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Growth-at-risk and macroprudential policy design

by
Javier Suarez

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Executive summary

This paper explores a potential application of the empirical growth-at-risk (GaR) approach to the assessment and design of macroprudential policies. In parallel to the concept of value-at-risk, the GaR of an economy over a given horizon is a low quantile of the distribution of the (projected) GDP growth rate over the same horizon. In contrast to the standard macroeconomic focus on the expected value (and, perhaps, the variance) of aggregate output growth, looking at low quantiles of such growth implies, as in risk management, a focus on the severity of potential adverse outcomes.

The recent impulse to use the concepts of GDP-at-risk (Cecchetti, 2008) and GaR (Adrian et al., 2018) is mainly empirical. It is related to the availability of econometric techniques that extend regression analysis (single dependent variable models, panel data models and vector auto-regressive models) to quantiles and their use in macroprudential applications by a growing number of authors. Conceptually, relative to other indicators of financial stability, GaR features the advantage of having an explicit and intuitive statistical interpretation and being measured in the same units as GDP growth, the most universal summary indicator of an economy's overall performance. However, existing empirical efforts still lack a clear fit with an explicit policy design problem of the type considered for other macroeconomic policies. This paper aims to fill this gap.

The analysis relies on a simple linear representation of the empirical GaR approach in combination with a linear-quadratic social welfare criterion that rewards expected GDP growth and penalises the gap between expected GDP growth and GaR. Akin to the mean-variance approach in portfolio theory, it is shown that, if the growth rate follows a normal distribution, this welfare criterion is consistent with expected-utility maximisation under preferences for GDP levels exhibiting constant absolute risk aversion.

The baseline formulation – which abstracts from the time dimension by considering cumulative growth over the relevant policy horizon – focuses on the case in which macroprudential policy design is facing a trade-off: the available policy instrument can linearly increase GaR but at the expense of reducing expected GDP growth. Under the baseline specification, the optimal policy rule is linear in a variable named the "risk indicator" which represents the exogenous drivers of systemic risk. The sensitivity of the optimal policy to changes in this risk indicator is independent of the risk preferences embedded in the welfare criterion. This sensitivity depends directly on the impact of risk on the gap between expected growth and GaR, and inversely on the effectiveness of policy in reducing this gap. Optimal macroprudential policy targets a gap between expected growth and GaR which does not depend on the level of the risk indicator but on the cost-effectiveness of macroprudential policy and the risk preference parameter.

The explored variations in the basic setup cover cases with non-linearities in the impacts of the policy variable and the risk variable on the relevant outcomes, multiple policy variables and discrete policy variables. An important extension shows the compatibility of the GaR framework with the view that macroprudential policy involves various well-identified intermediate objectives, each of which can be associated with one or a subset of targeted policy tools. Additional discussions deal with the case of policies which seem to involve no trade-off between mean growth and GaR, the treatment of country heterogeneity, the interaction with other policies and the possibility of reformulating the analysis around the concept of growth-given-stress rather than GaR.

Under the postulated representation of preferences, the policy design problem yields a quantitative-based policy target and a metric for the assessment of policy stance similar to that of other macroeconomic policies. The main challenges for the applicability of this framework are more empirical and political than conceptual. On the empirical side, the main challenge resides in the consistent and sufficiently precise estimation of the causal effects of risk and policy variables on the relevant moments (mean and GaR) of the growth distribution. Properly detecting relevant non-linearities and interactions between policies is also important. Thus, the framework will develop at the speed with which data on the applied policies accumulate and econometric efforts succeed in providing reliable estimates of their effects on growth outcomes.

On the political side, once data and estimation provide a reliable description of the policy trade-offs, the main challenge will be to define society's aversion for financial instability on which optimal policies should be based. Additionally, given the uncertainty surrounding the relevant parameters implied by the empirical challenges, policymakers may need to be guided on how to expand the type of framework sketched in this paper to account for model uncertainty (that is, for the imperfect knowledge of the specification and parameters of the relevant quantile regressions) and the potential policy mistakes that could stem from this uncertainty.

JEL Classification: G01, G20, G28

Keywords: macroprudential policy, policy stance, growth-at-risk, quantile regressions

1 Introduction

This paper is motivated by the growing attention paid to growth-at-risk (GaR) in the assessment of macroprudential policies. The concept arose as a natural extension to the assessment of systemic risk of value-at-risk – a popular risk management concept. In risk management, the value-at-risk of a given portfolio position is the critical level of the estimated distribution of possible losses over a reference horizon that realised losses will not exceed with a high probability (such as 95% or 99%), known as the confidence level of the assessment. Estimating the value-at-risk allows the portfolio holder to assess, for example, the capital position that would be needed to absorb the potential losses over the reference horizon with this confidence level of probability. From a statistical viewpoint, the value-at-risk of a portfolio is just the estimate of a low quantile (5% or 1% in the above examples) of the distribution of the value of the portfolio by the end of the reference horizon.

In parallel to the concept of value-at-risk, the GaR of an economy over a given horizon is a low quantile of the distribution of the (projected) GDP growth rate over the same horizon. In other words, the growth rate at which the probability of the realised growth rate falling below it equals a low benchmark level such as 10% or 5%.[1] In contrast to the standard macroeconomic focus on the expected value (and, perhaps, the variance) of aggregate output growth, looking at low quantiles of such growth implies, as in risk management, a focus on the severity of potential adverse outcomes. In addition to measuring this severity, the approach can provide information on the variables that determine the probability or severity of bad outcomes, including policy variables that might then be used to influence or “manage” the aggregate risk.

The rising popularity of GaR in financial stability and macroprudential policy assessments is driven by demand and supply factors. From the demand side, macroprudential policy assessment and design is in need of a quantitative framework that provides a baseline for policymakers’ discussions, decision-making and communication with the public similar to those provided by standard macroeconomic models, targets and indicators in the fields of monetary or fiscal policy. For macroprudential policies, the multiplicity of tools, the multidimensional nature (and still vaguely defined concept) of systemic risk, data limitations and the relatively short historical experience with the use of most policy tools pose significant challenges for the development of such a framework. As a result, macroprudential policy is largely assessed and developed following a piecemeal approach (that is, splitting the task by sector, tool, risk or detected vulnerability, or by a combination of approaches) and relying on expert judgement for the qualitative integration of the pieces. While the aim is to cover the whole financial system, the resulting assessment is often less complete, integrated, systematic and quantitative than in other policy fields.

From the supply side, the impulse to use the concepts of GDP-at-risk (Cecchetti, 2008) and GaR (Adrian et al., 2018) is mainly empirical. It is related to the availability of econometric techniques

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1 The use of lower implied confidence levels (90%, 95% in the examples above) in GaR than in value-at-risk is partly related to the fact that GDP is not observed at frequencies that allow for an accurate estimation of extremely low quantiles.

that extend regression analysis (single dependent variable models, panel data models and vector auto-regressive models) to quantiles and their use in macroprudential applications by a growing number of authors. Quantile-regression techniques allow the focus to be shifted from modelling the conditional mean of the dependent variable to modelling the conditional quantiles, and thus the whole conditional distribution of the dependent variables.

Quantile regressions allowed Cecchetti and Li (2008) to use the concept of GDP-at-risk as an empirically-viable summary measure of the impact of asset price booms on financial stability. This approach was further developed and promoted by the influential paper published by Adrian et al. (2019), which shows that the lower quantiles of the distribution of the US GDP growth rate fluctuate more and are more influenced by financial conditions than the upper quantiles, thus supporting the focus of macroprudential surveillance and policies on the lower quantiles. Adrian et al. (2018) documented the "term structure" of GaR and suggested the existence of an intertemporal trade-off whereby some policies might improve GaR at medium and long horizons but at the cost of damaging GaR (or expected growth) at shorter horizons.

Other contributions following a quantile-regression approach to the analysis of growth vulnerabilities and their relationship with financial conditions and macroprudential policies include Caldera-Sánchez and Röhn (2016), De Nicolo and Lucchetta (2017), Prasad et al. (2019), Arbatli-Saxegaard et al. (2020), Chavleishvili et al. (2020), Duprey and Ueberfeldt (2020), Franta and Gambacorta (2020), Figueres and Jarociński (2020), Galán (2021) and Aikman et al. (2021). The empirical approach to GaR has also been embraced in part of the work undertaken by the Expert Group on "Macroprudential Stance – Phase II" of the European Systemic Risk Board (ESRB).

Most of this empirical work puts the emphasis on the capacity of financial variables to forecast low quantiles of GDP growth (and not necessarily high quantiles in a symmetric manner), thus suggesting a connection between financial factors or financial stability indicators and downside risk to output growth.[2] Other contributions focus on the impact of macroprudential policies on GaR. For instance, Duprey and Ueberfeldt (2020), with data from Canada, find that the growth of credit to households contributes to tail risk and that the tightening of macroprudential policy (as captured by a qualitative index of policy actions) reduces tail risk but possibly at the cost of reducing mean GDP growth.[3]

Franta and Gambacorta (2020) also find positive financial stability implications of policy actions relating to the tightening of loan-to-value ratios and provisioning of loan losses in a sample of 52 countries but they find no evidence of a cost in terms of mean growth outcomes. Likewise, Aikman et al. (2021) find that higher bank capitalisation improves GaR over a three-year horizon without

[2] However, Plagborg-Møller et al. (2020) question the short-term forecasting capacity gained by considering variables such as the national financial conditions index (NFCI) in the prediction of GDP growth moments other than the conditional mean, while Brownlees and Souza (2021) challenge the out-of-sample short-term forecasting performance of quantile regressions relative to standard volatility models such as GARCH.

[3] The empirical analysis in Duprey and Ueberfeldt (2020) is complemented by a simple macroeconomic model that provides a microfoundation for the trade-off between mean growth and tail risk faced by macroprudential policy.

significantly reducing mean growth. In contrast, the results in Galán (2021) are consistent with the view that the positive effect of macroprudential policy on tail outcomes over the medium term might come at the expense of a negative effect of tightening actions on mean growth in the short term.[4]

Conceptually, relative to other indicators of financial stability, GaR features the advantage of having an explicit and intuitive statistical interpretation and being measured in the same units as GDP growth, the most universal summary indicator of an economy's overall performance. Hence, quantitative contributions relating to the concept of GaR are followed with great interest (and some scepticism too) by the institutions involved in the assessment and design of macroprudential policies. Many see in the GaR approach a promising step in the development of an integrated quantitative framework for macroprudential policy assessment and design.[5] However, as further discussed in Cecchetti and Suarez (2021), existing empirical efforts still lack a clear fit with an explicit policy design problem of the type considered for other macroeconomic policies (e.g. in the derivation of an optimal monetary policy rule).

This paper aims to fill this gap by digging into the potential application of the empirical GaR approach to the design and assessment of macroprudential policies. Relying on a stylised representation of the type of equations that the quantile-regression approach may deliver, the paper studies how macroprudential policy could be designed and evaluated using a linear-quadratic social welfare criterion that rewards expected GDP growth and penalises the gap between expected GDP growth and GaR. It shows that, in specific environments, this welfare criterion can be microfounded as consistent with expected-utility maximisation under risk-averse preferences for GDP levels. The paper characterises the properties of optimal macroprudential policy rules in the basic setup and a number of relevant extensions. Implications are drawn on the possibility of assessing macroprudential policy stance with a metric emanating from the estimated equations of the empirical GaR approach.

The baseline formulation – which abstracts from the time dimension by considering cumulative growth over the relevant policy horizon – focuses on the case in which macroprudential policy design is facing a trade-off: the available policy instrument can linearly increase GaR but at the expense of reducing expected GDP growth (e.g. because the tightening of some prudential requirement reduces, within the policy horizon, medium-term vulnerabilities but has a contractive

[4] All this evidence must be taken with caution because of the hard-to-treat endogeneity of macroprudential policy actions, the short time span over which authorities have applied active macroprudential policies so far and the measurement difficulties associated with the diversity of macroprudential tools (whose activation or deactivation in many cases can only be captured as changes in binary variables or counting processes).

[5] Some skeptics have doubts about the feasibility and/or desirability of such an integrated approach. They think that the multidimensionality of macroprudential policy cannot be subsumed by looking at a single aggregate indicator such as GaR. Instead, a policymaker in this field might keep track of a welfare criterion that directly combines (intermediate) objectives along the many dimensions of systemic risk and takes into account how (potentially interacting) policies affect all such (intermediate) objectives. In the EU, Recommendation ESRB/2013/1 establishes five intermediate objectives for macroprudential policy.

short-term impact on economic activity).[6] Under the baseline formulation, the optimal policy rule is linear in a variable named the "risk indicator" which represents the exogenous drivers of systemic risk. The sensitivity of the optimal policy to changes in this risk indicator is independent of the risk preferences embedded in the welfare criterion. This sensitivity depends directly on the impact of risk on the gap between expected growth and GaR, and inversely on the effectiveness of policy in reducing this gap. Optimal macroprudential policy targets a gap between expected growth and GaR which does not depend on the level of the risk indicator but on the cost-effectiveness of macroprudential policy and the risk preference parameter.

The explored variations in the basic setup cover cases with non-linearities in the impacts of the policy variable and the risk variable on the relevant outcomes, multiple policy variables and discrete policy variables. An important extension shows the compatibility of the GaR framework (and the main insights from the basic formulation) with the view that macroprudential policy involves various well-identified intermediate objectives, each of which can be associated with one or a subset of targeted policy tools. Additional discussions deal with the case of policies which seem to involve no trade-off between mean growth and GaR, the treatment of country heterogeneity, the interaction with other policies and the possibility of reformulating the analysis around the concept of growth-given-stress rather than GaR.

The paper is structured as follows. Section 2 provides a basic linear formulation of the type found in the empirical GaR approach. Section 3 develops the welfare criterion used for optimal policy design under this formulation, and derives and establishes the properties of the optimal macroprudential policy rule. Section 4 discusses the implications of the results for the assessment of macroprudential policy stance (that is, how the estimates associated with the empirical counterpart of the model equations could help inform about the stance of macroprudential policy). Section 5 develops several extensions of the basic setup, generalising its results for a variety of empirically- and policy-relevant cases, including the situation in which macroprudential policy comprises several intermediate objectives which can be addressed with targeted tools. Section 6 contains further discussion of the proposed analytical framework and results. Section 7 concludes the paper. The Appendix contains the microfoundations of the GaR-based welfare criterion used in the design of the optimal policies and discusses the extent to which, when departing from normality, a focus on the low tail of the GDP growth distribution over a given horizon could have advantages over an alternative focus only on the conditional mean and conditional variance of the growth distribution.

6 The description of the macroprudential policy problem as one in which the policymaker faces a frontier in the mean growth vs. GaR (or tail risk) space can also be explicitly found in existing literature, including Aikman et al. (2018) and Duprey and Ueberfeldt (2020). However, these contributions do not elaborate on the social welfare criterion that is relevant in such a setting or on the properties of the implied optimal policies. Previously, Poloz (2014) referred in purely narrative/graphical terms to a policy frontier between financial stability risk and inflation target risk.

2 A basic formulation of the empirical GaR approach

A quantile-regression approach can deliver equations for arbitrary quantiles of GDP growth over relevant horizons. Let us consider a stylised representation of this approach that consists of two estimated equations: one for the mean (or perhaps the median) of the (cumulative) GDP growth over the policy horizon, denoted by $\bar{y}$, and another for a relevant low quantile of the (cumulative) GDP growth over the same horizon, y_c.[7] The subscript c in y_c identifies the threshold probability (or confidence level) at which GaR is measured. By definition, y_c satisfies:

$$\Pr(y \leq y_c) = c. \tag{1}$$

This means that the probability of experiencing growth rates that are lower than y_c over the relevant horizon is just c. The confidence level c can be thought to be 5% or 10% so that y_c reflects how bad growth may be under adverse circumstances typically associated with systemic distress.

To start with, we consider the simple case in which the quantile-regression approach delivers conditional forecast equations for GaR y_c and expected growth $\bar{y}$ of the form:

$$y_c = \alpha_c + \beta_c x + \gamma_c z, \tag{2}$$

and

$$\bar{y} = \alpha + \beta x + \gamma z, \tag{3}$$

where x is a unidimensional risk indicator or exogenous driver of systemic risk (e.g. a driver of excessive credit growth or any other factor that potentially contributes to the accumulation of financial imbalances) and z is a unidimensional macroprudential policy variable (e.g. a bank capital-based measure such as the countercyclical capital buffer (CCyB) in Basel III).[8] Let us further assume that the endogeneity of z has been treated well enough to allow for γ_c and γ to be interpreted as the causal impact of variations in z on GaR and expected growth, respectively.

We also assume that:

$$\beta_c < \min\{0, \beta\} \text{ and } \gamma < 0 < \gamma_c. \tag{4}$$

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7 As mentioned in the introduction, this formulation abstracts from the exact shape of the path followed by GDP growth within the policy horizon by focusing on the cumulative growth over the whole horizon. Practical applications may consider quarterly variations within a multi-quarter horizon, making it possible to capture the policy trade-off referred to below as one in which policy can improve GaR in a distant quarter only at the cost of reducing mean growth in a closer quarter.

8 An advanced reader might easily extend some of the derivations and claims contained in this note to the cases in which x and z are vectors of risk drivers and policy variables, respectively. See Section 5 for extensions of the basic formulation that deal with multiple policy variables. Section 6.3 considers the interaction with policies other than macroprudential policies.

In other words, risk driver x has a negative impact on GaR and a less negative (or even positive) impact on expected growth, while policy variable z has a positive impact on GaR but a negative impact on expected GDP growth.[9] The last properties imply that the policy measured by z involves a trade-off.[10] For example, if x measures a driver of excessive credit growth and z is the CCyB rate, a trade-off can arise because increasing the CCyB rate reduces the final systemic risk implied by, for instance, a credit boom (e.g. the probability and implications of an abrupt reversal) but at the same time has a contractive impact on aggregate demand and, hence, on the central outlook.[11]

Finally, we assume that the variation ranges of y and z, together with the values of intercepts α and α_c, guarantee $y_c < \bar{y}$ over the relevant range (otherwise the linearity in (2) and (3) might lead to $\bar{y} < y_c$ which would not make sense for low values of c).

[9] The linear specification implies that z monotonically affects y and y_c. What really matters for the validity of the analysis below is that this is locally true over the relevant range of variation in z. Otherwise the specification could be modified by redefining z as a suitable non-monotonic transformation of the policy variable.

[10] Empirical findings in Adrian et al. (2018), Duprey and Ueberfeldt (2020) and Galán (2021) are consistent with the existence of this trade-off but findings of other authors are not (e.g. because they imply $\gamma = 0$). Section 6.1 discusses the case in which policy involves, or seems to involve, no trade-off.

[11] The risk indicator x should be thought of as an exogenous driver of risk and not the final systemic risk faced by the economy. Systemic risk would be the result of the interaction of the risk driver x and policy z put in place to mitigate or counter its impact on tail outcomes. Thus, in the linear formulation above, systemic risk would be proportional to $\beta_c x + \gamma_c z$ rather than directly and solely x. In a recursive context, x could also be interpreted as the predetermined value (at the time of deciding on policy) of a risk indicator whose evolution over the policy horizon is affected by z.

3 Social preferences and the optimal policy rule

To further illustrate the policy trade-offs that derive from the empirical GaR formulation, let us suppose that the policymaker has preferences that can be represented by the social welfare function:

$$W = \bar{y} - \frac{1}{2} w(\bar{y} - y_c)^2, \tag{5}$$

where $w > 0$ measures the aversion for financial instability, which is here proxied by the magnitude of the quadratic deviations of GaR with respect to expected growth.

As shown in Section A.1 of the Appendix, in the particular case in which GDP growth follows a normal distribution, the welfare criterion in (5) can be justified as consistent with the maximisation of the expected utility of a representative risk-averse agent whose utility depends on GDP levels. Specifically, if the agent has preferences for GDP levels that exhibit a constant absolute risk aversion (CARA) coefficient λ, then (5) provides an exact representation of such preferences under a value of w which is directly proportional to λ.

Of course, if y is normally distributed, social preferences and the policy problem could have also been formulated in the usual mean-variance terms of portfolio theory, with an equation describing the dependence of the standard deviation of the growth rate σ_y on x and z replacing (2) (see Section A.2 of the Appendix for details). What this means is that the true advantages of adopting a GaR approach (instead of a mean-variance approach) in the formulation of the macroprudential policy problem must derive from that fact that in reality: (i) the conditional distribution of y is not Gaussian, and (ii) as documented in recent empirical work, the financial factors and policy tools on which macroprudential policy focuses affect the conditional low quantiles of the true growth distribution in a stronger and more clearly identifiable manner than its conditional variance. From this perspective, an advantage of the quantile-regression approach to the modelling of the quantile y_c is that it does not require the assumption of a specific distribution for the conditional quantile. In other words, nothing prevents the estimated version of equations (2) and (3) from capturing features such as the potential left skewness of the true conditional distribution of the GDP growth rate.[12]

Beyond the exact expected-utility microfoundations of the specific normal case, the welfare criterion in (5) could also be defended in heuristic or axiomatic terms as the representation of the

[12] A draft policy report by Cecchetti and Suarez (2021) explores the accuracy with which the welfare measure in (5) approximates the underlying expected utility of a representative agent in a number of empirically motivated examples in which (i) the GDP growth rate is not normally distributed, and (ii) preferences on GDP levels do not exhibit CARA but rather constant relative risk aversion (CRRA), as typically assumed in other applications in economics and finance. For realistic levels of variability of cumulative GDP growth over three-year periods, the accuracy of the metric provided by (5) is very good.

preferences of a policymaker that is facing a trade-off between improving mean outcomes and reducing the severity very bad outcomes. An interesting feature of (5) from such a perspective is that the dislike for "very bad outcomes" is proportional to the square of the distance between the bad outcomes y_c and the mean outcomes $\bar{y}$, where the latter would play the role of a reference level (or status quo point) similar to those emphasised in some non-expected-utility formulations of agents' preferences for risk. Specifically, from the perspective of prospect theory (Kahneman and Tversky, 1979), the coefficient w in (5) could be interpreted as capturing loss aversion rather than risk aversion.[13]

3.1 The optimal policy rule

An optimal macroprudential policy that is conditional on a risk level x would thus maximise W given x. That is, it would be characterised by the policy rule:

$$z(x) = \underset{z}{\operatorname{argmax}} W(x, z), \tag{6}$$

where $W(x, z)$ describes W as a function of the risk indicator x and the policy variable z after taking into account (2) and (3).

If the optimal policy is interior, it must solve the following first order condition (FOC):

$$\frac{\partial \bar{y}}{\partial z} - w(\bar{y} - y_c)\left(\frac{\partial \bar{y}}{\partial z} - \frac{\partial y_c}{\partial z}\right) = 0, \tag{7}$$

which uses the chain rule in (5). From (2) and (3), this FOC can be written as:

$$\gamma - w(\alpha + \beta x + \gamma z - \alpha_c - \beta_c x - \gamma_c z)(\gamma - \gamma_c) = 0. \tag{8}$$

Solving for z leads to the macroprudential policy rule:

$$z(x) = \phi_0 + \phi_1 x, \tag{9}$$

with

$$\phi_0 = \frac{\alpha - \alpha_c}{\gamma_c - \gamma} + \frac{\gamma}{w(\gamma_c - \gamma)^2} \tag{10}$$

and

$$\phi_1 = \frac{\beta - \beta_c}{\gamma_c - \gamma}. \tag{11}$$

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13 The asymmetric focus on low tail losses can also be related to Fishburn (1977), who explores preferences in which the decision maker is averse to obtaining below-target payoffs. Kilian and Manganelli (2008) analyse the decision problem of a central banker using that approach. In a similar vein, Svensson (2003) considers a monetary policy problem under preferences that asymmetrically penalise extreme events.

Under our assumptions, the intercept of the policy rule ϕ_0 can, in principle, have any sign since it is the sum of a first term which will most typically be positive (specifically if $\alpha - \alpha_c > 0$) and a second term which is negative (since $\gamma < 0$). However, ϕ_0 is intuitively increasing in the policymaker's preference for financial stability w (since the absolute size of the negative term declines with w) and also increasing in the difference $\gamma_c - \gamma > 0$, which measures the effectiveness of the policy variable in reducing the gap between expected growth and GaR, $\bar{y} - y_c$.[14]

Interestingly, the parameter ϕ_1 which measures the responsiveness of the optimal policy to variations in risk indicator x is positive and independent of the preference parameter w. So in this setup, policymakers with different preferences for financial stability would differ in the level at which they use macroprudential policy but not in the extent to which they modify their policies in response to changes in the risk assessment. This optimal policy responsiveness is directly proportional to the impact of risk x on the gap between expected growth and GaR ($\beta - \beta_c$, which is positive under (4)) and inversely proportional to the effectiveness of policy z in reducing the gap ($\gamma_c - \gamma$, which is also positive under (4)).

While the empirical GaR approach (namely, estimating equations (2) and (3)) does not, per se, allow the policy parameter w to be estimated, it might allow a direct estimate to be made of the optimal policy responsiveness parameter ϕ_1 and its components $\beta - \beta_c$ and $\gamma_c - \gamma$. It might also allow the optimal policy rule to be represented for different illustrative values of the preference parameter w.[15]

[14] For instance, in the polar case in which the policymaker has absolute preference for financial stability ($w \to \infty$), the intercept would become just $(\alpha - \alpha_c)/(\gamma_c - \gamma)$ and lead to a solution with $\bar{y} = y_c$ (which, although unrealistic in practice, is mathematically feasible given the linearity of (2) and (3) in x and z). In the other polar scenario with no preference for financial stability ($w \to 0$), we would have $\phi_0 \to -\infty$, implying that the policymaker would choose the lowest possible value of z, since under the linear specification of (3) this is the way to maximise expected growth (albeit at the cost of minimising GaR).

[15] The use of the conditional "might" is a reminder of the importance of relying on estimates of parameters γ and γ_c that reflect the causal impact of policy on growth outcomes and not just partial correlations between historical realisations of policy and outcomes.

3.2 Graphical illustration

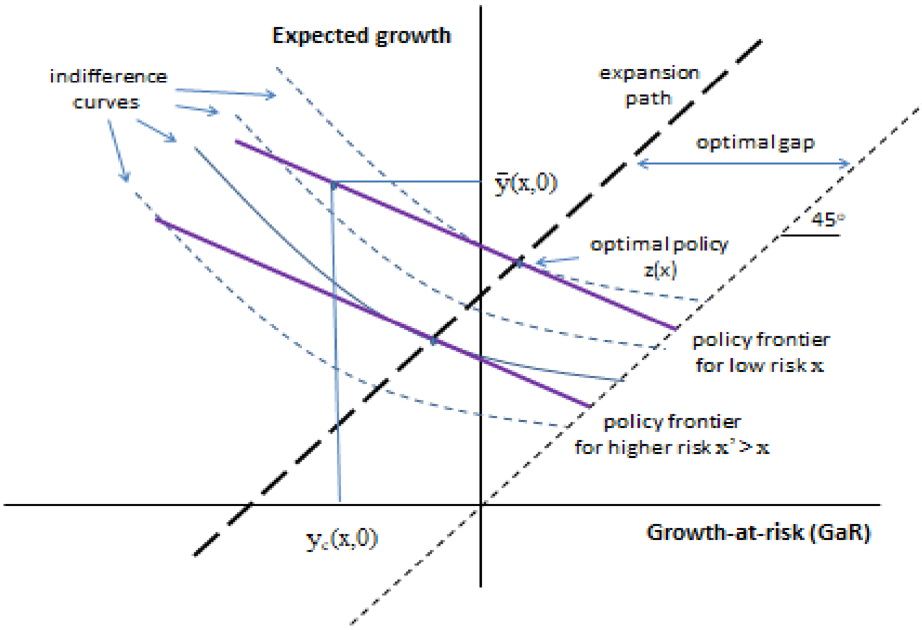
Further understanding of the interaction between the policy trade-offs implied by (2) and (3) and the preferences reflected in (5) can be obtained by depicting the frontier of pairs of y_c and $\bar{y}$ that can be reached, for a given value of risk variable x, by varying the policy variable z. Mathematically, this conditional policy frontier (for a given x) is defined by the line:

$$\bar{y} = \left(\alpha - \frac{\gamma}{\gamma_c}\alpha_c\right) + \left(\beta - \frac{\gamma}{\gamma_c}\beta_c\right)x + \frac{\gamma}{\gamma_c}y_c, \tag{12}$$

which is downward sloping in $(y_c, \bar{y})$ space. Figure 1 depicts the policy frontier for a given value of x. The point $(y_c(x,0), \bar{y}(x,0))$ corresponds to the case in which $z = 0$. Intuitively choosing $z > 0$ allows higher values of y_c to be obtained, but at the cost of lowering $\bar{y}$.[16]

Figure 1

Graphical illustration of the policy design problem



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[16] Under the assumed linearity, there is nothing special about $z = 0$ but in specific applications the policy variable could be normalised so that it means something, e.g. the historical mean or "normal stance" of the corresponding policy (then $z < 0$ would represent a stance that is looser than normal and $z > 0$ a stance that is tighter than normal). For some policy instruments there may be a natural lower bound to z, e.g. a CCyB rate under Basel III cannot be negative (although in practice there are instances of capital forbearance that might be similar to having $z < 0$ for this instrument). Explicit consideration of these bounds would raise complications regarding occasionally binding constraints that are familiar in other contexts.

The preferences in (5) describe a map of indifference curves in $(y_c, \bar{y})$ space that are convex parabolas which reach their minima on the ray $y_c = \bar{y}$. The map makes economic sense to the left of this ray. Intuitively, for $w > 0$, on each indifference curve any decline in y_c should be compensated with an increase in $\bar{y}$ in order to keep the welfare level unchanged. Moreover, for a given decline in y_c, the required compensating increase in $\bar{y}$ increases with the distance from the ray $\bar{y} = y_c$. This explains why the FOC (7) includes the term $w(\bar{y} - y_c)$, which accounts for the marginal cost of financial instability.

Optimal policy $z(x)$ is the choice of z that leads to maximum welfare on the corresponding conditional policy frontier. In other words, $z(x)$ is the policy level that leads to the point where the conditional policy frontier is tangent to the map of indifference curves. From the determinants of the slopes of these curves it follows that, all other things being equal, a policymaker with a stronger preference for financial stability will choose combinations of $(y_c, \bar{y})$ on the frontier that involve a lower $\bar{y}$ and a higher y_c, that is, a lower gap between expected growth and GaR.

3.3 Optimal target gap property

What happens when the risk indicator x moves? From (12), changes in the risk indicator x cause a parallel shift in the policy frontier (just another implication of the linear formulation). In the most plausible situation, in which risk does not increase expected growth too much (formally, when $\beta < \gamma\beta_c/\gamma_c$, where $\gamma\beta_c/\gamma_c$ is positive under (4)), a rise in x shifts the policy frontier down, necessarily worsening the terms of choice for the policymaker. In the alternative scenario, where $\beta > \gamma\beta_c/\gamma_c$, risk has such a strong effect on expected growth that it moves the policy frontier up.

In both cases, however, the optimal policy rule (9) implies that an increase in risk will lead to a tightening of policy decision z, indicating that the fall in y_c that would occur if policy were not adjusted is at least partly offset by increasing z. When risk has a positive marginal impact on expected growth ($\beta > 0$), the optimal policy response will diminish the raw positive effect of risk indicator x on mean growth $\bar{y}$. When risk has a negative marginal impact on expected growth ($\beta < 0$), the optimal policy response will still aim to offset its even more negative effect on y_c by lowering mean growth $\bar{y}$ beyond the implications of the raw negative effect of risk indicator x.

Mathematically, the final impact of changes in risk indicator x on y_c and $\bar{y}$ can be seen by substituting the optimal policy rule $z(x)$ in (2) and (3), which leads to:

$$y_c = (\alpha_c + \beta_c\phi_0) + (\beta_c + \gamma_c\phi_1)x = (\alpha_c + \beta_c\phi_0) + \frac{\gamma_c\beta - \gamma\beta_c}{\gamma_c - \gamma}x, \qquad (13)$$

and

$$\bar{y} = (\alpha + \beta\phi_0) + (\beta + \gamma\phi_1)x = (\alpha + \beta\phi_0) + \frac{\gamma_c\beta - \gamma\beta_c}{\gamma_c - \gamma}x. \qquad (14)$$

Interestingly, the coefficient of risk indicator x is identical in these two equations, which implies that the optimal policy rule would keep the gap between expected growth and GaR constant:

$$\bar{y} - y_c = (\alpha - \alpha_c) + (\beta - \beta_c)\phi_0 = \frac{1}{w}\frac{(-\gamma)}{\gamma_c - \gamma} = \frac{1}{w}\frac{1}{1 + \gamma_c/(-\gamma)} \qquad (15)$$

Notice that this target gap is positive under (4) since $\gamma < 0$. The target gap decreases in the preference for financial stability w and increases in the marginal growth-gap rate of transformation implied by the policy frontier, $(-\gamma)/(\gamma_c - \gamma)$, which can be rewritten as $1/[1 + \gamma_c/(-\gamma)]$ to better visualise its negative dependence with respect to the marginal cost-effectiveness of the policy variable: the ratio of the c-quantile-improving effect $\gamma_c > 0$ to the mean-reducing effect $-\gamma < \gamma_c$.

In fact, this "constant target gap" property can be directly obtained from the FOC in (7), which can be rearranged as:

$$\bar{y} - y_c = -\frac{1}{w}\frac{\frac{\partial \bar{y}}{\partial z}}{\frac{\partial \bar{y}}{\partial z} - \frac{\partial y_c}{\partial z}} = \frac{1}{w}\frac{1}{1+\gamma_c/(-\gamma)}. \tag{16}$$

Graphically, this implies that changes in risk indicator x and the optimal policy response under $z(x)$ describe a linear expansion path in $(y_c, \bar{y})$ with a slope equal to one. Thus, starting from the optimal policy identified in Figure 1 for a particular risk level x, changes in x will lead to combinations $(y_c, \bar{y})$ on the line with slope one that goes through that point.

More specifically, when risk does not increase expected growth too much (that is, in the case of $\beta < \gamma\beta_c/\gamma_c$ described above), the coefficient of x in the reduced-form equations (13) and (14) is negative. Thus, when the risk indicator increases and policy responds optimally, GaR and expected growth deteriorate by the same amount, keeping the gap between expected growth and GaR, $\bar{y} - y_c$ constant. This is the case depicted in Figure 1, where the policy frontier under $x' > x$ lies on the left of the policy frontier for x.

Otherwise (when $\beta > \gamma\beta_c/\gamma_c$), the coefficient of x in (13) and (14) is positive, and rises in x and the optimal policy response lead GaR and expected growth to improve by the same amount, but once again keeping $\bar{y} - y_c$ constant.

An important corollary to these findings is that, under the specified preferences, macroprudential policy should not target a constant GaR or keep GaR above a certain lower bound, but should allow the GaR target to comove (actually by the same amount) with the expected growth estimate. In other words, these derivations suggest that the $\bar{y} - y_c$ gap is a more useful indicator of stance than each of its components separately.

4 A framework for policy assessment?

A tentative list of policy-relevant outputs that this approach can deliver is set out below.

1. Estimating (2) and (3) allows us to positively describe the direct impact of risk x and policy z on GaR and expected growth, as well as the policy trade-offs involved.

2. The policy trade-offs can be further illustrated using a policy frontier as shown in Figure 1.

   (a) If it is evaluated at the historical mean value of x, this frontier could be called the mean policy frontier. If a practical application involves a discrete x, then a reference value could be selected to represent a "normal" situation.

   (b) Under the linear specification, the conditional policy frontier is just a parallel shift of the mean (or "normal") policy frontier. The relative position of the conditional frontier relative to the historical mean (or "normal") frontier may indicate whether the economy is facing a state of above-normal or below-normal risk exposure.

3. If social preferences (or the preferences of the policymaker) can be described with a mean growth versus GaR welfare criterion as in (5), then the following points should be taken into consideration.

   (a) The optimal policy responsiveness to the risk indicator can be measured using $\phi_1 = \frac{\beta-\beta_c}{\gamma_c-\gamma}$, as in (11). This measure is independent of parameter w that describes the policymaker's preference for financial stability.

   (b) If the policymaker follows the optimal policy rule, it will implicitly target a constant gap between expected growth and GaR, as in (16). The optimal policy gap will be decreasing in the preference parameter w and in the cost-effectiveness ratio of the policy tool $\gamma_c/(-\gamma)$. From this gap, and the estimates of the empirical GaR model, the implicit preference parameter could be recovered (inferred) from the condition:

   $$w = \frac{1}{\bar{y}-y_c}\frac{1}{1+\gamma_c/(-\gamma)}. \quad (17)$$

   (c) Conditional on a reference value of w, the optimal policy rule can be fully described using (6). Graphically, it can be described using the expansion path previously illustrated in Figure 1.

   (d) Conditional on a reference value of w and an assessment of risk x, a graphical counterpart of the optimal policy choice can be described by depicting the conditional policy frontier and the point on it that is associated with the optimal policy (given by the intersection between the policy frontier and the expansion path).

   (e) Conditional on an assessment of risk x, a policy stance could be deemed inefficient if it leads to points sufficiently far away from the policy frontier. However, when the policy variable x is unidimensional (as in all the derivations above), all choices of x are "efficient", so the concept of inefficiency is only useful when there are two or more policy variables (as in some of the extensions discussed below).

(f) Conditional on the reference value of w and an assessment of risk $x,$ a policy stance could be deemed suboptimal if it is sufficiently far away from the expansion path. This corresponds to an excessive distance between z and $z(x)$ or, in terms of outcomes, a $\bar{y} - y_c$ gap that is sufficiently far away from its target. Thus, policy would be too tight if z is sufficiently higher than $z(x)$ and equivalently if the $\bar{y} - y_c$ gap is well below the target. Conversely, policy would be too loose if z is sufficiently lower than $z(x)$ and equivalently if the $\bar{y} - y_c$ gap is well above target.

5 Extensions

This section considers several specific extensions of the basic formulation. It demonstrates the capacity of the framework to accommodate multiple variations and checks the robustness of the properties of optimal macroprudential policies in each of them.

5.1 Policy variable with non-linear effects

A particularly relevant non-linearity may be related to the diminishing effectiveness of the policy variable (or variables, if there are several) in improving GaR. Another interesting case may emerge if the impact of the policy variable on expected growth is marginally increasing. Let us therefore consider a generalised version of (2) and (3) in which

$$y_c = \alpha_c + \beta_c x + \Gamma_c(z), \tag{18}$$

and

$$\bar{y} = \alpha + \beta x + \Gamma(z), \tag{19}$$

where the functions $\Gamma_c(z)$ and $\Gamma(z)$ satisfy $\Gamma' < 0 < \Gamma_c'$ and $\Gamma_c'' < \Gamma'' < 0$. In this case, the FOC solved by an interior optimal policy can be written as:

$$\Gamma'(z) - w[\bar{y}(x,z) - y_c(x,z)][\Gamma'(z) - \Gamma_c'(z)] = 0, \tag{20}$$

where the dependence of $\bar{y}$ and y_c on x and z has been made explicit to emphasise the type of non-linear equation that would have to be solved to find the optimal policy rule $z(x)$.

By rearranging (20), we obtain an expression for the gap associated with the optimal policy that is very similar to (16):

$$\bar{y}(x,z) - y_c(x,z) = \frac{1}{w}\frac{1}{1+\Gamma_c'(z)/(-\Gamma'(z))}. \tag{21}$$

However, in this case the target gap is not invariant to risk indicator x. If x increases, all other things being equal, the left hand side of (21) increases, calling for an offsetting increase in z. But the right hand side of (21) is now increasing in z because, intuitively, the policy trade-off measured by the marginal cost-effectiveness of the policy (here $\Gamma_c'(z)/(-\Gamma'(z))$) worsens at higher levels of z. This implies that optimal policy $z(x)$ in this case grows less than linearly with x and the optimal gap increases with x. In other words, as risk deteriorates, the policymaker would accommodate the diminishing cost-effectiveness of the policy tool by widening the targeted gap between expected growth and GaR.

5.2 Risk variable with non-linear effects

Let us now consider a situation in which risk variable x has a non-linear impact on expected growth and GaR, captured by the following modification of (2) and (3):

$$y_c = \alpha_c + B_c(x) + \gamma_c z, \quad (22)$$

and

$$\bar{y} = \alpha + B(x) + \gamma z, \quad (23)$$

where $B_c(x)$ and $B(x)$ are functions satisfying $B'(x) - B_c'(x) > 0$ and $B''(x) - B_c''(x) > 0$. In other words, increases in x widen the gap between expected GDP and GaR at an increasing rate (e.g. by making the financial system more and more likely to reach a tipping point of full meltdown). In this case, the FOC of the welfare maximisation problem becomes:

$$\gamma - w[\bar{y}(x,z) - y_c(x,z)](\gamma - \gamma_c) = 0, \quad (24)$$

which implicitly defines the policy rule $z(x)$. In this case, the FOC, and consequently the policy rule, are only non-linear because of the non-linear effect of x on $\bar{y}(x,z) - y_c(x,z)$. Rearranging this to solve for the optimal gap, we obtain:

$$\bar{y}(x,z) - y_c(x,z) = \frac{1}{w}\frac{1}{1+\gamma_c/(-\gamma)}, \quad (25)$$

where the right hand side is invariant to x, thus implying the same gap as when x had a linear impact on y_c and $\bar{y}$. Interestingly, in this case, when the risk indicator increases, the left hand side increases more than proportionally to x, also calling for a more than proportional increase in the policy variable. In this setup, the policy response to rises in risk should be increasingly aggressive as risk increases.[17]

5.3 A vector of policy variables

Let us consider an extended version of (2) and (3) with M different continuous policy variables z_j with $j = 1,2, \ldots M$ linearly affecting y_c and $\bar{y}$ with coefficients γ_{cj} and γ_j, respectively. We assume that these coefficients satisfy $\gamma_j < 0 < \gamma_{cj}$ as in (4) and further assume that the variables are scaled so that $z_j = 0$ is the lowest bound applicable to all of them. In this linear world, as seen by exploring the relevant first order conditions, there will generally be one variable dominating the others in the maximisation of W. This most efficient or preferred policy tool j^* would be the one featuring the lowest value of what was referred to as the marginal growth-gap rate of transformation in the single policy variable benchmark:

[17] The opposite situation, in which optimal policy is increasingly less aggressive as x increases. emerges if $B''(x) - B_c''(x) < 0$.

$$\frac{\frac{\partial \bar{y}}{\partial z_j}}{\frac{\partial \bar{y}}{\partial z_j}-\frac{\partial y_c}{\partial z_j}} = \frac{1}{1+\gamma_{cj}/(-\gamma_j)} > 0, \qquad (26)$$

that is, the policy tool with the best marginal cost-effectiveness as measured by $\gamma_{cj}/(-\gamma_j)$. Intuitively, when $\gamma_{cj}/(-\gamma_j)$ is higher, the same reduction in the gap between expected growth and GaR can be achieved at a lower cost in terms of expected growth. For the most efficient tool, the optimal value of z_{j^*} would be the value that satisfies the counterpart of equation (7). The associated policy rule would be the same as in (9) with ϕ_0 and ϕ_1 particularised to the preferred tool j^*. All elements in Figure 1 remain valid if the policy frontiers are also particularised to those obtained using the preferred tool.

All the other policy variables should remain at their lowest bound of zero. In terms of Figure 1, using an inferior tool would imply moving over policy "frontiers" that also go through the point $(y_c(x,0), \bar{y}(x,0))$ but with steeper slopes, confirming that such a tool would only be able to increase y_c by causing larger declines in $\bar{y}$. Conditional on using a less cost-effective tool, equation (16) would imply that the target gap should be larger, thus accommodating the harder trade-off faced along the corresponding policy frontier.

5.4 Optimal policy mixes

The optimality of using non-trivial combinations of tools in macroprudential policy would only emerge in the event of departures from linearity. For example, optimal policies could be obtained that involve using several tools at the same time if the effectiveness of each policy tool in reducing GaR is marginally decreasing, for instance, as given by some functions $\Gamma_{cj}(z_j)$ with $\Gamma'_{cj} > 0$ and $\Gamma''_{cj} < 0$, or if there are complementarities between tools under a general quasi-concave function $\Gamma_c(z_1, z_2, \ldots z_M)$ that replaces the terms $\Sigma_{j=1}^{M}\gamma_{cj}z_j$ in the extended version of (2).

In such a non-linear world, all policy variables activated at a strictly positive level at the optimum would satisfy a properly modified version of (7) and, consequently, (16), implying:

$$\bar{y} - y_c = -\frac{1}{w}\frac{\frac{\partial \bar{y}}{\partial z_j}}{\frac{\partial \bar{y}}{\partial z_j}-\frac{\partial y_c}{\partial z_j}} = \frac{1}{w}\frac{1}{1+\frac{\partial \Gamma_c}{\partial z_j}/(-\gamma_j)}. \qquad (27)$$

Thus, optimal policy mixes would feature equalisation of the marginal cost-effectiveness ratios, $\frac{\partial \Gamma_c}{\partial z_j}/(-\gamma_j)$, across all the activated policy tools. The optimal gap between expected GDP growth and GaR would be decreasing in both the common cost-effectiveness ratio and the aversion to financial instability.

In terms of interactions between tools, the optimal gap may no longer be constant since the compound effectiveness of a given policy mix may depend on the intensity with which policies are activated. For example, if a rise in the risk variable x calls for a more intensive use of two complementary policies that jointly exhibit decreasing returns to intensity (akin to when complementary inputs are combined in a production function with decreasing returns to scale), then the optimal policy will accommodate (as in the case of the single policy variable with decreasing marginal effectiveness discussed above) the decreasing effectiveness by tolerating a larger gap when the risk is high than when the risk is low.

5.5 Intermediate objectives and targeted policy tools

Current practice of macroprudential policy largely involves a piecemeal approach. Authorities around the globe, as well as research in the field of macroprudential policy, often address the design and assessment of macroprudential policy by splitting it into separate silos. As in microprudential regulations, these silos are commonly determined by the nature of the underlying source of systemic risk (e.g. liquidity vs. solvency risk) or by the sector that originates, transmits or suffers the risk (e.g. banks vs. non-banks, commercial vs. residential real estate, etc.). The resulting silos are typically associated with one or several dedicated policy tools (e.g. "capital-based tools for the banking sector", "liquidity-based tools for the investment management sector" or "borrower-based tools for residential real estate risk"). The practical attractiveness of the piecemeal approach stems from the difficulties of integrating under a common general equilibrium perspective and with a common ultimate goal the multiple dimensions of systemic risk, the multiple potential factors contributing to financial stability (or the lack of it) and the multiple policy tools available to address these dimensions and factors. The purpose of this subsection is to show that such an approach is not incompatible with the analytical framework and empirical efforts associated with the GaR approach. In fact, the latter can contribute to integrating, adding up or at least bringing under a common umbrella sectoral macroprudential policies that might otherwise be difficult to relate to each other when trying to obtain an overall notion of macroprudential policy stance.

The illustration shown below of the capability of the GaR approach to integrate prior piecemeal approaches into macroprudential policy design relies on simplifying assumptions that are directed at making the problem analytically tractable. In the spirit of keeping things simple and close to some of the extensions described above, let us assume that macroprudential policy involves M different dimensions, $j = 1,2,..M$ and that each dimension can now be associated with an intermediate objective I_j and a targeted policy tool z_j.[18]

Let us further assume that intermediate objectives can be represented as linear functions of their targeted tools:

$$I_j = \lambda_{0j} + \lambda_{1j} z_j, \tag{28}$$

where λ_{0j} is an autonomous component of the intermediate objective and $\lambda_{1j} > 0$ measures the marginal impact of the targeted policy variable on the intermediate objective.[19] The baseline equations (2) and (3) could then be reformulated as follows:

$$y_c = \alpha_c + \Gamma_c(I_1, I_2, \dots I_M), \tag{29}$$

and

[18] Advanced readers may further expand the proposed setup to accommodate additional generalisations of the problem.

[19] Without loss of generality, we impose a sign convention for I_j and z_j such that increasing I_j is good for financial stability (i.e. it increases y_c) and increasing the policy variable z_j is good for the intermediate objective j (i.e. it increases I_j).

$$\bar{y} = \alpha + \Sigma_{j=1}^{M} \gamma_j z_j, \tag{30}$$

where Γ_c is an increasing and strictly concave function of the vector of intermediate objectives and $\gamma_j < 0$ so that, as in the baseline setup, macroprudential policies involve a trade-off: increasing policy z_j improves the intermediate objective I_j but at the cost of reducing mean growth at the margin.[20]

As in the non-linear world described in the above extension on optimal policy mixes, all targeted policy variables that are activated at a strictly positive level at the optimum would satisfy a modified version of (7) and, consequently, (16), implying:

$$\bar{y} - y_c = -\frac{1}{w}\frac{\frac{\partial \bar{y}}{\partial z_j}}{\frac{\partial \bar{y}}{\partial z_j} - \frac{\partial y_c}{\partial z_j}} = \frac{1}{w}\frac{1}{1+\frac{\partial \Gamma_c}{\partial I_j}\lambda_j/(-\gamma_j)}. \tag{31}$$

Thus, the optimal vector of targeted policies would again feature equalisation of the marginal growth-gap rates of transformation and hence equalisation of the marginal cost-effectiveness ratios across all activated policy tools. Moreover, the optimal gap between expected GDP growth and GaR would be decreasing in both the aversion to financial instability and the common ratio. The cost-effectiveness ratio of targeted policy j is the ratio between the marginal effectiveness of the policy, that is, its marginal capability to improve GaR by affecting intermediate objective j ($\frac{\partial \Gamma_c}{\partial I_j}\lambda_j$) and the marginal cost of the policy in terms of mean growth ($-\gamma_j$).

Depending on the degree to which intermediate objectives may feature complementarity in their compound impact on GaR, as captured by the cross derivatives of function Γ_c, the setup with multiple intermediate objectives might imply increasing or decreasing the target gap as well as varying the optimal policy mix in response to changes, for instance, in the autonomous component of one of the intermediate objectives. For example, in the simple case in which Γ_c is additively separable across intermediate objectives (so that they do not directly interact in affecting GaR), the policy response to a deterioration in the autonomous component of one objective j would be the tightening of policy across all intermediate objectives (so that $\frac{\partial \Gamma_c}{\partial I_{j'}}$ declines across all policy dimensions j' and the equality in (31) is restored at a higher target gap).

5.6 A discrete policy variable

Intuitively, if the policy variable is discrete and yet enters the problem as assumed in (2) and (3), the left hand side of the FOC in (7) must be replaced by its finite differences counterpart and its sign checked to discover whether there are gains from increasing (or keeping on increasing) the variable or, conversely, whether there could be gains from reducing it.

[20] Notice that, for purposes of simplicity and in contrast to the baseline specification, (29) and (30) do not explicitly contain any risk variable x. However, it would be trivial to introduce one or a vector of them affecting y_c and $\bar{y}$ linearly as in the baseline model. We could also consider risk variables that affect the autonomous component λ_{0j} of each intermediate objective.

More formally, if we first consider the general case in which the policy variable can take N different values: $z \in \{z_1, z_2, \dots z_N\}$ with $N \geq 2$. Let

$$\Delta W(x, z_i) = W(x, z_{i+1}) - W(x, z_i) \tag{32}$$

represent the welfare gain from increasing the discrete policy variable by one notch when starting from z_i. Using the definition of W in (5) and the expressions for (2) and (3), we obtain the following expression:

$$\Delta W(x, z_i) = \gamma(z_{i+1} - z_i) - \frac{w}{2}(\gamma_c - \gamma)^2(z_{i+1}^2 - z_i^2) + w(\gamma_c - \gamma)A(x)(z_{i+1} - z_i), \tag{33}$$

with $A(x) = (\alpha - \alpha_c) + (\beta - \beta_c)x > 0$. Under the assumptions in (4), the first two terms in this expression are negative, reflecting the direct expected GDP cost of tightening macroprudential policy and the impact of this cost in reducing the gap between expected growth and GaR, which diminishes the marginal gains from further tightening. The third term is positive and increasing in risk variable x and captures the gap-reducing gains from tightening the policy. In a typical case, $\Delta W(x, z_i)$ will be positive at low values of i and turn negative at higher values of i, identifying the optimal policy as the highest i for which $\Delta W(x, z_i)$ is positive. Intuitively, as $A(x)$ is increasing, the optimal level of activation of the discrete policy will generally be higher for higher values of the risk variable x.

A particular case of interest in some applications is that in which the possible values of the policy variable are equally spaced (e.g. when using a cumulative index of macroprudential policy actions). If the scale of the variable is normalised to make the space between any two consecutive values one and sets $z_1 = 0$, then $z_i = i - 1$ and we can use $z_{i+1}^2 - z_i^2 = 2i - 1$ to write:

$$\Delta W(x, z_i) = \gamma - \frac{w}{2}(\gamma_c - \gamma)^2(2i - 1) + w(\gamma_c - \gamma)A(x), \tag{34}$$

whose negative second term depends linearly on i, reflecting, all other things being equal, diminishing marginal welfare gains from the activation of discrete policy at higher and higher levels.

In the even more special case where the policy variable z is binary and can only take values of 0 (inactive) or 1 (active), the welfare gain from activating the policy can be found setting $i = 0$ in (34):

$$\Delta W(x, 0) = \gamma - \frac{w}{2}(\gamma_c - \gamma)^2 + w(\gamma_c - \gamma)A(x), \tag{35}$$

whose interpretation is the same as that provided for the more general case.

In terms of Figure 1, the discreteness of the policy variable does not alter the indifference curves and the location of the “hypothetical” policy frontier that would emerge if z were continuous. The difference is that the effective frontier now only includes as many points on the hypothetical frontier as possible values for z_i. Heuristically, it is still correct to think about the optimal policy as the one that brings the gap between expected growth and GaR as close as possible to the gap in (16) that would be targeted if z were a continuous variable.

6 Further discussion

6.1 What if the policy variable involves no trade-off?

Let us suppose the quantile-regression methodology yields an estimate of γ equal to zero. In this case, under the remaining assumptions of the baseline model, the policy variable z should be increased up to the point in which either the gap between expected growth and GaR is zero or the policy variable reaches its upper bound, whichever happens first. The first implication (being able to use the policy to make GaR equal to expected growth) does not seem plausible or economically meaningful: it is too good to be true. In this case the emergence of $\gamma = 0$ in the estimation of the parameters of (3) may point to the existence of some relevant non-linearity (e.g. a negative effect which is observable only when z is sufficiently large) that the linear specification fails to capture. In the field of macroprudential policy this can easily happen as many policies have not been used historically at all relevant ranges of activation, so identifying those negative effects in the data may simply be impossible.

Practical solutions to the problem may involve running non-linear specifications of (3) or, if the available data do not allow the conjectured non-linearity to be captured, introducing the suspected missing cost of the policy using an auxiliary calculation. For instance, if the policy variable is a borrower-based measure that has never being tried at a very high level but there are reasonable theoretical arguments to believe that its activation might have negative implications for welfare, a negative term capturing the estimated (otherwise missing) marginal certainty-equivalent cost of the policy could be added in the equation for expected GDP growth, expressed as a fraction of initial GDP. After introducing such an adjustment, if it is consistent with the mandate of the macroprudential authority, the design and assessment of macroprudential policy could proceed as indicated in the previous sections.

If the policy variable has a natural upper limit (e.g. it is a binary or discrete variable measuring the quality of institutions, such as resolution regimes or policy coordination), then the implication that it should be activated at its maximum level may be meaningful and require no further adjustment in the analysis.

6.2 Country heterogeneity

In a multi-country environment, the empirical framework considered in this paper may involve country-specific versions of equations (2) and (3) as well as cross-country differences in the risk preference parameter w. Obtaining the former does not necessarily mean running quantile regressions country by country, which, in the absence of sufficiently long time series for each country, could imply a lack of accuracy in the required estimates. Alternatives include running panel quantile regressions that allow for country fixed effects or include coefficients for the risk indicators or the policy variables that vary with some country-specific characteristics (e.g. variables intended to capture differences in the structure of countries' financial or legal systems). In the context of the "single country" baseline specification explored in this paper, these country differences can be thought of as just having different values of the involved parameters and their implications can be easily extracted from the expressions for the policy rule and the target gap provided for the baseline case. In particular, the following points should be noted.

4. If countries structurally differ in aspects that only alter intercepts α_c and α and/or the risk sensitivity parameters β_c and β in (2) and (3), then the target gap in (16) will not differ across countries. Yet, as reflected in the expressions for ϕ_0 in (10) and ϕ_1 in (11), their optimal policy rules may differ in intensity and risk responsiveness to accommodate their structural differences in each of these sets of parameters. For instance, a country with a larger value of $\alpha - \alpha_c$ (a larger "structural gap") will, all other things being equal, have to activate its macroprudential policy at a higher level (higher ϕ_0), while a country with a larger value of $\beta - \beta_c$ (a larger "gap vulnerability to risk") will have to be systematically more responsive to changes in risk indicator x (higher ϕ_1).

5. If countries structurally differ in the effect of policy on GaR γ_c and/or expected growth γ, their target gap and the parameters of their optimal policy rule will differ. Specifically, countries featuring the hardest trade-offs, as measured by the size of the marginal cost-effectiveness of the policy tool (that is, the steepness of the policy frontiers depicted in Figure 1) will target a larger gap between expected growth and GaR and adapt their policy rules accordingly.

6. If countries structurally differ in their risk preferences as captured by w (not a very plausible source of heterogeneity under the microfoundation provided in the Appendix of this paper), then their target gap and the intercept ϕ_0 of their optimal policy rule will also differ. However, as previously mentioned when commenting on the determinants of ϕ_1, differences in w would not translate into a different responsiveness of their policies to changes in risk variable x.

6.3 Interaction with other policies

The discussion so far has not explicitly dealt with the case in which policies other than macroprudential policies have an impact on expected growth and GaR. An immediate way to integrate these into the framework considered in this paper would be to add variables representing those policies in a vector version of risk variable x. Under this reformulation, x would then account not only for risk variables in a narrow sense but also for other relevant elements of the economic situation at the time of designing macroprudential policy and that the macroprudential policymaker takes as given. Under this formulation (which resembles other treatments in which policies under the control of different authorities interact as in a non-cooperative game), at the time of designing macroprudential policy the state of other relevant non-macroprudential policies (e.g. monetary policy) would appear as part of x in vector versions of (2) and (3), and consequently in the macroprudential policy rule (9). The policy rule could then be interpreted as the macroprudential policy reaction function (reflecting the reaction of macroprudential policy to the current settings of other policies).

A more general discussion covering the issue of optimal policy coordination would require further extensions. For instance, to consider optimal coordination with monetary policy, the objective function W might have to include terms that reflect the goals of this policy, such as price stability, that might not be fully reflected in the terms currently included in (5).[21] A policymaker optimising on the two policies at the same time would, in that case, treat the non-macroprudential policy under consideration as an element of a vector version of policy variable z giving rise to issues similar to those discussed in Sections 5.3 and 5.4 when considering multiple macroprudential policy tools. These extensions might also allow an analysis to be made of the outcome of several authorities acting in a non-cooperative manner under different mandates and with separate policy tools and to assess the extent to which those outcomes differ from those achieved under a more centralised solution (and thus the potential gains from policy coordination).

6.4 Reformulation in terms of growth-given-stress

Interestingly, in a normal distribution, the distance between the mean and the c-quantile is proportional to the distance between the mean and the expectation of the random variable conditional on being below the c-quantile. Section A.3 of the Appendix uses this property to show that under the baseline assumption that the GDP growth rate is normally distributed, the welfare criterion in (5) can be re-expressed in terms of growth-given-stress (that is, the expectation of the growth rate conditional on being below the c-quantile of the growth rate), retaining the microfoundation provided in Section A.1 of this Appendix. Consequently, the constant target gap property of the optimal policy rule in (16) could also be expressed in terms of the distance between expected growth and growth-under-stress. Therefore, in normal circumstances, the formulation of the macroprudential policy problem and assessment of macroprudential stance on the basis of GaR or growth-given-stress would make no difference.

21 See Cecchetti and Kohler (2014) for an example that considers coordination between conventional monetary policy and capital regulation in a related reduced-form setup.

7 Concluding remarks

Using the concept of GaR in the measurement of the downside risks that macroprudential policy aims to address opens very interesting avenues for the use of empirical quantitative models in the design of macroprudential policies and the development of concrete notions of macroprudential policy stance. The setup allows us to explicitly consider, using a similar modelling methodology, the effects of risk and policy variables on expected GDP growth (arguably, a succinct measure of what other macroeconomic policies care about) and the risk of sufficiently adverse GDP growth outcomes (arguably, a promising concrete measure of what macroprudential policy cares about). This paper has explored the foundations for the design and assessment of macroprudential policies using this setup.

The paper starts with a stylised description of the setup in the context of its implementation using the outcome of a quantile-regression approach. A welfare criterion for the design of the optimal policies has been proposed that can be microfounded as consistent with the maximisation of the expected utility of a representative agent in some contexts. The properties of the optimal policies have been explored in the basic setup as well as in several extensions and modifications covering cases with non-linearities in the impacts of policy variables and risk variables on the relevant outcomes, multiple policy variables and discrete policy variables. An important extension shows the compatibility of this framework with the view that macroprudential policy involves various well-identified intermediate objectives, each of which can be associated with one or a subset of targeted policy tools. Additional discussions deal with policies that seem to involve no trade-off between mean growth and GaR, the treatment of country heterogeneity and the possibility of reformulating the analysis around the concept of growth-given-stress rather than GaR.

Under the postulated representation of preferences, the policy design problem yields a quantitative-based policy target and a metric for the assessment of policy stance similar to that of other macroeconomic policies. The main challenges for the applicability of this framework are more empirical and political than conceptual. On the empirical side, the main challenge resides in the consistent and sufficiently precise estimation of the causal effects of risk and policy variables on the relevant moments (mean and GaR) of the growth distribution. Properly detecting relevant non-linearities and interactions between policies is also important. In the absence of proper estimates of the relevant parameters and relationships, the mechanical application of this framework could produce misguided policy advice. Therefore, the framework will develop at the speed with which data on the applied policies accumulate and econometric efforts succeed in providing reliable estimates of their effects on growth outcomes.

On the political side, once data and estimation provide a reliable description of the policy trade-offs, the main challenge will be to define society's aversion for financial instability on which optimal policies should be based. Given the uncertainty surrounding the relevant parameters implied by the empirical challenges, policymakers may need to be guided on how to expand the type of framework sketched in this paper to account for model uncertainty (that is, for the imperfect knowledge of the specification and parameters of the relevant quantile regressions) and the potential policy mistakes that could stem from this uncertainty.

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Appendix. Microfoundations of the GaR-based welfare criterion

A.1 CARA preferences and normally distributed growth rates

Let Y denote GDP and let y describe the implied (geometric) GDP growth rate relative to a benchmark level Y_0 so that:

$$Y = (1 + y)Y_0. \tag{36}$$

Suppose also that there is a representative agent whose preferences for GDP levels are represented by a utility function $U(Y)$ with a local coefficient of absolute risk aversion $\lambda(Y_0)$ at $Y = Y_0$ and that the utility function can be (locally) described as one exhibiting CARA with parameter $\lambda(Y_0)$, so that:

$$U(Y) = -\exp(-\lambda(Y_0)Y). \tag{37}$$

Using (36), we can write:

$$U(Y) = -\exp(-\lambda(Y_0)Y_0(1 + y)) = -\exp(-\lambda(Y_0)Y_0)\exp(-\lambda(Y_0)Y_0 y). \tag{38}$$

For fixed Y_0, since affine monotonic transformations of a utility function will represent exactly the same preferences, we can replace $U(Y)$ with:

$$u(y) = -\exp(-\lambda(Y_0)Y_0 y) = -\exp(-\rho_0 y), \tag{39}$$

where $\rho_0 = \lambda(Y_0)Y_0$ describes the agent's coefficient of relative risk aversion at Y_0. Thus, this utility function describes CARA preferences directly on the growth rate y but the parameter ρ_0 in this specification measures the relative risk aversion of the agent (in terms of their preferences for GDP levels) at the initial GDP level Y_0.

Let us now suppose that GDP growth is normally distributed, so $y \sim N(\bar{y}; \sigma_y^2)$. From the well-known properties of normal distribution, the moment generating function of the distribution of y is then:

$$M(t) = E(\exp(ty)) = \exp(\bar{y}t + \tfrac{1}{2}\sigma_y^2 t^2) \tag{40}$$

for any t. In particular,

$$M(-\rho_0) = E(\exp(-\rho_0 y)) = \exp(-\rho_0 \bar{y} + \tfrac{1}{2}\rho_0^2 \sigma_y^2). \tag{41}$$

Hence, from (39) and (41), we can write the agent's expected utility as:

$$E[u(y)] = -E[\exp(-\rho_0 y)] = -\exp(-\rho_0 \bar{y} + \tfrac{1}{2}\rho_0^2 \sigma_y^2). \tag{42}$$

Further, since monotonic transformations of expected utility will represent exactly the same preferences, these preferences can be equivalently described by the (indirect) utility function

$$v = \bar{y} - \frac{\rho_0}{2}\sigma_y^2. \tag{43}$$

that is, a simple linear expression in the mean $\bar{y}$ and the variance σ_y^2 of the growth rate y.

Growth-at-risk (GaR) for a given confidence level c is the c-quantile of the probability distribution of y, that is, the value y_c so that:

$$\Pr(y \leq y_c) = c. \tag{44}$$

Under the properties of normal distributions, $(y - \bar{y})/\sigma_y$ is a standard normal random variable, $N(0,1)$. If $\Phi(\cdot)$ is the cumulative distribution function of a standard normal, we can write:

$$\Pr(y \leq y_c) = c \Leftrightarrow \Pr((y - \bar{y})/\sigma_y \leq (y_c - \bar{y})/\sigma_y) = c \Leftrightarrow \Phi((y_c - \bar{y})/\sigma_y) = c. \tag{45}$$

Solving for y_c in the last expression yields:

$$y_c = \bar{y} + \sigma_y \Phi^{-1}(c). \tag{46}$$

Alternatively, solving for σ_y yields:

$$\sigma_y = \frac{y_c - \bar{y}}{\Phi^{-1}(c)}, \tag{47}$$

which, plugged into (43), leads to the indirect utility function:

$$v(\bar{y}, y_c; \rho_0, c) = \bar{y} - \frac{\rho_0}{2(\Phi^{-1}(c))^2}(\bar{y} - y_c)^2, \tag{48}$$

which expresses the agent's expected utility as a function of expected growth, GaR at a confidence level c, the relative risk aversion coefficient of the agent at the initial level of GDP ρ_0, and the confidence level c.

Hence, maximising a welfare criterion of the form:

$$W = \bar{y} - \frac{w}{2}(\bar{y} - y_c)^2, \tag{49}$$

as assumed in the main text, would be equivalent to the maximisation of the expected utility of the representative agent for

$$w = \frac{\rho_0}{(\Phi^{-1}(c))^2}. \tag{50}$$

For instance, for $c = 0.05$, we have $\Phi^{-1}(c) = -1.6449$, so, with a coefficient $\rho_0 = 2$ of relative risk aversion at Y_0, both criteria would coincide under $= 2(1.6449)^{-2} = 0.7392$.

Intuitively, the policymaker's preference for financial stability should increase with the agent's relative risk aversion parameter ρ_0 as well as for any $c < 0.5$, with the level of confidence c at which GaR is calculated.[22]

A.2 Modelling GaR vs. growth volatility and departing from normality

Under the normality assumption sustaining the interpretation of the welfare criterion W as consistent with expected-utility maximisation, modelling a lower quantile such as y_c and expected growth $\bar{y}$ is no different from modelling the standard deviation and the mean of the growth rate and focusing on a welfare criterion that directly depends on those moments of the growth distribution.

Moreover, under normality, if expected growth is determined as in (3) and the standard deviation of the growth rate is linear in x and z, for instance:

$$\sigma_y = \alpha_\sigma + \beta_\sigma x + \gamma_\sigma z, \tag{51}$$

then (46) implies that (51) is exactly compatible with the specification of y_c in (2) if, and only if, $\alpha_c = \alpha + \Phi^{-1}(c)\alpha_\sigma$, $\beta_c = \beta + \Phi^{-1}(c)\beta_\sigma$, and $\gamma_c = \gamma + \Phi^{-1}(c)\gamma_\sigma$, where for $c < 0.5$ we have $\Phi^{-1}(c) < 0$. So, the prior assumption that the policy variable has a positive effect on y_c ($\gamma_c > 0$) and a negative effect on $\bar{y}$ ($\gamma < 0$) would require that it also has a sufficiently large negative impact on σ_y ($\gamma_\sigma < -\gamma/\Phi^{-1}(c) < 0$).

While the ability to structurally interpret the analysis in the main text under the assumption of normality as exactly compatible with expected-utility maximisation is reassuring, the normal case would not justify a strict preference for the quantile-regression approach. A quantile-regression approach to the analysis of macroprudential policies is typically defended on the grounds that there are variables whose impact on extreme low quantiles of the growth distribution is empirically detectable, while its impact on the standard deviation of the growth rate (or on high quantiles of the growth distribution) might not be (or at least not so clearly). For instance, it is likely that empirical measures of GDP volatility are dominated by what happens at business cycle frequencies, while what happens at a sufficiently low growth quantile may better capture the impact of infrequent financial crises.

However, representing the world in which lower quantiles are disproportionately affected by one variable or infrequent discrete events have non-linear implications for growth implies departing from the normality assumption and, hence, from the setup in which the interpretation of the welfare criterion in expected-utility terms is exactly valid. In other words, while the normal world provides a useful benchmark to help connect the preference for financial stability reflected into the welfare criterion W with a standard way of representing the agent's preferences in economics, it is probably

[22] Note that for $c < 0.5$, $\Phi^{-1}(c)$ is negative and approaches zero as c increases, so $(\Phi^{-1}(c))^2$ is decreasing in c.

not the most practically relevant one. Box D in Cecchetti and Suarez (2021) describes a number of simulation exercises in which GDP growth is drawn from empirically-motivated non-normal distributions and examines the accuracy with which a GaR-based criterion such as W approximates the true expected utility (measured in certainty equivalent terms). The message from those simulations is that GaR-based metrics provide a reasonably good approximation to the expected-utility-based metrics even when the growth distribution deviates substantially from normality, as well as under constant relative risk-aversion (CRRA) preferences.

Additionally, as discussed in the main text, in the non-normal world, we could interpret W as a heuristic representation of the preferences of a policymaker who cares about the gap $\bar{y} - y_c$, for a suitably low value of c, rather than the standard deviation of GDP growth, for instance, because of some form of loss aversion. Under this perspective, the focus on the trade-off between maximising $\bar{y}$ and minimising the gap $\bar{y} - y_c$ could reflect that the policymaker cares more about the relative output losses incurred at the low tail of the growth rate distribution than the potentially offsetting (in expected terms) relative output gains obtained at the high tail.

A.3 Reformulation using a growth-given-stress criterion[23]

Let us define the growth-given-stress (GgS) for a given reference probability c as the expected value of the GDP growth rate y conditional on this rate being lower than the c-quantile of its distribution y_c, that is:

$$GgS_c = E(y|y \leq y_c). \tag{52}$$

When y is a normal random variable, GgS_c is just the mean of an upper-truncated normal random variable with truncation point at y_c. The well-known expression for this mean implies:

$$GgS_c = \bar{y} - \frac{\phi\left(\frac{y_c - \bar{y}}{\sigma_y}\right)}{\Phi\left(\frac{y_c - \bar{y}}{\sigma_y}\right)}\sigma_y, \tag{53}$$

where $\phi(\cdot)$ is the density function of standard normal. But, since y_c is the c-quantile of the distribution of y, the term $(y_c - \bar{y})/\sigma_y$ can be written as $\Phi^{-1}(c)$. This allows us to write:

$$GgS_c = \bar{y} - \frac{\phi(\Phi^{-1}(c))}{c}\sigma_y. \tag{54}$$

Using (47) to substitute for σ_y and rearranging, we can now express:

$$\bar{y} - GgS_c = \frac{-\phi(\Phi^{-1}(c))}{c\Phi^{-1}(c)}(\bar{y} - y_c), \tag{55}$$

[23] I wish to thank Stephen Cecchetti for making me aware of the possibility of this reformulation.

where, for a given c, the ratio $-\phi(\Phi^{-1}(c))/(c\Phi^{-1}(c))$ is proportionality constant (which is positive for $c < 0.5$).

In other words, when the growth rate y is normally distributed, the gap between expected growth and GgS is proportional to the gap between expected growth and GaR. Therefore, maximising the welfare criterion W specified in (5) would be equivalent to maximising a similar linear-quadratic criterion whose quadratic term contains the square of the distance between expected growth and GgS and where the instability aversion parameter w is replaced by

$$w_{GgS} = \left(\frac{c\Phi^{-1}(c)}{\phi(\Phi^{-1}(c))}\right)^2 w. \tag{56}$$

This criterion would thus have the same microfoundation as the criterion provided in Section A1 of this Appendix for W. Using this microfoundation, the parameter w_{GgS} would become, using (50),

$$w_{GgS} = \frac{c^2 \rho_0}{\phi(\Phi^{-1}(c))} \tag{57}$$

The optimal policy rule resulting from solving the baseline policy problem under the GaR-based welfare criterion would be equivalent to the one that emerges under the equivalent GgS-based criterion and would also satisfy the constant target gap property in (16). This property could be translated into targeting a gap between expected growth and GgS, given by:

$$\bar{y} - GgS_c = \frac{1}{w_{GgS}} \frac{1}{1+\gamma_c/(-\gamma)}. \tag{58}$$

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Javier Suarez
CEMFI, Casado del Alisal 5, 28014 Madrid, Spain; email: **suarez@cemfi.es**

Postal address	60640 Frankfurt am Main, Germany
Telephone	+49 69 1344 0
Website	**www.esrb.europa.eu**

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