Working Paper Series

No 124 / August 2021

A Multi-level Network Approach to Spillovers Analysis: An Application to the Maltese Domestic Investment Funds Sector

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Abstract

In this paper we present a new approach to analyse the interconnectedness between a macro-level network and a local-level network. Our methodology is developed on the Diebold and Yilmaz connectedness measure and it considers the presence of entities within a global network which can influence other entities within their own local network but are not relevant enough to influence the entities which do not belong to the same local network. This methodology is then applied to the Maltese domestic investment funds sector and we find that a high-level correlation between the domestic funds can transmit higher spillovers to the local stock exchange index and to the government bond secondary market prices. Moreover, a high correlation among the Maltese domestic investment funds can increase their vulnerability to shocks stemming from financial indices, and therefore, investment funds may potentially become a shock transmission channel.

JEL Classification: C32, C58, G10, G23
Keywords: Network model, investment funds, interconnectedness, contagion, systemic risk, herding behaviour.

Introduction

The analysis of interconnectedness and contagion, with the latter defined as an increase in interconnectedness attributable to a turmoil in the markets (Forbes & Rigobon, 1999), became an established topic of studies in the academic field by the end of the 1990s. Initially the focus was on cross-correlations and how these change in extreme events (Boyer, et al., 1997; Loretan & English, 2000; Longin & Solnik, 2001). The global financial crisis and the renewed focus on systemic risk and financial stability exposed the weaknesses of the existing methodologies, thus requiring new and more sophisticated methodologies to analyse and assess properly the interlinkages between financial institutions. One of the methodologies which received significant attention in the field of contagion analysis is the CoVaR approach of Adrian and Brunnermeier (2011), which measures the change in the Value-at-Risk of the financial system conditioned on one or more institutions being in distress.

The interconnectedness analysis took an alternative approach to the usual analysis of the correlation coefficients behaviour, as it started being applied to the identification of financial networks. Each node in the network represents one financial entity, so that the interconnectedness analysis focuses on the strength and the direction of the edges linking these nodes. This network analysis mainly follows two approaches.

The first approach follows the seminal work of Allen and Gale (2000) and is based on the identification of the direct linkages between institutions using granular data such as direct exposures through deposits, borrowings and investments in equity shares. Initially, this strand of research was constrained by the fact that the amount of granular data required for this approach was limited. However, in the aftermath of the global financial crisis, regulators introduced new and more comprehensive reporting requirements for financial institutions. This facilitated the publishing of new studies on network analysis in banks (among others Espinosa-Vega and Solé (2010) and Covi, Gorpe and Kok (2019), in different financial institutions (Abad, et al., 2017) and in derivatives (D'Errico & Roukny, 2019; Rosati & Varica, 2019). In the Maltese scenario, Fenech and Zahra (2020) follow this approach to reconstruct and analyse the underlying network of Maltese financial institutions (namely banks, insurance undertakings and mutual funds) using granular data submitted by the entities licensed with the MFSA.

The second approach is based on the use of market data to identify, usually through statistical methodologies, indirect linkages between entities. The most popular methodology in this strand of research was developed by Diebold and Yilmaz (2009, 2012, 2014). This methodology analyses the equity returns or volatility data through a Vector Autoregressive (VAR) Model, and it uses the forecast error variance decomposition to study the interlinkages between entities in the network. Similar to Diebold and Yilmaz (2009), Billio et al. (2012) developed a network of financial companies starting from a VAR model but the linkages between these companies were studied using a pairwise Granger-causality analysis.

Our study builds on the Diebold and Yilmaz methodology and presents a new approach to analyse multi-level networks. In particular, our methodology considers financial entities which, despite belonging to the same global financial network, may form part of different sub-networks in terms of relevance or geographical area. The main novelty of our methodology is that it considers the presence of a 'macro-level' network and a 'local-level' network. The 'macro-level' network is composed of various large indices whose developments influence the whole global network. The 'local-level' network is composed of entities whose developments can impact the other entities in the same local network but are not important enough to influence the whole global network.

We apply our new methodology to the Maltese domestic investment funds. The application of such interconnectedness analysis to investment funds is particularly suitable as a large quantity of market data is available for building satisfactory time series. Moreover, investment funds tend to have limited direct exposure to each other, with the interconnectedness arising mainly from their joint exposure to the same market factors and to the same underlying individual securities forming part of their investment portfolio. Spillovers between funds and financial markets could be especially relevant in the case where the latter are not particularly liquid, and in those market niches where funds play a dominant role. In the Maltese scenario, where only few transactions occur in the stock exchange, if investment funds start receiving significant redemptions due to poor performance, the fund managers may need to sell off local equities and bonds to meet these redemptions, depressing prices in the Maltese financial markets. Through the Diebold and Yilmaz methodology it is possible to analyse the interlinkages in the investment funds sector even when granular data on exposures is not available. The application of the Diebold and Yilmaz methodology in the context of investment funds is relatively new, with just a few examples in recent years (among the few, Manicaro and Falzon (2017), Meglioli (2019) and ESMA (2020)). In this paper we study the relationship between the correlation structure of domestic funds and the spillovers which they receive and transmit. We find that if funds become more correlated for a prolonged period of time, they could become more vulnerable to external shocks and they tend also to transmit more shocks to the local bond and equity markets.

The paper is structured as follows: the next section presents the methodologies applied to the generalised case and to the specific case studied in this paper. Then, we present the empirical application to the Maltese domestic investment funds. Finally, we analyse our findings from a financial stability perspective.

Methods and Approach

Generalised Case

Consider a global network formed by two different levels: a macro-level and a (or several) local-level network(s). The macro-level network is composed of variables which can influence all the variables in the global network, such as the US GDP or the S&P500 index given their size and importance. However, it is assumed that the variables in the macro-level network are affected only by other variables in the macro-level network, but not by the variables in the local-level network. Conversely, the local-level network comprises of relatively small variables, such as the GDP and the stock indices of small or closed-economies, which are interconnected between themselves and may be affected by developments in the macro-level network. However, these variables are not big enough to have an impact on the macro-level variables or on variables in the other local networks.

Consider m variables w_i in the macro-level network and n variables y_i belonging to the local-level network. Starting from the general case in which w_i is modelled through a VAR(p) model while y_i is modelled as a VAR(q) model, we will have:

$$\boldsymbol{w}_t = \boldsymbol{c} + \sum_{i=1}^{p} \boldsymbol{\Phi}_i \boldsymbol{w}_{t-i} + \boldsymbol{\epsilon}_t \tag{1}$$

and

$$\boldsymbol{y}_t = \boldsymbol{a} + \sum_{i=1}^{q} \boldsymbol{\Theta}_i \boldsymbol{y}_{t-i} + \boldsymbol{\beta} \boldsymbol{w}_t + \boldsymbol{\eta}_t = \boldsymbol{a} + \sum_{i=1}^{q} \boldsymbol{\Theta}_i \boldsymbol{y}_{t-i} + \boldsymbol{\beta} \left(\boldsymbol{c} + \sum_{i=1}^{p} \boldsymbol{\Phi}_i \boldsymbol{w}_{t-i} + \boldsymbol{\epsilon}_t \right) + \boldsymbol{\eta}_t \tag{2}$$

where $\boldsymbol{w}_t$ is an $m \times 1$ vector, $\boldsymbol{\epsilon}_t$ is serially uncorrelated with distribution $N(\boldsymbol{0}, \boldsymbol{\Sigma}_\epsilon)$, $\boldsymbol{y}_t$ is an $n \times 1$ vector where the residual $\boldsymbol{\eta}_t$ is also assumed to be serially uncorrelated with distribution $N(\boldsymbol{0}, \boldsymbol{\Sigma}_\eta)$ and $\boldsymbol{\beta}$ is an $n \times m$ matrix.

This can be written in a single VAR(max(p,q)) framework as:

$$\begin{pmatrix} \boldsymbol{y}_t - \boldsymbol{\mu}_y \\ \boldsymbol{w}_t - \boldsymbol{\mu}_w \end{pmatrix} = \sum_{i=1}^{\max(p,q)} \begin{pmatrix} \boldsymbol{\Theta}_i & \boldsymbol{\beta}\boldsymbol{\Phi}_i \\ \boldsymbol{0} & \boldsymbol{\Phi}_i \end{pmatrix} \begin{pmatrix} \boldsymbol{y}_{t-i} - \boldsymbol{\mu}_y \\ \boldsymbol{w}_{t-i} - \boldsymbol{\mu}_w \end{pmatrix} + \begin{pmatrix} \boldsymbol{I} & \boldsymbol{\beta} \\ \boldsymbol{0} & \boldsymbol{I} \end{pmatrix} \begin{pmatrix} \boldsymbol{\eta}_t \\ \boldsymbol{\epsilon}_t \end{pmatrix} \tag{3}$$

Let's define $\boldsymbol{\xi}_t$ as:

$$\boldsymbol{\xi}_t = \begin{bmatrix} \boldsymbol{\eta}_t \\ \boldsymbol{\epsilon}_t \end{bmatrix}$$

Since the noise term $\boldsymbol{\epsilon}_t$ enters in $\boldsymbol{y}_t$'s equation as an exogenous variable, then the covariance matrix of $\boldsymbol{\xi}_t$ can be defined as:

$$\boldsymbol{\Sigma}_\xi = \begin{bmatrix} \boldsymbol{\Sigma}_\eta & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Sigma}_\epsilon \end{bmatrix}.$$

Moreover,

$$\begin{pmatrix} \boldsymbol{\mu}_y \\ \boldsymbol{\mu}_w \end{pmatrix} = \left[\boldsymbol{I} - \sum_{i=1}^{\max(q,p)} \begin{pmatrix} \boldsymbol{\Theta}_i & \boldsymbol{\beta}\boldsymbol{\Phi}_i \\ \boldsymbol{0} & \boldsymbol{\Phi}_i \end{pmatrix} \right]^{-1} \begin{pmatrix} \boldsymbol{a} \\ \boldsymbol{\beta}\boldsymbol{c} \end{pmatrix} \tag{4}$$

At this point, substituting for the term $\begin{pmatrix} \boldsymbol{I} & \boldsymbol{\beta} \\ \boldsymbol{0} & \boldsymbol{I} \end{pmatrix} \begin{pmatrix} \boldsymbol{\eta}_t \\ \boldsymbol{\epsilon}_t \end{pmatrix}$ with the noise term $\begin{pmatrix} \boldsymbol{\eta}_t^* \\ \boldsymbol{\epsilon}_t \end{pmatrix}$, we can rewrite Eq. 3 in a more traditional VAR model as:

$$\begin{pmatrix} \boldsymbol{y}_t - \boldsymbol{\mu}_y \\ \boldsymbol{w}_t - \boldsymbol{\mu}_w \end{pmatrix} = \sum_{i=1}^{\max(p,q)} \begin{pmatrix} \boldsymbol{\Theta}_i & \boldsymbol{\beta}\boldsymbol{\Phi}_i \\ \boldsymbol{0} & \boldsymbol{\Phi}_i \end{pmatrix} \begin{pmatrix} \boldsymbol{y}_{t-i} - \boldsymbol{\mu}_y \\ \boldsymbol{w}_{t-i} - \boldsymbol{\mu}_w \end{pmatrix} + \begin{pmatrix} \boldsymbol{\eta}_t^* \\ \boldsymbol{\epsilon}_t \end{pmatrix} \tag{5}$$

or,

$$\boldsymbol{z}_t = \sum_{i=1}^{\max(p,q)} \boldsymbol{\Phi}_i^* \boldsymbol{z}_{t-i} + \boldsymbol{\xi}_t^* \tag{6}$$

with $\boldsymbol{z}_t = \begin{pmatrix} \boldsymbol{y}_t - \boldsymbol{\mu}_y \\ \boldsymbol{w}_t - \boldsymbol{\mu}_w \end{pmatrix}$, $\boldsymbol{\Phi}_i^* = \begin{pmatrix} \boldsymbol{\Theta}_i & \boldsymbol{\beta\Phi}_i \\ \boldsymbol{0} & \boldsymbol{\Phi}_i \end{pmatrix}$ and $\boldsymbol{\xi}_t^* = \begin{pmatrix} \boldsymbol{\eta}_t^* \\ \boldsymbol{\epsilon}_t \end{pmatrix}$. Moreover, the covariance of $\boldsymbol{\xi}_t^*$ becomes:

$$\boldsymbol{\Sigma}_{\xi^*} = \begin{pmatrix} \boldsymbol{I} & \boldsymbol{\beta} \\ \boldsymbol{0} & \boldsymbol{I} \end{pmatrix} \begin{pmatrix} \boldsymbol{\Sigma}_\eta & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Sigma}_\epsilon \end{pmatrix} \begin{pmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{\beta}' & \boldsymbol{I} \end{pmatrix} = \begin{bmatrix} \boldsymbol{\Sigma}_\eta + \boldsymbol{\beta\Sigma}_\epsilon \boldsymbol{\beta}' & \boldsymbol{\beta\Sigma}_\epsilon \\ \boldsymbol{\Sigma}_\epsilon \boldsymbol{\beta}' & \boldsymbol{\Sigma}_\epsilon \end{bmatrix}$$

Through Eq. 6 it is now possible to derive the moving average representation:

$$\boldsymbol{z}_t = \sum_{i=1}^{\infty} \boldsymbol{\omega}_i^* \boldsymbol{\xi}_t^* \tag{7}$$

with

$$\boldsymbol{\omega}_i^* = \begin{cases} \boldsymbol{0}, & i < 0 \\ \boldsymbol{I}, & i = 0 \\ \sum_{j=1}^{\max(q,p)} \boldsymbol{\Phi}_j^* \boldsymbol{\omega}_{i-j}^*, & i > 0 \end{cases} \qquad \text{and} \qquad \boldsymbol{\omega}_i^* = \begin{pmatrix} \boldsymbol{\omega}_i^{*11} & \boldsymbol{\omega}_i^{*12} \\ \boldsymbol{0} & \boldsymbol{\omega}_i^{*22} \end{pmatrix}$$

Using the moving average representation, if the information available is only till time t, the forecast error of the value of $\boldsymbol{z}_{t+h}$ is equal to:

$$\begin{aligned} \boldsymbol{v}_{t+h} = \boldsymbol{z}_{t+h} - \boldsymbol{z}_{(t+h|t)} &= \sum_{j=0}^{h-1} \boldsymbol{\omega}_j^* \boldsymbol{\xi}_{t+h-j}^* = \sum_{j=0}^{h-1} \begin{pmatrix} \boldsymbol{\omega}_j^{*11} & \boldsymbol{\omega}_j^{*12} \\ \boldsymbol{0} & \boldsymbol{\omega}_j^{*22} \end{pmatrix} \begin{pmatrix} \boldsymbol{\eta}_{t+h-j}^* \\ \boldsymbol{\epsilon}_{t+h-j} \end{pmatrix} \\ &= \sum_{j=0}^{h-1} \begin{pmatrix} \boldsymbol{\omega}_j^{*11} & \boldsymbol{\omega}_j^{*12} \\ \boldsymbol{0} & \boldsymbol{\omega}_j^{*22} \end{pmatrix} \begin{pmatrix} \boldsymbol{\eta}_{t+h-j} + \boldsymbol{\beta\epsilon}_{t+h-j} \\ \boldsymbol{\epsilon}_{t+h-j} \end{pmatrix} \\ &= \sum_{j=0}^{h-1} \begin{pmatrix} \boldsymbol{\omega}_j^{*11} \boldsymbol{\eta}_{t+h-j} + \left(\boldsymbol{\omega}_j^{*11} \boldsymbol{\beta} + \boldsymbol{\omega}_j^{*12}\right) \boldsymbol{\epsilon}_{t+h-j} \\ \boldsymbol{\omega}_j^{*22} \boldsymbol{\epsilon}_{t+h-j} \end{pmatrix} \end{aligned} \tag{8}$$

Once the matrix $\boldsymbol{\Omega}_i$ is defined as:

$$\boldsymbol{\Omega}_i = \begin{pmatrix} \boldsymbol{\omega}_i^{*11} & \boldsymbol{\omega}_i^{*11} \boldsymbol{\beta} + \boldsymbol{\omega}_i^{*12} \\ \boldsymbol{0} & \boldsymbol{\omega}_i^{*22} \end{pmatrix} \tag{9}$$

the forecast error of the value of $\boldsymbol{z}_{t+h}$ can be written simply as:

$$\boldsymbol{v}_{t+h} = \sum_{j=0}^{h-1} \boldsymbol{\Omega}_j \boldsymbol{\xi}_{t+h-j} \tag{10}$$

Therefore, the covariance matrix of the forecast error can be written as:

$$\mathrm{Cov}(\boldsymbol{v}_{t+h}) = \sum_{j=0}^{h-1} \boldsymbol{\Omega}_j \boldsymbol{\Sigma}_\xi \boldsymbol{\Omega}'_j \tag{11}$$

At this point, it is possible to follow step by step the framework of Pesaran and Shin (1998) for the generalised variance decomposition. Assuming a shock in the k^{th} variable (were k goes from 1 to $n+m$), the new conditional forecast error becomes equal to:

$$\boldsymbol{v}_{t+h}^k = \sum_{j=0}^{h-1} \boldsymbol{\Omega}_j (\boldsymbol{\xi}_{t+h-j} - E(\boldsymbol{\xi}_{t+h-j} | \xi_{k,t+h-j})) \tag{12}$$

and the covariance matrix of the conditional forecast error becomes:

$$\mathrm{Cov}(\boldsymbol{v}_{t+h}^k) = \sum_{j=0}^{h-1} \boldsymbol{\Omega}_j \boldsymbol{\Sigma}_\xi \boldsymbol{\Omega}_j' - \sigma_{kk}^{-1} \sum_{j=0}^{h-1} \boldsymbol{\Omega}_j \boldsymbol{\Sigma}_\xi \mathbf{e}_k \mathbf{e}_k' \boldsymbol{\Sigma}_\xi \boldsymbol{\Omega}_j' \tag{13}$$

where σ_{kk} is the k^{th} diagonal element of $\boldsymbol{\Sigma}_\xi$ and $\boldsymbol{e}_k$ is a selection vector which takes the value of 1 in the k^{th} element and 0 elsewhere.

Therefore, the covariance matrix of the difference in the unconditional and conditional forecast is simply:

$$\boldsymbol{\Delta}_{kh} = \mathrm{Cov}(\boldsymbol{v}_{t+h}^k - \boldsymbol{v}_{t+h}) = \sigma_{kk}^{-1} \sum_{j=0}^{h-1} \boldsymbol{\Omega}_j \boldsymbol{\Sigma}_\xi \mathbf{e}_k \mathbf{e}_k' \boldsymbol{\Sigma}_\xi \boldsymbol{\Omega}_j' \tag{14}$$

A shock in the k^{th} variable explains an amount of forecast error variance in the i^{th} equation equal to:

$$\Psi_{i \leftarrow k}^h = \frac{\sigma_{kk}^{-1} \sum_{j=0}^{h-1} \mathbf{e}_i' \boldsymbol{\Omega}_j \boldsymbol{\Sigma}_\xi \mathbf{e}_k \mathbf{e}_k' \boldsymbol{\Sigma}_\xi \boldsymbol{\Omega}_j' \mathbf{e}_i}{\sum_{j=0}^{h-1} \mathbf{e}_i' \boldsymbol{\Omega}_j \boldsymbol{\Sigma}_\xi \boldsymbol{\Omega}_j' \mathbf{e}_i} \tag{15}$$

The term $\Psi_{i \leftarrow k}^h$ represents the generalised forecast error variance decomposition (GFEVD) as defined in Pesaran and Shin (1998). Following the Diebold and Yilmaz methodology, to build the global network of the model in consideration, we use as an interconnectedness measure the quantity:

$$\lambda_{i \leftarrow k} = \frac{\Psi_{i \leftarrow k}^h}{\sum_{k=1}^{(n+m)} \Psi_{i \leftarrow k}^h} \tag{16}$$

which explains the proportion of the variance attributed to the k^{th} component on the i^{th} component h periods in the future. In non-technical terms, this measure explains the percentage of the variance for one variable expected in the future that is due to a shock which occurred in another variable. The reason for which the GFEVD is divided by the sum of the matrix values in the same row is that, differently from the orthogonalized forecast error variance decomposition, it uses the original errors, and therefore, it would add up to one only in the case in which the original errors are orthogonal.

Finally, we compute some summary connectedness measures as per Diebold and Yilmaz, such as:

Total directional connectedness from others to i:

$$C_{i\leftarrow\blacksquare}^{H} = \sum_{\substack{k=1 \\ k\neq i}}^{n+m} \lambda_{i\leftarrow k}^{h}$$

Total directional connectedness from the macro network to i:

$$C_{i\leftarrow M}^{H} = \sum_{\substack{k=n+1 \\ k\neq i}}^{n+m} \lambda_{i\leftarrow k}^{h}$$

Total directional connectedness to the local network from k:

$$C_{L\leftarrow k}^{H} = \sum_{\substack{i=1 \\ i\neq k}}^{n} \lambda_{i\leftarrow k}^{h}$$

Total directional connectedness to the local network from the macro network:

$$C_{L\leftarrow M}^{H} = \frac{1}{n}\sum_{i=1}^{n}\sum_{k=n+1}^{n+m} \lambda_{i\leftarrow k}^{h}$$

Total connectedness in the local network:

$$LC^{H} = \frac{1}{n}\sum_{\substack{i,k=1 \\ i\neq k}}^{n} \lambda_{i\leftarrow k}^{h}$$

Total directional connectedness from the local network to i:

$$C_{i\leftarrow L}^{H} = \sum_{\substack{k=1 \\ k\neq i}}^{n} \lambda_{i\leftarrow k}^{h}$$

Total directional connectedness to others from k:

$$C_{\blacksquare\leftarrow k}^{H} = \sum_{\substack{i=1 \\ i\neq k}}^{n+m} \lambda_{i\leftarrow k}^{h}$$

Total directional connectedness to the macro network from k:

$$C_{M\leftarrow k}^{H} = \sum_{\substack{i=n+1 \\ i\neq k}}^{n+m} \lambda_{i\leftarrow k}^{h}$$

Total connectedness:

$$C^{H} = \frac{1}{n+m}\sum_{\substack{i,k=1 \\ i\neq k}}^{n+m} \lambda_{i\leftarrow k}^{h}$$

Total connectedness in the macro network:

$$MC^{H} = \frac{1}{m}\sum_{\substack{i,k=n+1 \\ i\neq k}}^{n+m} \lambda_{i\leftarrow k}^{h}$$

The generalised case presents various difficulties from a practical point of view. Namely, in order to estimate the model, both the parameters and the covariance matrix of the error terms need to be constrained. While most of the statistical software and/or packages allow you to insert linear constraints on the coefficient parameters, the same cannot be said about the covariance matrix. Moreover, the idea behind a network is to analyse several nodes at once, especially if one wants to build several levels of a network. When more than one lag is used, the number of parameters which one needs to estimate could add up to several hundreds. Therefore, in the next section we derive a simple and practical two-step estimation methodology for a multi-level network, in which both the macro-level network and local-level network are modelled through a VAR(1).

Derivation of the VAR(1) case

In this section, we present a two-step approach to estimate a multi-level network in the particular case where both the macro-level and local-level networks are modelled using only one lag.

In the first step, the macro-level parameters $\boldsymbol{c}$, $\boldsymbol{\Phi}_1$ and $\boldsymbol{\phi}_j$ are estimated independently. Therefore, considering again the m variables w_i in the macro-level network, the following VAR(1) model, together with its moving-average representation, is estimated:

$$\boldsymbol{w}_t = \boldsymbol{c} + \boldsymbol{\Phi}_1 \boldsymbol{w}_{t-1} + \boldsymbol{\epsilon}_t = \left(\sum_{j=0}^{\infty} \boldsymbol{\phi}_j\right)\boldsymbol{c} + \sum_{j=0}^{\infty} \boldsymbol{\phi}_j \boldsymbol{\epsilon}_{t-j} \tag{17}$$

where $\boldsymbol{w}_t$ is an $m \times 1$ vector, and $\boldsymbol{\epsilon}_t$ is serially uncorrelated with distribution $N(\boldsymbol{0}, \boldsymbol{\Sigma}_\epsilon)$. Thus, it is possible to obtain the estimates $\hat{\boldsymbol{c}}$, $\widehat{\boldsymbol{\Phi}}_1$ and $\widehat{\boldsymbol{\phi}}_j$.

In the second step, the VAR(1) for the n variables y_i belonging to the local-level network is fit, using the macro-level network VAR model, $\boldsymbol{w}_t$, as an exogenous variable:

$$\boldsymbol{y}_t = \boldsymbol{a} + \Theta_1 \boldsymbol{y}_{t-1} + \boldsymbol{\beta} \boldsymbol{w}_t + \boldsymbol{\eta}_t = \left(\sum_{j=0}^{\infty} \boldsymbol{\theta}_j\right)\boldsymbol{a} + \sum_{j=0}^{\infty} \boldsymbol{\theta}_j \boldsymbol{\eta}_{t-j} + \sum_{j=0}^{\infty} \boldsymbol{\Pi}_j \boldsymbol{\epsilon}_{t-j} + \sum_{j=0}^{\infty} \boldsymbol{\mu}_j \tag{18}$$

where $\boldsymbol{y}_t$ is an a $n \times 1$ vector and the residual $\boldsymbol{\eta}_t$ is serially uncorrelated with distribution $N(\boldsymbol{0}, \boldsymbol{\Sigma}_\eta)$. The matrices $\boldsymbol{\Pi}_\mathrm{j}$ and $\boldsymbol{\mu}_j$ model the moving average representation of the macro-level network within the local-network's moving average representation. Their computation is calculated using the estimates $\hat{\boldsymbol{c}}$, $\widehat{\boldsymbol{\Phi}}_1$, $\widehat{\boldsymbol{\phi}}_j$ which were obtained in the first step. The derivation of these two matrices is provided in Appendix A.2. In particular, the matrix $\boldsymbol{\Pi}_\mathrm{j}$ corresponds to the term $\boldsymbol{\omega}_i^{*11}\boldsymbol{\beta} + \boldsymbol{\omega}_i^{*12}$ in Eq. 9.

Now it is possible to follow the same structure of the generalised model. Therefore, using the moving average representation, if the information available is till time t, the best forecast of the value of $\boldsymbol{y}_{t+h}$ is equal to:

$$\boldsymbol{y}_{t+h\,|t} = \left(\sum_{j=0}^{\infty} \boldsymbol{\theta}_j\right)\boldsymbol{a} + \sum_{j=h}^{\infty} \boldsymbol{\theta}_j \boldsymbol{\eta}_{t+h-j} + \sum_{j=h}^{\infty} (\boldsymbol{\Pi}_j \boldsymbol{\epsilon}_{t+h-j}) + \sum_{j=0}^{\infty} \boldsymbol{\mu}_j \tag{19}$$

while, if the information available is up to time $t+h$, it would be possible to compute the true value of $\boldsymbol{y}_{t+h}$ as:

$$\boldsymbol{y}_{t+h} = \left(\sum_{j=0}^{\infty} \boldsymbol{\theta}_j\right)\boldsymbol{a} + \sum_{j=0}^{\infty} \boldsymbol{\theta}_j \boldsymbol{\eta}_{t+h-j} + \sum_{j=0}^{\infty} (\boldsymbol{\Pi}_j \boldsymbol{\epsilon}_{t+h-j}) + \sum_{j=0}^{\infty} \boldsymbol{\mu}_j \tag{20}$$

Therefore, the forecast error $\boldsymbol{v}_{t+h}$ from time t to time $t+h$ is equal to:

$$\boldsymbol{v}_{t+h} = \boldsymbol{y}_{t+h} - \boldsymbol{y}_{t+h\,|t} = \sum_{j=0}^{h-1} (\boldsymbol{\theta}_j \boldsymbol{\eta}_{t+h-j} + \boldsymbol{\Pi}_j \boldsymbol{\epsilon}_{t+h-j}) \tag{21}$$

The covariance matrix can be computed (given the independence between $\boldsymbol{\eta}_t$ and $\boldsymbol{\epsilon}_t$) as:

$$\mathrm{Cov}(\boldsymbol{v}_{t+h}) = \sum_{j=0}^{h-1} \boldsymbol{\theta}_j \boldsymbol{\Sigma}_\eta \boldsymbol{\theta}_j' + \boldsymbol{\Pi}_j \boldsymbol{\Sigma}_\epsilon \boldsymbol{\Pi}_j' \tag{22}$$

It is convenient, at this point, to redefine the forecast error, $\boldsymbol{v}_{t+h}$, in Eq. 21 as:

$$\boldsymbol{v}_{t+h} = \sum_{j=0}^{h-1} \boldsymbol{\Gamma}_j \boldsymbol{\xi}_{t+h-j} \tag{23}$$

with the $n \times (n+m)$ matrix $\boldsymbol{\Gamma}_j$ equal to:

$$\boldsymbol{\Gamma}_j = [\boldsymbol{\theta}_j \quad \boldsymbol{\Pi}_j]$$

while the $(n+m) \times 1$ vector $\boldsymbol{\xi}_{t+h-j}$ is equal to:

$$\boldsymbol{\xi}_{t+h-j} = \begin{bmatrix} \boldsymbol{\eta}_{t+h-j} \\ \boldsymbol{\epsilon}_{t+h-j} \end{bmatrix}$$

and which follows $N(0, \boldsymbol{\Sigma}_\xi)$. The covariance matrix $\boldsymbol{\Sigma}_\xi$ is defined as:

$$\boldsymbol{\Sigma}_\xi = \begin{bmatrix} \boldsymbol{\Sigma}_\eta & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Sigma}_\epsilon \end{bmatrix}$$

Moreover, the covariance matrix of the forecast error can be rewritten as:

$$\mathrm{Cov}(\boldsymbol{v}_{t+h}) = \sum_{j=0}^{h-1} \boldsymbol{\Gamma}_j \boldsymbol{\Sigma}_\xi \boldsymbol{\Gamma}_j' \tag{24}$$

Now it is easy to draw a parallel between Eq. 23-24 with Eq. 10-11. Therefore, following Eq. 12 to Eq.14, we get that the generalised variance decomposition for Eq.18 is equal to (with i taking any value from 1 to n and k taking any value from 1 to $n+m$):

$$\Psi_{i \leftarrow k}^{h} = \frac{\sigma_{kk}^{-1} \sum_{j=0}^{h-1} \mathbf{e}_i' \boldsymbol{\Gamma}_j \boldsymbol{\Sigma}_\xi \mathbf{e}_k \mathbf{e}_k' \boldsymbol{\Sigma}_\xi \boldsymbol{\Gamma}_j' \mathbf{e}_i}{\sum_{j=0}^{h-1} \mathbf{e}_i' \boldsymbol{\Gamma}_j \boldsymbol{\Sigma}_\xi \boldsymbol{\Gamma}_j' \mathbf{e}_i} \tag{25}$$

Now it is possible to combine the macro-level and the local-level network by substituting $\boldsymbol{\Gamma}_\mathrm{j}$ with $\boldsymbol{\Omega}_j$, such that $\boldsymbol{\Omega}_j = \begin{bmatrix} \boldsymbol{\theta}_j & \boldsymbol{\Pi}_j \\ \mathbf{0} & \boldsymbol{\phi}_j \end{bmatrix}$, and letting i take any value between 1 to $n+m$. Substituting, the GFVED becomes equal to Eq. 15, i.e.:

$$\Psi_{i \leftarrow k}^{h} = \frac{\sigma_{kk}^{-1} \sum_{j=0}^{h-1} \mathbf{e}_i' \boldsymbol{\Omega}_j \boldsymbol{\Sigma}_\xi \mathbf{e}_k \mathbf{e}_k' \boldsymbol{\Sigma}_\xi \boldsymbol{\Omega}_j' \mathbf{e}_i}{\sum_{j=0}^{h-1} \mathbf{e}_i' \boldsymbol{\Omega}_j \boldsymbol{\Sigma}_\xi \boldsymbol{\Omega}_j' \mathbf{e}_i}$$

In the traditional Diebold and Yilmaz methodology, the use of an orthogonalized forecast error variance decomposition would allow you to downplay the role of a variable within a network by re-ordering it. However, this new approach solves this issue when it is not just one variable that is in a lower hierarchical level but a whole sub-network. Moreover, this approach allows you to include as many sub-networks as needed, ignoring the problem to find theoretical support for the ordering. Another important benefit is that this approach reduces considerably the number of parameters which need to be estimated compared to the traditional Diebold and Yilmaz methodology. Inserting the macro-level network as exogenous within the local-level network equation allows you to estimate $m \times n$ less parameters (with m being the number of macro-level variables and n the number of local-level variables).

Application to the Maltese domestic investment fund industry

The methodology outlined in this paper is applied to study the financial stability implications of herding behaviour between domestic Maltese investment funds. The Maltese domestic investment funds industry is potentially susceptible to herding behaviour. One of the main reasons is that a handful of asset management companies are managing all the domestic funds. Therefore, it is likely that during a period of distress, a fund manager would take similar investment decisions across the different fund strategies that it manages. Moreover, due to most of the Maltese domestic investment funds being retail and the limited number of Maltese households, it is likely that managers will try to meet the investment preferences of the same pool of investors, thus adopting the same investment behaviour.

The herding behaviour is proxied through the correlation among funds. The financial stability implications are analysed by testing the following two hypotheses:

H_1: a stronger herding behaviour in the fund industry results in the Maltese equity index and long-term Government yields being more vulnerable to shocks in the funds.

H_2: a stronger herding behaviour in the funds industry results in Maltese funds being more vulnerable to external shocks.

We build a global network composed of a macro network and a local Maltese network. The macro network is comprised of six indices which represent developments in the equity and bond markets in Europe and the US. The local Maltese network consists of the Maltese Equity Total Return Index, the Malta Government Bond 10 Year Yield[1] and five time-series representing the different types of domestic investment funds, namely bond funds, diversified funds, equity funds, mixed funds and other funds[2, 3]. The weekly logged returns of the variables in the global network are taken for the period 1st January 2010 - 3rd January 2020 with a total of 523 observations. Consistent with the Diebold and Yilmaz (2014) methodology, we compute the realised volatility for each series. Due to the lack of intra-weekly observations for some variables, the realised volatility is computed as the moving sum of four weekly squared returns. Accordingly, the realised volatility can be seen as the rolling monthly realised volatility observed at a weekly frequency. Consequently, 520 realised volatilities are calculated for each variable. Finally, the log of these realised volatilities is considered in order to approximate normality for each of the observations (Andersen, et al., 2003). The descriptive statistics of both the log returns and the log realised volatilities can be found in Appendix A.5.

Static Multi-Level Network

Initially, we fit a global network using all the available observations, calculating an unconditional static interconnectedness matrix (Table 1). The time horizon selected to compute the interconnectedness matrix is equal to four weeks, which represents how much the shocks suffered by one variable at time t would impact the forecast error variance of another variable one month ahead. This matrix is also represented graphically through the multi-level network chart (Figure 1), where the local network is presented at the top of the figure (blue edges), the macro network at the bottom of the figure (sand edges) and the spillovers from the macro to the local network are illustrated by the grey edges[4]. The bolder the edges are, the higher the spillovers and interconnectedness.

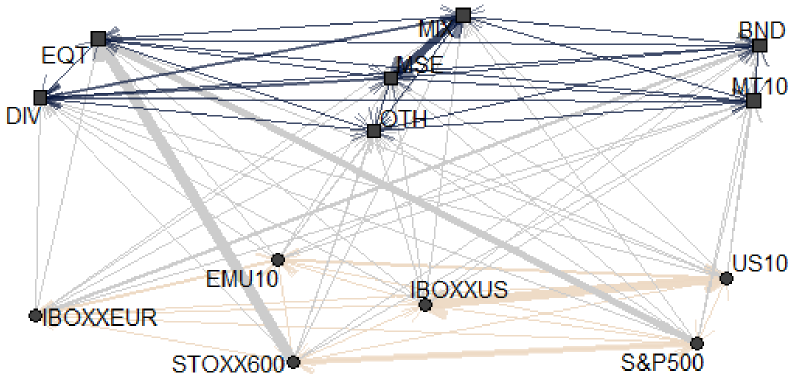

Figure 1: Multi-level Network

[1] The Malta Government Bond Index and the Malta Corporate Bond Index published by the Malta Stock Exchange were not considered because the data are available only from 2017 onwards. Instead, the Government Bond 10 Year Yield is used to proxy the behaviour of the Malta government bond prices due to the inverse relationship between the two.

[2] 'Other funds' is a residual category.

[3] Despite these funds being classified as domestic, their investment funds' portfolios do not only comprise of Maltese assets.

[4] A list of the variables' abbreviations can be found in Appendix A.5.

An analysis of the multi-level network provides some initial important observations. Firstly, it appears that for most of the variables, the forecast error variance is mainly explained by a shock within the same variable, which is reflected by the diagonal elements of the interconnectedness matrix. However, this statement does not stand for equity funds (EQT). Indeed, it appears that most of the spillovers are coming from the variables in the macro network, namely STOXX EUR 600 and S&P 500. Another important point is that the equity funds node is the only one in the local network which is highly affected by shocks in the macro network. Interestingly, for the MSE Equity Total Return Index (MSE), the spillovers received are almost all from within the local network. The main source of external spillover for this variable are the mixed funds (MIX) which represent also the main spillover transmitters within the local network. Instead, the spillovers between the domestic equity funds and the local stock exchange are negligible. Despite appearing counterintuitive, the reason behind this could be that domestic equity funds are mainly investing into European and international stocks which therefore are the main drivers of the funds' volatility. In relation to the macro variables, STOXX EUR 600 is the variable transmitting the largest amount of spillovers followed by the US 10 Year Government Index. The latter is also the macro variable that is mostly vulnerable to shocks from the macro network. The main links in the macro network occur between the US government (US10) and corporate bonds (IBOXX US), and between the STOXX EUR 600 (STOXX600) and the S&P500.

Table 1: Interconnectedness Matrix[5]

		Local							Macro								
		MSE	**MT 10**	**BND**	**DIV**	**EQT**	**MIX**	**OTH**	**IBOXX EUR**	**IBOXX US**	**STOXX 600**	**S&P 500**	**US 10**	**EMU 10**	**Spillovers from others**	**Spillovers from local**	**Spillovers from macro**
Local	**MSE**	62.1	0.2	0.1	7.3	0.8	28.5	0.0	0.0	0.0	0.3	0.0	0.0	0.6	37.9	36.9	1.1
	MT 10	0.3	78.2	9.7	0.5	0.2	1.1	1.3	2.4	0.2	0.8	0.0	0.2	5.1	21.8	13.2	8.7
	BND	0.3	6.1	65.1	1.2	0.8	0.3	1.6	13.0	4.0	0.2	0.4	1.7	5.3	34.9	10.3	24.6
	DIV	10.3	0.7	1.9	66.9	3.3	10.1	0.2	1.5	0.1	1.1	1.0	0.4	2.6	33.1	26.5	6.7
	EQT	0.1	0.1	0.0	1.2	21.4	0.3	1.7	0.4	1.8	42.7	24.2	2.8	3.2	78.6	3.5	75.1
	MIX	26.8	0.5	0.1	7.1	1.6	61.4	0.1	0.2	0.0	0.2	0.2	0.1	1.6	38.6	36.3	2.4
	OTH	0.8	1.3	0.4	0.7	2.0	0.7	80.5	0.1	1.2	4.4	7.3	0.2	0.2	19.5	6.0	13.4
Macro	**IBOXX EUR**	0.0	0.0	0.0	0.0	0.0	0.0	0.0	73.7	6.1	0.4	0.6	4.2	14.9	26.3	0.0	26.3
	IBOXX US	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4.4	51.7	1.6	1.3	34.0	7.1	48.3	0.0	48.3
	STOXX 600	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.5	72.5	14.0	5.2	5.9	27.5	0.0	27.5
	S&P 500	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	2.3	25.0	71.5	0.4	0.8	28.5	0.0	28.5
	US 10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.8	32.8	1.6	0.3	51.4	10.1	48.6	0.0	48.6
	EMU 10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	15.0	8.8	2.3	0.2	12.3	61.4	38.6	0.0	38.6
	Spillovers to others	38.7	8.9	12.3	18.0	8.8	41.0	5.0	41.9	58.9	80.6	49.5	61.4	57.3	$\boldsymbol{C^H = 37.1}$		
	Net Spillovers	0.8	-13.0	-22.6	-15.1	-69.8	2.3	-14.5	15.6	10.6	53.1	21.0	12.8	18.8	$\boldsymbol{C^H_{L \leftarrow M} = 18.8}$		
	Spillovers to local	38.7	8.9	12.3	18.0	8.8	41.0	5.0	17.6	7.4	49.8	33.2	5.4	18.5	$\boldsymbol{LC^H = 18.9}$		
	Spillovers to macro	0.0	0.0	0.0	0.0	0.0	0.0	0.0	24.3	51.5	30.8	16.3	56.0	38.8	$\boldsymbol{MC^H = 36.3}$		

[5] The full variable names can be found in Appendix A.5.

Dynamic Multi-Level Network

Next, we study how the spillovers from and to the fund industry evolved over a period of time. We fit the same model used earlier but now using rolling samples instead of the full sample to capture the connectedness dynamics within the network. We use rolling estimation windows equal to 104 weekly observations, which is equivalent to a period of two years, given the large number of variables that need to be estimated. Accordingly, we compute two interconnectedness indices. The first index represents the average spillovers received by the MSE Equity Total Return Index and the Malta Government Bond 10 Year Yield originating from the domestic funds. The second index represents the amount of spillovers received by investment funds over time.

The spillover plots (Figures 2a and 2b) show that the connectedness between funds and the global network changed significantly over time. From Figure 2a, we observe an increasing trend in spillovers from funds to the local network starting in 2014 and reaching its peak at the end of 2015. This was followed by a sharp drop in the spillovers and a downward trend which persisted till mid-2018. One possible explanation for this observation can be the operation of new funds in the domestic fund industry which were investing in international markets. In Figure 2b, we observe two strong uptrends, the first one starting in January 2014 and reaching its peak in November 2014 while the second one starting in April 2015 till August 2016. Thereafter, a decreasing trend spanning to the beginning of 2018 is observed. In general, one can identify that after 2015, the spillover index level is generally above the levels observed in the previous period.

Figure 2a: Spillover from funds to the local network

Figure 2b: Spillover to funds

In the next stage we analyse the dynamics in the conditional correlation among fund strategies to study the hypothesis question on whether the evolution of the spillovers was affected by the strength of the herding behaviour among fund managers.

We start by estimating the univariate and pairwise parameters of the Dynamic Conditional Correlation (DCC) GARCH (1,1)[6]. The univariate models show that in most of the fund strategies the assumption of a GARCH effect is supported by the data. Most of the parameters are statistically significant apart from the heteroscedasticity parameter α in the mixed funds category.

[6] Further details are provided in Appendix A.3.

The β's are all statistically significant indicating that the univariate volatilities of the fund strategies appear to form clusters.

Looking at the multivariate models, the pairwise parameter estimates (Table 2) show that the parameter '$\boldsymbol{b}$' is significant for most of the equations. This means that there is persistency in the conditional correlation among the strategies. The only exception is for the pair mixed funds and other funds where the parameter '$\boldsymbol{a}$', being also non-statistically significant, appears to indicate that the correlation remains stable across time. The parameter '$\boldsymbol{a}$' is also statistically insignificant for the pairs diversified and equity funds and diversified and other funds. This indicates that a shock at a particular point in time does not seem to have a short-term effect on the conditional correlation coefficients.

Table 2: Univariate and Pairwise parameter estimates

Strategy	μ	ω	α	β
Bond	0.07***	0*	0.1***	0.86***
Diversified	0.06***	0.01	0.1*	0.88***
Equity	0.13**	0.12**	0.12***	0.82***
Mixed	0.08***	0.01	0.09	0.86***
Other	0.03***	0	0.16***	0.84***

Pairs	$\boldsymbol{a}$	$\boldsymbol{b}$
Bond_Diversified	0.01**	0.98***
Bond_Equity	0.02*	0.96***
Bond_Mixed	0.02***	0.98***
Bond_Other	0.03***	0.95***
Diversified_Equity	0.01	0.98***
Diversified_Mixed	0.04*	0.92***
Diversified_Other	0	0.98***
Equity_Mixed	0.01	0.83***
Equity_Other	0.04***	0.95***
Mixed_Other	0	0.95

*** significant at the 0.01 level, ** significant at the 0.05 level, * significant at the 0.1 level.

These estimated parameters are then used to obtain the dynamic conditional correlations across the sample period which are illustrated in Figure 3.

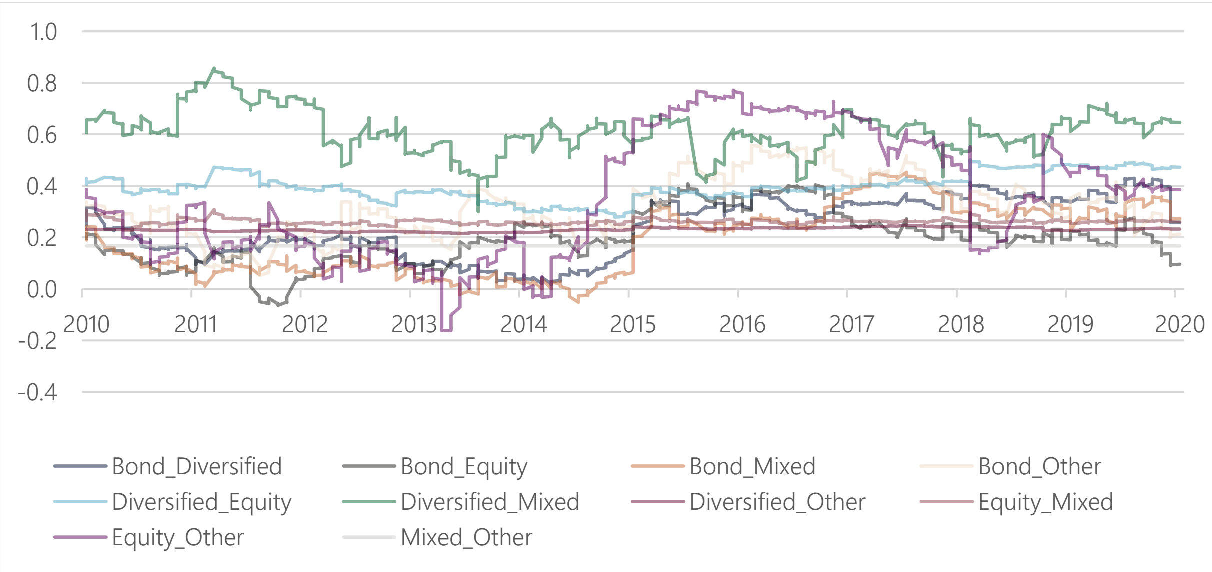

Figure 3: Dynamic conditional correlations across the sample period

The strongest correlation appears to be between the two categories diversified funds and mixed funds. Unexpectedly, the linear relationship between equity and other funds changes significantly over time, from being positive from 2010 till 2013, turning negative over the period spanning from 2013 till 2014, thereafter turning significantly positive for the period 2015 - 2017.

Then the overall correlation among the funds is proxied by averaging all the pairwise correlations at each point in time to obtain a unique series. In this way, it is possible to analyse the short and long-run dynamics among spillovers and correlations. From Figure 4, it appears that there is a strong co-trending between spillover from funds and the correlation up to the beginning of 2016. However, the relationship between DCC and spillover to funds cannot be clearly observed from Figure 4. As a result, we run two Autoregressive Distributed Lag (ARDL) (1,1) models[7]. For one of the models we use the spillover from funds as an endogenous variable while for the other model we use as endogenous the spillover to funds. In both models, the DCC is used as an exogenous variable.

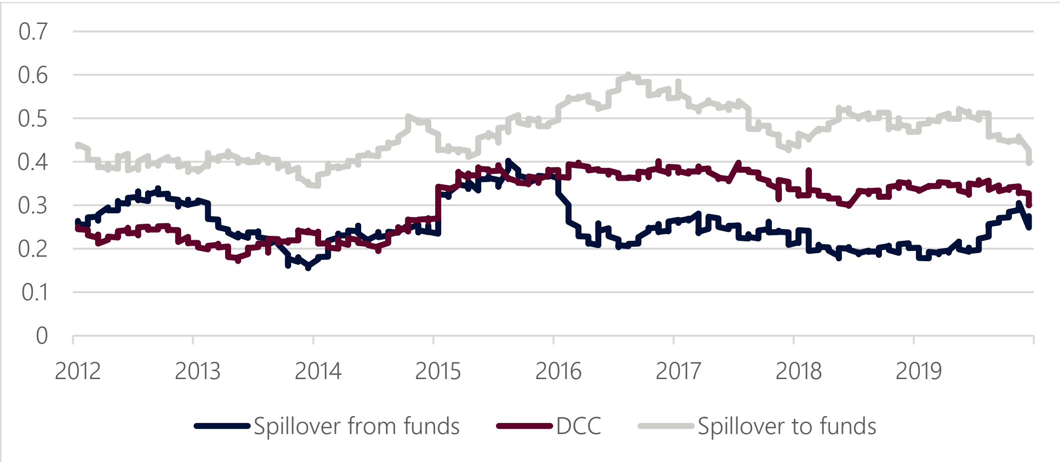

Figure 4: Average DCC and spillovers

The first step is to check that the variables that we want to analyse in the ARDL model are integrated of order 1 (I(1)), that is, difference stationary. Accordingly, we run an Augmented Dickey-Fuller (ADF) test for the average DCC, the spillovers from funds and the spillovers to funds. The results are presented in Table 3. The unit root hypothesis cannot be rejected for the variables in levels but is then rejected for their first difference. Therefore, this confirms that they are I(1).

Table 3: ADF Test

	ADF Test p-value
Spillovers from funds	0.69
Spillovers to funds	0.93
Average DCC	0.82
First difference_ Spillover from funds	0.01
First difference_ Spillover to funds	0.01
First difference_ Average DCC	0.01

[7] Further details are provided in Appendix A.4.

The next step is to run the two ARDL(1,1) regressions. The results in Table 4 show that while all the short-run parameters are not significant, the long-run parameters β's are statistically significant at the 0.01 level. This means that while a change in the correlation among funds does not have any immediate effect on the spillovers transmitted and received by funds, a prolonged high correlation among funds could result to a higher amount of spillovers. This confirms both the H_1 and H_2 hypotheses, meaning that when the herding behaviour in the domestic fund industry is stronger, the local financial market is more vulnerable to shocks in the domestic funds, as well as the domestic funds become more vulnerable to shocks in the financial markets. From a financial stability point of view, this means that the domestic funds can potentially become a transmission channel of shocks to the Maltese equities and bonds when investment managers start behaving in the same way.

Table 4: ARDL parameter estimates

	Parameters	Coeff.	st_error	t_val	p-val
Spillover from funds	α	-0.006	0.006	-1.007	0.314
	θ	0.005	0.005	0.961	0.337
	ψ	-0.023	0.054	-0.420	0.675
	β	0.789	0.227	3.475	0.001
Spillover to funds	α	-0.007	0.006	-1.102	0.270
	θ	0.010	0.010	1.068	0.286
	ψ	-0.004	0.049	-0.087	0.931
	β	1.446	0.185	7.835	0.000

Conclusion

The main objective of this study is to present a new approach to the Diebold and Yilmaz methodology which is applied to a complex network of entities with different hierarchical levels. Given the popularity that the Diebold and Yilmaz methodology gained during recent years, this new approach could support international institutions to analyse more accurately the interconnectedness within the global financial sector. Our approach was applied to the Maltese investment fund industry to analyse the interconnectedness across the domestic fund industry, the local bond and equity markets and the global financial markets. Furthermore, to show the relevance from a financial stability perspective, we analysed how herding behaviour among investment fund managers may influence the spillover transmission mechanism. From our study, we found a long-run relationship between the correlations within the domestic funds and the amount of spillovers which they can transmit and receive.

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Appendix

Appendix A.1 – Multi-level complex model

The framework developed above can be extended to more complex scenarios. Let us assume for instance that in a country there are two indices, namely a small cap index and a large cap index. Moreover, let us assume that the companies in the small cap index ($\boldsymbol{y}_t$) are influenced by the companies in the large cap index ($\boldsymbol{x}_t$) and by various global indices ($\boldsymbol{w}_t$). Finally, let us also assume that the companies in the small cap index do not have a direct effect on the global indices. However, they have a potentially indirect effect due to spillovers between the small cap and large cap index, since the companies in the large cap index are considered relevant enough to affect the global indices.

This scenario can be represented by the following system of equations (for simplicity all the VAR models are taken with lag 1, but the same can be applied with higher lags):

$$\begin{cases} \boldsymbol{y}_t^* = \boldsymbol{\Phi}_{11}\boldsymbol{y}_{t-1}^* + \boldsymbol{\Phi}_{12}\boldsymbol{x}_{t-1}^* + \boldsymbol{\beta}\boldsymbol{w}_t^* + \boldsymbol{\epsilon}_t^1 \\ \boldsymbol{x}_t^* = \boldsymbol{\Phi}_{21}\boldsymbol{y}_{t-1}^* + \boldsymbol{\Phi}_{22}\boldsymbol{x}_{t-1}^* + \boldsymbol{\Phi}_{23}\boldsymbol{w}_{t-1}^* + \boldsymbol{\epsilon}_t^2 \\ \boldsymbol{w}_t^* = \boldsymbol{\Phi}_{32}\boldsymbol{x}_{t-1}^* + \boldsymbol{\Phi}_{33}\boldsymbol{w}_{t-1}^* + \boldsymbol{\epsilon}_t^3 \end{cases}$$

which become:

$$\begin{cases} \boldsymbol{y}_t^* = \boldsymbol{\Phi}_{11}\boldsymbol{y}_{t-1}^* + (\boldsymbol{\Phi}_{12} + \boldsymbol{\beta}\boldsymbol{\Phi}_{32})\boldsymbol{x}_{t-1}^* + \boldsymbol{\beta}\boldsymbol{\Phi}_{33}\boldsymbol{w}_{t-1}^* + \boldsymbol{\beta}\boldsymbol{\epsilon}_t^3 + \boldsymbol{\epsilon}_t^1 \\ \boldsymbol{x}_t^* = \boldsymbol{\Phi}_{21}\boldsymbol{y}_{t-1}^* + \boldsymbol{\Phi}_{22}\boldsymbol{x}_{t-1}^* + \boldsymbol{\Phi}_{23}\boldsymbol{w}_{t-1}^* + \boldsymbol{\epsilon}_t^2 \\ \boldsymbol{w}_t^* = \boldsymbol{\Phi}_{32}\boldsymbol{x}_{t-1}^* + \boldsymbol{\Phi}_{33}\boldsymbol{w}_{t-1}^* + \boldsymbol{\epsilon}_t^3 \end{cases} \tag{26}$$

with $\boldsymbol{y}_t^* = \boldsymbol{y}_t - \boldsymbol{\mu}_y$, $\boldsymbol{x}_t^* = \boldsymbol{x}_t - \boldsymbol{\mu}_x$ and $\boldsymbol{w}_t^* = \boldsymbol{w}_t - \boldsymbol{\mu}_w$.

This can be written in matrix form as:

$$\boldsymbol{z}_t = \begin{pmatrix} \boldsymbol{y}_t^* \\ \boldsymbol{x}_t^* \\ \boldsymbol{w}_t^* \end{pmatrix} = \begin{pmatrix} \boldsymbol{\Phi}_{11} & \boldsymbol{\Phi}_{12} + \boldsymbol{\beta}\boldsymbol{\Phi}_{32} & \boldsymbol{\beta}\boldsymbol{\Phi}_{33} \\ \boldsymbol{\Phi}_{21} & \boldsymbol{\Phi}_{22} & \boldsymbol{\Phi}_{23} \\ \boldsymbol{0} & \boldsymbol{\Phi}_{32} & \boldsymbol{\Phi}_{33} \end{pmatrix} \begin{pmatrix} \boldsymbol{y}_{t-1}^* \\ \boldsymbol{x}_{t-1}^* \\ \boldsymbol{w}_{t-1}^* \end{pmatrix} + \begin{pmatrix} \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{\beta} \\ \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \end{pmatrix} \begin{pmatrix} \boldsymbol{\epsilon}_t^1 \\ \boldsymbol{\epsilon}_t^2 \\ \boldsymbol{\epsilon}_t^3 \end{pmatrix}$$

The noise term $\boldsymbol{\epsilon}_t = \begin{pmatrix} \boldsymbol{\epsilon}_t^1 \\ \boldsymbol{\epsilon}_t^2 \\ \boldsymbol{\epsilon}_t^3 \end{pmatrix}$ is assumed to follow the distribution $\boldsymbol{\epsilon}_t \sim \boldsymbol{N}(\boldsymbol{0}, \boldsymbol{\Sigma})$, with $\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} & \boldsymbol{0} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} & \boldsymbol{\Sigma}_{23} \\ \boldsymbol{0} & \boldsymbol{\Sigma}_{32} & \boldsymbol{\Sigma}_{33} \end{bmatrix}$. The noise term can be redefined as $\boldsymbol{\epsilon}_t^* = \begin{pmatrix} \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{\beta} \\ \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \end{pmatrix} \begin{pmatrix} \boldsymbol{\epsilon}_t^1 \\ \boldsymbol{\epsilon}_t^2 \\ \boldsymbol{\epsilon}_t^3 \end{pmatrix}$ so that the covariance matrix becomes equal to $\boldsymbol{\Sigma}^* = \begin{pmatrix} \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{\beta} \\ \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \end{pmatrix} \begin{bmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} & \boldsymbol{0} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} & \boldsymbol{\Sigma}_{23} \\ \boldsymbol{0} & \boldsymbol{\Sigma}_{32} & \boldsymbol{\Sigma}_{33} \end{bmatrix} \begin{pmatrix} \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{\beta} \\ \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \end{pmatrix}'$.

Similar to before, the vector autoregressive model becomes equal to:

$$\boldsymbol{z}_t = \boldsymbol{\Phi}_i^* \boldsymbol{z}_{t-1} + \boldsymbol{\epsilon}_t^*$$

with $\mathbf{\Phi}_i^* = \begin{pmatrix} \mathbf{\Phi}_{11} & \mathbf{\Phi}_{12} + \boldsymbol{\beta}\mathbf{\Phi}_{32} & \boldsymbol{\beta}\mathbf{\Phi}_{33} \\ \mathbf{\Phi}_{21} & \mathbf{\Phi}_{22} & \mathbf{\Phi}_{23} \\ \mathbf{0} & \mathbf{\Phi}_{32} & \mathbf{\Phi}_{33} \end{pmatrix}$. The moving average representation becomes equal to:

$$\boldsymbol{z}_t = \sum_{i=1}^{\infty} \boldsymbol{\omega}_i^* \boldsymbol{\epsilon}_t^* \tag{27}$$

with

$$\boldsymbol{\omega}_i^* = \begin{cases} \mathbf{0}\,, & i < 0 \\ \boldsymbol{I}\,, & i = 0 \\ \mathbf{\Phi}_i^* \boldsymbol{\omega}_{i-j}^*, & i > 0 \end{cases} \qquad \text{and} \qquad \boldsymbol{\omega}_i^* = \begin{pmatrix} \boldsymbol{\omega}_i^{*11} & \boldsymbol{\omega}_i^{*12} & \boldsymbol{\omega}_i^{*13} \\ \boldsymbol{\omega}_i^{*21} & \boldsymbol{\omega}_i^{*22} & \boldsymbol{\omega}_i^{*23} \\ \boldsymbol{\omega}_i^{*31} & \boldsymbol{\omega}_i^{*32} & \boldsymbol{\omega}_i^{*33} \end{pmatrix}.$$

Therefore,

$$\boldsymbol{z}_t = \sum_{i=1}^{\infty} \boldsymbol{\omega}_i^* \boldsymbol{\epsilon}_t^* = \sum_{i=1}^{\infty} \begin{pmatrix} \boldsymbol{\omega}_i^{*11} & \boldsymbol{\omega}_i^{*12} & \boldsymbol{\omega}_i^{*13} \\ \boldsymbol{\omega}_i^{*21} & \boldsymbol{\omega}_i^{*22} & \boldsymbol{\omega}_i^{*23} \\ \boldsymbol{\omega}_i^{*31} & \boldsymbol{\omega}_i^{*32} & \boldsymbol{\omega}_i^{*33} \end{pmatrix} \begin{pmatrix} \boldsymbol{I} & \mathbf{0} & \boldsymbol{\beta} \\ \mathbf{0} & \boldsymbol{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{I} \end{pmatrix} \begin{pmatrix} \boldsymbol{\epsilon}_t^1 \\ \boldsymbol{\epsilon}_t^2 \\ \boldsymbol{\epsilon}_t^3 \end{pmatrix} = \sum_{i=1}^{\infty} \mathbf{\Omega}_i \boldsymbol{\epsilon}_t \tag{28}$$

with $\mathbf{\Omega}_i = \begin{pmatrix} \boldsymbol{\omega}_i^{*11} & \boldsymbol{\omega}_i^{*12} & \boldsymbol{\omega}_i^{*11}\boldsymbol{\beta} + \boldsymbol{\omega}_i^{*13} \\ \boldsymbol{\omega}_i^{*21} & \boldsymbol{\omega}_i^{*22} & \boldsymbol{\omega}_i^{*21}\boldsymbol{\beta} + \boldsymbol{\omega}_i^{*23} \\ \boldsymbol{\omega}_i^{*31} & \boldsymbol{\omega}_i^{*32} & \boldsymbol{\omega}_i^{*31}\boldsymbol{\beta} + \boldsymbol{\omega}_i^{*33} \end{pmatrix}$. From this point onwards it is possible to proceed as per the generalised case.

Appendix A.2 – Moving Average Representation

For this model, both p and q are taken to be equal to 1. Therefore, the moving average representation of $\boldsymbol{y}_t$ becomes:

$$\boldsymbol{y}_t = \left(\sum_{j=0}^{\infty} \boldsymbol{\theta}_j\right)\boldsymbol{a} + \sum_{j=0}^{\infty} \boldsymbol{\theta}_j \boldsymbol{\eta}_{t-j} + \sum_{j=0}^{\infty} \boldsymbol{\theta}_j \boldsymbol{\beta} \boldsymbol{w}_{t-j}$$

with $\boldsymbol{\theta}_j = \boldsymbol{\Theta}^j$ and $\boldsymbol{\theta}_0 = \mathbf{I}$. Moreover, the moving average representation of $\boldsymbol{w}_t$ can be written as:

$$\boldsymbol{w}_t = \left(\sum_{j=0}^{\infty} \boldsymbol{\phi}_j\right)\boldsymbol{c} + \sum_{j=0}^{\infty} \boldsymbol{\phi}_j \boldsymbol{\epsilon}_{t-j}$$

with $\boldsymbol{\phi}_j = \boldsymbol{\Phi}^j$ and $\boldsymbol{\phi}_0 = \mathbf{I}$. Expanding the $\boldsymbol{\beta}\boldsymbol{w}_{t-j}$'s term in the $\boldsymbol{y}_t$ equation, we get:

$$\begin{array}{lcl}
\boldsymbol{\beta}\boldsymbol{w}_t & = & \boldsymbol{\beta} \times \left(\boldsymbol{\epsilon}_t + \boldsymbol{\Phi}^1\boldsymbol{\epsilon}_{t-1} + \boldsymbol{\Phi}^2\boldsymbol{\epsilon}_{t-2} + \boldsymbol{\Phi}^3\boldsymbol{\epsilon}_{t-3} + \boldsymbol{\Phi}^4\boldsymbol{\epsilon}_{t-4} + \cdots + \left(\sum_{j=0}^{T} \boldsymbol{\phi}_j\right)\boldsymbol{c} \right. \\
+ & & \\
\boldsymbol{\Theta}^1\boldsymbol{\beta}\boldsymbol{w}_{t-1} & = & \boldsymbol{\Theta}^1\boldsymbol{\beta} \times \left(\boldsymbol{\epsilon}_{t-1} + \boldsymbol{\Phi}^1\boldsymbol{\epsilon}_{t-2} + \boldsymbol{\Phi}^2\boldsymbol{\epsilon}_{t-3} + \boldsymbol{\Phi}^3\boldsymbol{\epsilon}_{t-4} + \cdots + \left(\sum_{j=0}^{T-1} \boldsymbol{\phi}_j\right)\boldsymbol{c} \right) \\
+ & & \\
\boldsymbol{\Theta}^2\boldsymbol{\beta}\boldsymbol{w}_{t-2} & = & \boldsymbol{\Theta}^2\boldsymbol{\beta} \times \left(\boldsymbol{\epsilon}_{t-2} + \boldsymbol{\Phi}^1\boldsymbol{\epsilon}_{t-3} + \boldsymbol{\Phi}^2\boldsymbol{\epsilon}_{t-4} + \cdots + \left(\sum_{j=0}^{T-2} \boldsymbol{\phi}_j\right)\boldsymbol{c} \right) \\
+ & & \\
\boldsymbol{\Theta}^3\boldsymbol{\beta}\boldsymbol{w}_{t-3} & = & \boldsymbol{\Theta}^3\boldsymbol{\beta} \times \left(\boldsymbol{\epsilon}_{t-3} + \boldsymbol{\Phi}^1\boldsymbol{\epsilon}_{t-4} + \cdots + \left(\sum_{j=0}^{T-3} \boldsymbol{\phi}_j\right)\boldsymbol{c} \right) \\
+ & & \\
\boldsymbol{\Theta}^4\boldsymbol{\beta}\boldsymbol{w}_{t-4} & = & \boldsymbol{\Theta}^4\boldsymbol{\beta} \times \left(\boldsymbol{\epsilon}_{t-4} + \cdots + \left(\sum_{j=0}^{T-4} \boldsymbol{\phi}_j\right)\boldsymbol{c} \right) \\
+\cdots & & \\
= & & \boldsymbol{\beta}\boldsymbol{\epsilon}_t + (\boldsymbol{\beta}\boldsymbol{\Phi}^1 + \boldsymbol{\Theta}^1\boldsymbol{\beta})\boldsymbol{\epsilon}_{t-1} + (\boldsymbol{\beta}\boldsymbol{\Phi}^2 + \boldsymbol{\Theta}^2\boldsymbol{\beta} + \boldsymbol{\Theta}\boldsymbol{\beta}\boldsymbol{\Phi})\boldsymbol{\epsilon}_{t-2} + (\boldsymbol{\beta}\boldsymbol{\Phi}^3 + \boldsymbol{\Theta}\boldsymbol{\beta}\boldsymbol{\Phi}^2 + \boldsymbol{\Theta}^2\boldsymbol{\beta}\boldsymbol{\Phi} + \boldsymbol{\Theta}^3\boldsymbol{\beta})\boldsymbol{\epsilon}_{t-3} \\
& & + (\boldsymbol{\beta}\boldsymbol{\Phi}^4 + \boldsymbol{\Theta}\boldsymbol{\beta}\boldsymbol{\Phi}^3 + \boldsymbol{\Theta}^2\boldsymbol{\beta}\boldsymbol{\Phi}^2 + \boldsymbol{\Theta}^3\boldsymbol{\beta}\boldsymbol{\Phi} + \boldsymbol{\Theta}^4\boldsymbol{\beta})\boldsymbol{\epsilon}_{t-4} + \cdots + \sum_{i=0}^{\infty} \boldsymbol{\mu}_i
\end{array}$$

where $\boldsymbol{\mu}_i = \boldsymbol{\Theta}^i\boldsymbol{\beta}\left(\sum_{j=0}^{t-i} \boldsymbol{\phi}_j\right)\boldsymbol{c}$. Finally, we can define the matrix $\boldsymbol{\Pi}_i = \sum_{j=0}^{i} \boldsymbol{\Theta}^{i-j}\boldsymbol{\beta}\boldsymbol{\Phi}^{\mathrm{j}}$ and rewrite the term $\sum_{j=0}^{\infty} \boldsymbol{\theta}_j \boldsymbol{\beta}\boldsymbol{w}_{t-j}$ as:

$$\sum_{j=0}^{\infty} \boldsymbol{\theta}_j \boldsymbol{\beta}\boldsymbol{w}_{t-j} = \sum_{j=0}^{\infty} \boldsymbol{\Pi}_j \boldsymbol{\epsilon}_{t-j} + \boldsymbol{\mu}_j$$

Appendix A.3 – Dynamic Conditional Correlation (DCC) GARCH Model

The correlation structure among the different domestic funds' strategies is modelled through the popular DCC GARCH Model proposed by Engle and Sheppard (2001). This is a multivariate GARCH model which can be estimated using a two-step approach. The univariate models for each variable are estimated in the first step, with the residuals which are then used as inputs for the second step, where the correlation structure is estimated.

More formally, given the set of information $\mathcal{F}_{t-1}$ available at time $t-1$, the returns of k variables can be modelled as:

$$r_t = \mu + \varepsilon_t$$
$$\varepsilon_t \mid \mathcal{F}_{t-1} \sim N(0, H_t)$$

with $\mathrm{H_t} \equiv \mathrm{D_t} \mathrm{R}_t \mathrm{D_t}$, where R_t is the time varying correlation matrix and D_t is a $k \times k$ diagonal matrix of time varying standard deviations from the univariate GARCH models, such that $D_t = diag\left(\sqrt{h_{1,1,t}}, \sqrt{h_{2,2,t}}, \dots, \sqrt{h_{k,k,t}}\right)$.

The elements of D_t can be determined by the univariate GARCH(p,q) model:

$$h_{i,t} = \omega_i + \sum_{p=1}^{P_i} \alpha_{i,p} \varepsilon_{i,t-p}^2 + \sum_{q=1}^{Q_i} \beta_{i,p}\, h_{i,t-q}$$

while the dynamic correlation structure is modelled as:

$$Q_t = \left(1 - \sum_{m=1}^{M} a_m - \sum_{n=1}^{N} b_n\right) \bar{Q} + \sum_{m=1}^{M} a_m \left(\epsilon_{t-m}\epsilon'_{t-m}\right) + \sum_{n=1}^{N} b_n Q_{t-n}$$
$$R_t = Q_t^{*-1} Q_t Q_t^{*-1}$$

where $\epsilon_t = \varepsilon_t D_t^{-1}$ are the residuals standardized by their conditional standard deviations. Moreover, the standardized residuals are trivially distributed as $\epsilon_t \sim \mathrm{N}(0, R_t)$. The unconditional covariance matrix of the standardized residuals is instead represented by the matrix $\bar{Q}$. Finally, the matrix Q_t^* is a diagonal matrix of the square root of the diagonal elements of Q_t: $Q_t^* = diag\left(\sqrt{q_{1,1,t}}, \sqrt{q_{2,2,t}}, \dots, \sqrt{q_{k,k,t}}\right)$.

The estimation of such model is done through a two-step quasi-likelihood estimation. The log-likelihood function of the estimator can be defined as:

$$\begin{aligned} \mathrm{L} &= -\frac{1}{2}\sum_{\mathrm{t}=1}^{\mathrm{T}} \left(k \log(2\pi) + \log(|\mathrm{H_t}|) + \varepsilon_t' \mathrm{H_t^{-1}} \varepsilon_t\right) \\ &= -\frac{1}{2}\sum_{\mathrm{t}=1}^{\mathrm{T}} \left(k \log(2\pi) + \log(|\mathrm{D_t} R_t D_t|) + \varepsilon_t' \mathrm{D_t^{-1}} R_t^{-1} D_t^{-1} \varepsilon_t\right) \\ &= -\frac{1}{2}\sum_{\mathrm{t}=1}^{\mathrm{T}} \left(k \log(2\pi) + 2\log(|\mathrm{D_t}|) + \log(|R_t|) + \epsilon_t' R_t^{-1} \epsilon_t\right) \end{aligned}$$

In the first step the univariate parameters are estimated replacing the matrix R_t with the identity matrix I_k.

$$QL_1(\phi|r_t) = -\frac{1}{2}\sum_{t=1}^{T}\left(k\log(2\pi) + \log(|D_t I_k D_t|) + \varepsilon_t' D_t^{-1} I_k^{-1} D_t^{-1}\varepsilon_t\right)$$
$$= -\frac{1}{2}\sum_{t=1}^{T}(k\log(2\pi) + 2\log(|D_t|) + \varepsilon_t' D_t^{-2}\varepsilon_t)$$
$$= -\frac{1}{2}\sum_{n=1}^{k}\left(T\log(2\pi) + \sum_{t=1}^{T}\left(\log(h_{it}) + \frac{r_{it}^2}{h_{it}}\right)\right)$$

with $\phi = \mu, \omega, \alpha_1, \dots, \alpha_P, \beta_1, \dots, \beta_P$. Then, the second stage is estimated using the correctly specified likelihood, condition on the parameters estimated in the first stage likelihood:

$$QL_1\left(\Phi \middle| \hat{\phi}, r_t\right) = -\frac{1}{2}\sum_{t=1}^{T}(k\log(2\pi) + 2\log(|D_t|) + \log(|R_t|) + \epsilon_t' R_t^{-1}\epsilon_t)$$

with $\Phi = a_1, \dots, a_P, b_1, \dots, b_P$. And since the second stage is conditioned on $\hat{\phi}$, the second step can be reduced to the maximisation of:

$$QL_1\left(\Phi \middle| \hat{\phi}, r_t\right) = -\frac{1}{2}\sum_{t=1}^{T}(\log(|R_t|) + \epsilon_t' R_t^{-1}\epsilon_t)$$

Appendix A.4 – Autoregressive Distributed Lag (ARDL) Model

To analyse the short-run and long-run dynamics between spillovers to and from the domestic funds and the correlation among the domestic funds, the Autoregressive Distributed Lag (ARDL) Model is used. Following Pesaran and Shin (1998), the ARDL Model is going to be used in the presence of two cointegrated $I(1)$ variables. Given two cointegrated $I(1)$ variables y and x, an ARDL (1,1) model can be defined as:

$$y_t = \phi_1 y_{t-1} + \theta_0 x_t + \theta_1 x_{t-1} + u_t$$
$$\text{with } x_t = x_{t-1} + \epsilon_t$$

Under the assumption that u_t is iid, ϵ_t is a general linear stationary process, $x_t$is strictly exogenous and $|\phi| < 1$, then the ARDL regression can be re-formulated as:

$$\Delta y_t = -(1-\phi_1)y_{t-1} + (\theta_0 + \theta_1)x_{t-1} + \theta_0\Delta x_{t-1} + u_t$$
$$= \alpha y_{t-1} + \theta x_{t-1} + \psi\Delta x_{t-1} + u_t$$

where $\alpha = -(1-\phi_1)$, $\theta = \theta_0 + \theta_1$ and $\psi = \theta_0$.

The long-run equilibrium is reached when $y_t = y_{t-1} = y^*$, $x_t = x_{t-1} = x^*$ and $u_t = 0$. Therefore, in long-run equilibrium the ARDL can be re-formulated as:

$$y^* = \phi_1 y^* + \theta_0 x^* + \theta_1 x^* = \beta x^*$$

with $\beta = \frac{\theta_0+\theta_1}{1-\phi_1} = -\frac{\theta}{\alpha}$. It is possible to sub-divide the parameters of the ARDL into two subgroups: α, θ, θ_0, and θ_1 control for the short-run dynamics, while β represents the long-run parameters on x. Moreover, if the assumptions listed before hold, Pesaran and Shin (1998) find that asymptotically:

$$\sqrt{\sum_{t-1}^{T} x_t^2}\ (\hat{\beta} - \beta) \sim N\left\{0, \frac{\sigma_u}{(1-\phi_1)^2}\right\}$$

Appendix A.5 – Descriptive Statistics

Table 5: List of Variables

Abbreviation	
MSE	MSE Equity Total Return Index
MT 10	Malta Government Bond 10 Year Yield
BND	Bond
DIV	Diversified
EQT	Equity
MIX	Mixed
OTH	Other
IBOXX EUR	IBOXX Euro Corporates (€)
IBOXX US	IBOXX US Corporates (€)
STOXX 600	STOXX Europe 600
S&P 500	S&P 500
US 10	US 10 Year Government Index
EMU 10	EMU 10 Year Government Index

Table 6: Returns

	Local							**Macro**						
	MSE	**MT 10**	**BND**	**DIV**	**EQT**	**MIX**	**OTH**	**IBOXX EUR**	**IBOXX US**	**STOXX 600**	**S&P 500**	**US 10**	**EMU 10**
Count	523	523	523	523	523	523	523	523	523	523	523	523	523
Mean	0.11	-0.01	0.06	0.09	0.11	0.08	0.02	0.08	0.15	0.16	0.29	0.13	0.10
Std	1.13	0.07	0.28	0.58	1.34	0.57	0.15	0.35	1.26	2.12	2.12	1.52	0.81
Skewness	0.12	1.26	-0.56	-0.31	-0.61	0.42	-1.56	-0.51	0.24	-0.63	-0.56	0.25	-0.40
Kurtosis	4.87	5.62	1.43	5.42	1.75	6.53	27.53	2.33	0.88	2.02	1.56	1.52	0.89
Min	-5.82	-0.24	-1.14	-3.96	-5.65	-2.90	-1.34	-1.75	-4.57	-10.35	-8.31	-4.94	-3.39
25%	-0.48	-0.05	-0.07	-0.22	-0.51	-0.24	0.00	-0.12	-0.66	-0.90	-0.83	-0.73	-0.38
50%	0.08	-0.01	0.08	0.07	0.26	0.06	0.02	0.10	0.10	0.44	0.42	0.09	0.23
75%	0.68	0.02	0.23	0.34	0.91	0.38	0.04	0.28	0.96	1.42	1.57	0.99	0.62
Max	5.93	0.47	0.84	2.11	4.42	3.31	0.97	1.33	5.15	8.34	6.24	6.88	2.92

Table 7: Log realized volatility

	Local							**Macro**						
	MSE	**MT 10**	**BND**	**DIV**	**EQT**	**MIX**	**OTH**	**IBOXX EUR**	**IBOXX US**	**STOXX 600**	**S&P 500**	**US 10**	**EMU 10**
Count	520	520	520	520	520	520	520	520	520	520	520	520	520
Mean	0.52	-2.22	-0.80	-0.17	0.75	-0.21	-2.19	-0.55	0.73	1.21	1.22	0.87	0.31
Std	0.53	0.52	0.53	0.57	0.49	0.55	0.93	0.47	0.48	0.51	0.50	0.54	0.44
Skewness	-0.14	0.09	-0.63	0.11	-0.01	-0.01	0.71	0.07	-0.47	-0.35	-0.32	-0.55	-0.21
Kurtosis	0.86	0.70	0.84	-0.36	-0.23	1.31	-0.45	0.05	0.81	0.63	0.54	1.41	0.02
Min	-1.50	-3.83	-3.18	-1.84	-0.66	-2.66	-3.61	-2.08	-1.15	-0.75	-0.52	-1.80	-1.10
25%	0.23	-2.54	-1.11	-0.63	0.44	-0.53	-2.97	-0.87	0.46	0.87	0.93	0.53	0.03
50%	0.54	-2.22	-0.77	-0.18	0.74	-0.18	-2.57	-0.55	0.72	1.23	1.20	0.89	0.32
75%	0.83	-1.93	-0.42	0.23	1.10	0.10	-1.54	-0.26	1.05	1.54	1.54	1.23	0.59
Max	2.12	-0.54	0.30	1.41	2.02	1.55	0.33	0.83	1.77	2.51	2.40	2.00	1.34
ADF p-value	≤0.01	≤0.01	≤0.01	≤0.01	≤0.01	≤0.01	≤0.01	≤0.01	≤0.01	≤0.01	≤0.01	≤0.01	≤0.01

Imprint and acknowledgements

The authors would like to thank Dr Fiona Sammut and Dr David Suda for their invaluable assistance and guidance in the development of the methodology as well as Mr. Tony Farrugia who was the main reviewer of this paper. Further appreciations go to Mr. Joseph Agius, Dr Mark Bamber, Mr. Ian Buttigieg, Mr. Jorge Guada Prada, and Mr. Ian Meli for their comments and suggestions in this paper. The report is principally based on data submitted to the Malta Financial Services Authority (MFSA) by the managers of the investment funds under analysis. Additional data has been collected from commercial sources. While every effort has been made in order to ensure that the information contained in this report is reliable and accurate at the time of publishing, no express or implied guarantees, representations or warranties are being made regarding the accuracy and/or completeness of the information contained in this report and any other material referred to in this report. The views expressed in this report are those of the authors and do not necessarily reflect the views of the MFSA. The MFSA and the authors of this report do not accept any liability: (i) for any loss or damage whatsoever which may arise in any way out of the use of any of the material contained in this report; (ii) for any errors in, or omissions from, the material contained in this report; or (iii) for any inaccuracy in any information contained in this report. The contents of this report are not to be relied upon as professional, legal and/or investment advice. The MFSA shall have no liability for any loss or damage arising out of negligence or otherwise as a result of the use of, or reliance on, any of the information contained in this report. If you have any doubt about a legal or other provision, or your rights and responsibilities, or other relevant requirements, you should seek appropriate advice from your legal or financial advisors.

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ISSN 2467-0677 (pdf)
ISBN 978-92-899-xxxx-x (pdf)
DOI 10.2866/xxxxxx (pdf)
EU catalogue No DT-AD-21-xxx-EN-N (pdf)