I}} \sum_{m \in \mathcal{I}} \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} \frac{\nabla_{\overline{x}^i} u^m(\cdot)}{\lambda^m} + \sum_{i \in \mathcal{I}} \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} \boldsymbol{\tau}^i, \end{aligned}$$

where the second line follows from the market-clearing condition, $\sum_{i \in \mathcal{I}} \left(\boldsymbol{x}^i - \boldsymbol{e}^i \right) = 0$, and the fact that Equation (68) implies that

$$\sum_{i \in \mathcal{I}} \frac{d\boldsymbol{\tau}^i}{d\boldsymbol{\tau}^j} \boldsymbol{x}^i + \sum_{i \in \mathcal{I}} \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} \boldsymbol{\tau}^i - \sum_{i \in \mathcal{I}} \frac{dT_0^i}{d\boldsymbol{\tau}^j} = 0.$$

Therefore, we can write the aggregate marginal welfare change in money metric terms as

$$\frac{dW}{d\boldsymbol{\tau}^j} = \sum_{i \in \mathcal{I}} \frac{\frac{dV^i}{d\boldsymbol{\tau}^j}}{\lambda^i} = \sum_{i \in \mathcal{I}} \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} \left(\boldsymbol{\tau}^i - \boldsymbol{\delta}^i \right),$$

where

$$\boldsymbol{\delta}^i = -\sum_{m \in \mathcal{I}} \frac{\nabla_{\overline{x}^i} u^m(\cdot)}{\lambda^m},$$

so Equation (69) follows immediately after stacking. $\square$

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