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Systemic illiquidity in the interbank network

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Abstract

We study systemic illiquidity using a unique dataset on banks' daily cash flows, short-term interbank funding and liquid asset buffers. Failure to roll-over short-term funding or repay obligations when they fall due generates an externality in the form of systemic illiquidity. We simulate a model in which systemic illiquidity propagates in the interbank funding network over multiple days. In this setting, systemic illiquidity is minimised by a macroprudential policy that skews the distribution of liquid assets towards banks that are important in the network.

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1 Introduction

Banks lend to each other at short maturities. In this interbank network, financial stress can spread via insolvency or illiquidity. In terms of insolvency, one bank's liability is another's asset. Default reduces the value of the lending bank's asset, moving it closer to insolvency, and potentially generating a cascade of counterparty defaults. In terms of illiquidity, one bank's outflow of cash is another's inflow. Failure to roll-over short-term funding or repay obligations when they fall due reduces counterparties' cash inflows, potentially generating a cascade of funding shortfalls—even if banks remain solvent.

Theoretical research suggests that the probability of default cascades is increasing in the size of interbank exposures (Nier, Yang, Yorulmazer & Alentorn, 2007). Empirically, however, interbank exposures tend to be small relative to bank equity. Defaults generate counterparty losses, but these losses are typically insufficient relative to equity to trigger further insolvencies (Upper, 2011).[1] This implies that default cascades are insufficient to characterise systemic risk in interbank networks.

Besides insolvency cascades, banks are vulnerable to funding shortfalls owing to their inherent liquidity mismatch (Gorton & Pennacchi, 1990). Bank runs can take place in both retail (Iyer & Peydro, 2011) and wholesale (Gorton & Metrick, 2012) markets. While interbank funding markets allow banks to share these idiosyncratic liquidity risks (Allen & Gale, 2000), they can also provide a channel by which funding shortfalls propagate through the network (Acemoglu, Ozdaglar & Tahbaz-Salehi, 2015). We refer to the latter phenomenon as "systemic illiquidity".

Our contribution to the analysis of systemic illiquidity is twofold. First, we model systemic illiquidity over multiple time periods by extending the single-period dynamic programming algorithm of Eisenberg & Noe (2001). This extension represents a substantive as well as methodological improvement on existing work. Although useful, single-period models are liable to underestimate systemic illiquidity as they do not capture cash flow dynamics. Illiquidity becomes more likely as stress persists: even if banks survive one day, they might become illiquid after multiple days of net outflows. This is pertinent

[1] Representative papers in the empirical literature studying solvency contagion in interbank markets are Degryse & Nguyen (2007) for Belgium; Upper & Worms (2004) and Memmel, Sachs & Stein (2012) for Germany; Mistrulli (2011) for Italy; and van Lelyveld & Liedorp (2006) for the Netherlands. Alves, Ferrari, Franchini, Heam, Jurca, Langfield, Laviola, Liedorp, Sanchez, Tavolaro & Vuillemey (2013) provide a cross-country analysis of contagion in a network of 53 large EU banks.

given that outflows tend to be serially correlated during banking crises: Acharya & Merrouche (2012) document that banks hoard liquid assets as a precautionary response to scarce external funding. Such hoarding behaviour exacerbates the cost and availability of liquidity for other banks.[2] Consequently, our multi-period extension provides a more precise quantification of systemic illiquidity in interbank networks.

Our second contribution is to bring the model to unique regulatory data. From Bank of England returns, we obtain information at daily frequency on 182 banks' expected future cash inflows and outflows at daily frequency. These granular cash flow data elicit novel insights when viewed through the lens of our multi-period model. Our results indicate that the potential for systemic illiquidity in the UK interbank network was low at our snapshot date at the end of 2013: in simulations, no bank falls short of funding because counterparties fail to repay obligations. However, systemic illiquidity does emerge when liquid asset holdings are envisaged to be less than half of their end-2013 levels, as was the case just before the 2008-09 financial crisis. This finding underscores the importance of post-crisis microprudential liquidity requirements in improving systemic stability.[3]

Furthermore, our model allows us to differentiate between individually illiquid banks (whose liquid asset buffers are insufficient to meet net cash outflows even in the absence of defaults by other banks) and systemically illiquid banks (which become illiquid owing to the failure of other banks to honour their obligations). Banks whose individual illiquidity generates systemic illiquidity are systemically important in the interbank funding network. We find that banks' systemic importance is not correlated with banks' total lending or borrowing within the interbank funding network. Instead, network structure—that is, the cross-sectional distribution of banks' lending and borrowing—determines banks' systemicity.

Finally, our model sheds light on the design of liquidity regulation. Existing liquidity requirements, including the liquidity coverage ratio (LCR) and net stable funding ratio (NSFR), are microprudential in the sense that they apply uniformly to all banks, regard-

[2] Shin (2009) describes how endogenous hoarding behaviour was at work in summer 2007, when the day-by-day deterioration of interbank funding markets eventually led to the UK government's nationalisation of Northern Rock. Similar dynamics led to the bankruptcy of Lehman Brothers (Duffie, 2010).

[3] Although the phase-in of LCR requirements only began in 2015 in the EU, following the entry into force of the Capital Requirements Regulation, the UK was an early adopter of similar requirements beginning in 2009.

less of their systemic importance. Under the LCR requirement, for example, all banks must hold enough liquid assets, such as cash or Treasury bonds, to cover net outflows over one month of stressed conditions. We compare this microprudential benchmark to macroprudential liquidity requirements which vary in the cross-section of banks.[4] In particular, we calculate the cross-sectional distribution of liquid assets that minimises systemic illiquidity for a given aggregate volume of liquid asset holdings. This constrained optimisation problem is solved by requiring systemically important banks to hold more liquid assets than other, less important, banks. Given that it is privately costly for banks to hold liquid assets, our findings reveal that macroprudential liquidity requirements would improve the efficiency of purely microprudential requirements in mitigating systemic illiquidity.

Related literature

Our paper is most closely related to the numerical literature on systemic liquidity risk. Gai, Haldane & Kapadia (2011) develop a rich model of the interbank funding market that encapsulates unsecured loans, secured loans and haircuts on collateral. Numerical simulations of their model show that shocks to haircuts could trigger widespread funding contagion, especially when the interbank network is concentrated. Similarly, Iori, Jafarey & Padilla (2006) find that bank heterogeneity can lead to systemic illiquidity. Lee (2013) simulates a simple model of hypothetical banking systems to study the effects of different network structures on systemic liquidity risk.

However, owing to a lack of data availability, these models of systemic illiquidity are not calibrated to real-world interbank funding networks. In earlier work, payments systems have proven a fruitful source of such data. Furfine (2003), for example, finds that the US federal funds market is robust to solvency contagion but vulnerable to liquidity contagion. Such analyses provide useful insights regarding vulnerabilities in payments systems, but suffer from the obvious drawback of focusing on a small subset of the interbank funding market. By contrast, our data cover the largest sources of interbank funding, namely unsecured loans and repurchase (repo) agreements.

[4] These macroprudential liquidity requirements can be interpreted as the quantity-based analogue to a price-based Pigovian tax on systemic importance (Perotti & Suarez, 2011).

In addition to our own extension, the seminal model of Eisenberg & Noe (2001) has previously been extended in various other directions. For example, Cifuentes, Ferrucci & Shin (2005) allow for liquidity considerations and Rogers & Veraart (2013) introduce costs of default. More recently, Feinstein, Pang, Rudloff, Schaanning, Sturm & Wildman (2018) test the sensitivity of the Eisenberg-Noe clearing vector to estimation errors in bilateral interbank liabilities. There are also studies on alternative clearing processes. For example, Csoka & Herings (2018) introduce a large class of decentralised clearing processes and shows that they would converge to a centralised clearing procedure when the unit of account is sufficiently small.

Another approach to the analysis of systemic illiquidity focuses on behavioural externalities within the banking system. Freixas, Parigi & Rochet (2000) study a setting in which banks are solvent but illiquid due to coordination failure. Using a global games approach, Morris & Shin (2008) likewise analyse coordination failures that can cause liquidity crises in a systemic context. Perotti & Suarez (2011) study the role of a Pigouvian tax to contain externalities arising from systemic illiquidity, although they do not explicitly define an interbank network structure. Taken together, these studies highlight the fragility of the interbank network in the context of coordination failures, and identify potential policy interventions to correct these externalities ex ante, including leverage constraints, liquidity requirements and Pigouvian taxes.

The paper is organised as follows. In Section 2, we introduce the model, before describing the dataset in Section 3. Section 4 evaluates systemic illiquidity under different network configurations. To infer policy implications, we calculate the socially optimal distribution of liquid assets in Section 5. Finally, Section 6 concludes.

2 Model

We build a multi-period model of an interbank funding network comprised of heterogeneous banks. Banks lend to and borrow from other banks in the network via unsecured loans and repo contracts. Banks also obtain wholesale funding from other financial institutions outside the interbank network. In this setting, we model a stress scenario in which all banks lose access to interbank and other wholesale funding. In the spirit of the LCR requirement, banks may use only their unencumbered high-quality liquid asset buffer to meet maturing obligations. A bank is illiquid (and defaults on its short-run liabilities) when its liquid asset buffer is insufficient to meet contractual obligations on any given day. This modelling framework, and the interbank payments model that it encompasses, is described below.

2.1 Framework

A common metric for liquidity risk used by microprudential bank regulators is the time period over which banks could survive a wholesale funding market freeze by converting their liquid asset buffer into cash (Basel Committee on Banking Supervision, 2013b). In the hypothetical scenario of a market freeze, banks are deemed unable to obtain new loans or roll-over existing debts. The adequacy of individual banks' liquid asset buffers is determined by assuming that all other banks are fully liquid, and therefore able to repay maturing debts. This microprudential assumption overlooks a negative externality: counterparties' cash inflows are impaired when a bank fails to repay obligations when they fall due. To account for this externality, our modelling framework allows for the fact that a defaulting bank will not repay its interbank obligations. If the externality is sufficiently strong, creditor banks will also become illiquid—even if they were deemed to have microprudentially adequate liquid asset buffers when negative externalities are not considered. We call such banks "individually liquid but systemically illiquid".

The negative externality could propagate further. Like individually illiquid banks, systemically illiquid ones will stop repaying interbank obligations, imposing a negative externality on other banks' liquid asset buffers, and generating further systemic illiquidity. Our model captures this propagation process within the interbank funding network in the following sequence.

1. On day 0, each bank has a given liquid asset buffer.
2. The stress scenario starts on day 1. By assumption, banks are unable to roll-over existing debt or obtain new loans. All loans with an open maturity are terminated on day 1.
3. Loans with a two-day maturity terminate on day 2, three-day loans on day 3, and so on for the duration of the stress scenario, which in our application has a one-

month horizon. Banks' liquid asset holdings deteriorate in accordance with their net outflows of cash over the stress scenario.

4. A bank defaults if it does not have enough liquid assets to meet its obligations.

5. If a bank defaults, its counterparties will not receive any future scheduled payments from that bank, but still need to repay any debt due to that bank.[5] This in turn affects other banks' liquid asset holdings.

6. If other banks default, repeat step 5.

Systemic illiquidity is measured in two ways. First, we count the number of banks that default, or default earlier than otherwise, due to systemic illiquidity. If a bank remains liquid throughout the stress period up to step 4 above, but defaults when systemic illiquidity is taken into account (in steps 5 and 6), we say that the bank defaults due to systemic illiquidity; it is "individually liquid but systemically illiquid". If a bank defaults due to individual liquidity mismatches (in step 2 or 3), but defaults earlier when systemic illiquidity is taken into account, we say that the bank defaults earlier due to systemic illiquidity.

Second, we calculate the proportion of the banking system that would default, or default earlier, due to systemic illiquidity. To do this, we infer banks' relative importance from their "impact score", which is a regulatory measure of the potential impact of idiosyncratic failure on the stability of the system as whole, taking into account balance sheet variables, such as total assets, as well as the provision of critical financial services.[6] To contextualise this impact score, the bank with the highest score accounts for 12% of the UK banking system's aggregate score. If this bank were to default in our simulations, we would say that 12% of the impact-weighted UK banking system is in default.

[5] For repos, we assume that all future payments are netted and settled immediately upon the default of the counterparty. Our results remain qualitatively similar if we assume that settlement takes place outside the 22-day period.

[6] The potential impact score is used by the Bank of England's Prudential Regulation Authority (PRA) to quantify the potential impact that a firm could have on financial stability. As such, it is a key component of the PRA's risk framework. We use the PRA's impact score—instead of a simpler measure such as total assets—because some firms, e.g. custodian banks, have small balance sheets but large potential impact.

2.2 Interbank payments model

The network is populated by N institutions, each of which is identified by $i, j \in [1, N]$. There are two types of liabilities: unsecured loans and repo contracts. In the single-period Eisenberg-Noe model, there are N nodes with contractual obligations to each other. Node i has an initial liquid asset buffer of e_i and total nominal liabilities p_i. The fraction of its total liabilities owed to node j is denoted by Π_{ij}. The terminal liquid asset buffer w_j of node j can be calculated by

$$w_j = e_j + \sum_{i \neq j} p_i \Pi_{ij} - p_j, \tag{2.1}$$

where $\sum_{i \neq j} p_i \Pi_{ij}$ represents the total payments received by j from other banks.

The algorithm finds a unique payment vector that is consistent with three regulatory requirements: (*i*) seniority of debt over equity, (*ii*) limited liability of equity, and (*iii*) proportional payment. In addition, we impose three common sense rules, namely that (*iv*) payments are non-negative, (*v*) payments do not exceed liabilities, and (*vi*) default is to be avoided if possible. These constraints and their implications are discussed in Appendix A.

The vectors $\boldsymbol{p^*}$ can be calculated as an optimal solution of the following optimisation problem:

$$\begin{aligned} & \underset{\boldsymbol{p^*}}{\operatorname{argmax}}\, 1 * \boldsymbol{p^*} \\ & \text{subject to} \\ & p_j^* \leq e_j + \sum_{i \neq j} p_i^* \Pi_{ij} \qquad \forall j \\ & 0 \leq p_j^* \leq p_j \qquad\qquad\quad \forall j. \end{aligned} \tag{2.2}$$

That is, $\boldsymbol{p^*}$ can be found by maximizing the sum of all nodes' payments, subject to the constraints that a bank's payment is non-negative, cannot exceed its promised payment, and cannot exceed what it receives from other banks by more than its initial stock of liquid assets.

In the multi-period extension of this canonical model, we introduce time, indexed by t, which is discrete (at daily frequency) and finite (with a horizon T, which in our application is equal to one calendar month, i.e. 22 business days). In this setting, we find a payment vector that is consistent with the six rules listed previously. The payment vectors $\boldsymbol{p}^*$ can then be calculated by optimising:

$$\begin{aligned}
&\underset{\boldsymbol{p}^*}{\operatorname{argmax}} \sum_{t=1}^{T} 1 * \boldsymbol{p}^{*t} && (2.3)\\
&\text{subject to} \\
&p_j^{*T} \leq e_j^T + \sum_{t=1}^{T} \sum_{i \neq j} p_i^{*t} \Pi_{ij}^t \qquad && \forall j \forall T \\
&0 \leq p_j^{*t} \leq p_j^t && \forall j \forall t.
\end{aligned}$$

In summary, we reformulate the single-period Eisenberg-Noe problem by introducing a time component in each equation. This model optimises banks' payments over multiple periods, and allows us to distinguish between periods in which a bank is a going concern, in the process of defaulting, or has defaulted, either due to individual illiquidity or systemic illiquidity. In Section 4,we solve this multi-period problem numerically. Appendix A provides a proof of the theorem that any numerical solution to the multi-period Eisenberg-Noe problem must be unique.

3 Data

Our second contribution to the analysis of systemic illiquidity concerns the application of the multi-period model to real-world data from the Bank of England. To apply the model, we combine three regulatory datasets. These datasets allow us to construct a snapshot (as of end-2013) of banks' liquid asset buffers (subsection 3.1), determine their expected cash inflows and outflows (subsection 3.2), and build a network of banks connected bilaterally to each other via payment obligations arising from unsecured loans and repos (subsection 3.3). Finally, subsection 3.4 presents summary statistics of the dataset used in subsequent analyses.

3.1 Liquid asset buffers

In regulatory returns, UK banks are obliged to report their liquid asset buffers as part of their Recovery and Resolution Planning (RRP) and other regulatory reporting. From these regulatory returns, we observe banks' liquid asset buffers on the reporting date at the end of 2013. This sets the initial condition for each bank at the start of the liquidity stress scenario.

In line with LCR requirements, we consider a stress scenario in which banks can use only their liquid asset buffer to meet maturing wholesale liabilities (i.e. they cannot obtain additional funding). A bank's liquid asset buffer at time $t+1$ is therefore given by its initial buffer at time t plus cash inflows net of outflows. In principle, banks with large lending positions could see their liquid asset buffer improve over the course of the stress scenario, but in most cases liquid asset buffers deteriorate since banks typically borrow short, including from non-banks, and lend long. To quantify the net change in the liquid asset buffer over time, we require additional data on cash inflows and outflows at daily frequency. We turn to this next.

3.2 Cash inflows and outflows

We require information on banks' wholesale cash inflows and outflows. On the cash inflow side, banks receive payments from their unsecured loans and reverse repo transactions (under the assumption that the counterparty is not in default). On the cash outflow side, banks repay their counterparties to unsecured loans and repos, and also pay wholesale deposits, bonds and notes that come due.

A second Bank of England dataset at our disposal, namely FSA047, fulfills this requirement. This dataset contains bank-level information on cash inflows and outflows at daily frequency from overnight onwards. This provides us with a complete picture of the cash inflows and outflows that banks can expect over the course of our liquidity stress scenario. Hence, we are able to populate an $N \times T$ matrix for banks' net cash inflows. In our application, the number of banks N is 182 and the total number of time periods T is 22 business days. By combining this $N \times T$ matrix with banks' initial conditions in terms of liquid asset buffers, we can also compute a companion $N \times T$ matrix which records the deterioration in banks' liquid asset buffers over the course of our liquidity stress scenario.

However, this companion matrix implicitly assumes that all counterparties repay their obligations when they come due. In applying our multi-period extension of the Eisenberg-Noe algorithm, we seek to interrogate this assumption by allowing banks to default on their obligations. For this, we require more granular bilateral data on interbank payments.

3.3 Bilateral payments

To apply our multi-period extension of the Eisenberg-Noe algorithm, we require a final ingredient: namely the bilateral nature of payments that take place between banks in the network. This ingredient is needed to model the externalities associated with a bank's failure to meet its payment obligations to other banks. We therefore extend our $N \times T$ matrix to a third dimension, namely $N-1$, which is the total number of potential counterparties for any given bank in the interbank funding network.

We define the interbank funding network as the interbank network of payment obligations related to outstanding unsecured loans and repos backed by low-quality assets (i.e. assets such as equities that are not eligible for liquid asset status under LCR requirements). Moreover, for consistency with LCR requirements, we focus on liabilities with a maturity of up to one calendar month, i.e. 22 business days. However, our general multi-period modelling framework can be applied to any arbitrary time horizon.

For overnight payments on unsecured loans, we observe bilateral obligations directly in the RRP dataset (Langfield, Liu & Ota, 2014). Empirically, a large fraction of unsecured loans have an overnight maturity: Table 1 shows that overnight unsecured loans represent just over one third of total unsecured loans over a one-month horizon. We therefore observe the bilateral nature of approximately one third of the payments on unsecured loans that occur during our liquidity stress scenario.

For subsequent payments on unsecured loans and for all repo payments, the $N \times T$ matrix imposes constraints on the free elements in the larger $N \times T \times (N-1)$ matrix. For cases in which a bank's cash outflow is known to be zero in aggregate from the FSA047 dataset, we can infer that all corresponding elements in the $N-1$ dimension of the matrix must also be zero. Owing to the relative sparseness of the interbank funding network, these constraints are binding in the vast majority of cases. In particular, 98.9% of the elements in the matrix of bilateral unsecured loan payments are known to be zero.

Similarly, 99.5% of the elements in the matrix of bilateral repo payments are known to be zero.

To further constrain the remaining free elements in the $N \times T \times (N-1)$ matrix, the RRP dataset contains information on interbank linkages with between two days and three months until maturity. These bilateral data set a maximum value on the row sum of the $T \times (N-1)$ matrix for each bank. Accordingly, they constrain each element in the $T \times (N-1)$ matrix to be no more than the total exposure of the counterparty bank vis-a-vis the reporting bank. Furthermore, we complement the bilateral data from the RRP dataset with a third dataset, namely FSA051, which contains information on bilateral exposures as well as their average maturity. This dataset indicates the existing bilateral relationships between banks and the maturity bucket in which these credit relationships come due on average. This information on maturity provides further guidance concerning the true inter-day interbank payments matrix.

Taken together, our datasets substantially constrain the elements in the observable $N \times T \times (N-1)$ matrix. For the remaining unconstrained elements, we deploy a modified maximum entropy algorithm which respects the constraints given by the aforementioned datasets. As Appendix B explains, the maximum entropy algorithm estimates all elements of a matrix from the vectors of column-sum and row-sum. Subject to these constraints and the additional constraints arising from our granular datasets, the algorithm spreads the elements as evenly as possible over the whole matrix as long as the elements are consistent with the column-sums and the row-sums. The use of maximum entropy to complete the inter-day matrices of liabilities implies that our results represent a lower bound on systemic illiquidity, given that maximum entropy tends to underestimate contagion effects.[7].

Since our use of maximum entropy in this context is substantially constrained, the extent to which our results concerning systemic illiquidity may be downwardly biased is minimal. As explained above, our combined constraints mean that approximately 99% of matrix elements are known to be zero. Therefore, the maximum entropy algorithm spreads the unobserved bilateral payments only among the remaining 1% of elements that are unconstrained. This largely prevents the algorithm from excessively smoothing

[7] See, for example, Mistrulli (2011); Anand, Craig & von Peter (2015); Anand, van Lelyveld, Banai, Friedrich, Garratt, Halaj, Fique, Hansen, Jaramillo, Lee, Molina-Borboa, Nobili, Rajan, Salakhova, Silva, Silvestri & de Souza (2017)

exposures across counterparties, which is the origin of the well-known underestimation problem associated with typical applications of maximum entropy.

One shortcoming of the data available to us is that they are collected from UK banks only. Consequently, our network does not cover connections with non-UK banks or non-banks. While our sample covers 91% of interbank unsecured loans, these represent just one-tenth of UK banks' unsecured borrowings from non-bank financial institutions. For repo, interbank transactions within the UK represent just 21% of transactions between one UK and one non-UK bank, since UK banks are reliant on foreign repo lenders for funding (Langfield et al., 2014). Due to these limitations, our results should be interpreted as a lower bound on the extent to which the interbank network is subject to systemic illiquidity.

3.4 Summary statistics

Table 1 summarises the unsecured loans and repo obligations among the 182 banks in our sample and over the 22 days of our liquidity stress scenario. On average, banks are the recipients of unsecured loans from 2.3 other banks and obtain repo funding backed by low-quality collateral from approximately 1.3 other banks. There is considerable cross-sectional variation around these averages, driven by a long right tail of banks that receive funding from many other banks. To provide a richer characterisation of network structure, we also compute a measure of strength, defined as the total value of a bank's payment obligations to all other banks. For unsecured loans, average strength is highest on day 1 at £25.5mn. Similarly, the largest single maturity bucket for repos is overnight at an average of £52.4mn per bank. In both cases, however, funding with a maturity longer than overnight represents the lion's share of short-term funding—namely 66% of unsecured loans and 80% of low-quality repo. This empirical insight underscores the empirical importance of our multi-period approach.

4 Results

4.1 Illiquidity in the observed interbank network

We first consider the case of liquid asset buffers as at the end of 2013. With these inputs, and in an adverse scenario in which banks are unable to roll-over their short-term wholesale funding, two banks would default owing to individual liquidity mismatches (as shown in Figure 1). No systemic illiquidity is present, however, since the two defaulting banks do not cause other banks to become illiquid during the 22-day period.

This result is not surprising, since the total size of the interbank funding network was only £64bn at the end of 2013—a fraction of the £724bn in liquid asset buffers held by banks. Under these conditions, most banks would withstand the adverse 22-day scenario envisaged by our simulations, and the only two banks that default in our first simulation do not generate any systemic illiquidity. Therefore, according to our model, liquidity contagion would not occur.

Nevertheless, we cannot safely conclude that the UK interbank system is immune to liquidity contagion. In 2013, UK banks' liquid asset holdings were well in excess of regulatory guidance (Bank of England, 2013). Before the crisis, however, UK banks held much lower levels of liquid assets, and interbank funding markets were more active (Figure 2). In addition, liquid asset holdings cover not only wholesale cash outflows but also other outflows such as retail deposits and margin calls, which are not modelled in this paper, but which are likely to further deteriorate the liquidity position of banks during a stress period. Taken together, these insights suggest that systemic illiquidity could be present when interbank lending is large relative to banks' liquid asset holdings, as was the case before the crisis.

4.2 Illiquidity in an enlarged interbank network

To shed further light on the sensitivity of systemic illiquidity, we model an interbank network in which the size of interbank lending is large relative to banks' liquid asset holdings. This can be done by increasing the size of interbank lending or reducing the liquid asset buffers (or both). When doing so, we multiply the true end-2013 interbank network and banks' liquid asset buffers by a constant scalar, so that network structure and the distribution of liquid asset buffers remain unchanged.

To replicate the pre-crisis situation with respect to liquid asset holdings and interbank markets, we set banks' liquid asset holdings at 50% of their end-2013 levels, and the size of the interbank funding network, defined as the sum of weighted links between all banks, at 300% of that which prevailed at the end of 2013. The simulation results under these assumptions are shown in Figure 3. Of the 182 sample banks, 12 default during the 22-day period due to individual liquidity mismatches. The default of these 12 banks generates substantial systemic illiquidity, as other banks do not receive scheduled payments from the individually illiquid banks. Consequently, the simulation reveals that a further 13 banks would default due to systemic illiquidity. In addition, three of the 12 individually illiquid banks would default earlier than otherwise due to systemic illiquidity.

Figure 4 illustrates a richer set of simulation results by varying the level of banks' liquid asset buffers as a percentage of end-2013 values, while keeping network size at 300% of the end-2013 level. Systemic illiquidity is present when liquid asset buffers are below two-thirds of end-2013 levels. However, the number of banks that default due to systemic illiquidity is not monotonically decreasing in the size of banks' liquid asset buffers, because banks may be individually illiquid when they have small liquid asset buffers. In these cases, systemic illiquidity can only hasten a demise that would have occurred anyway due to individual illiquidity.

Figure 5 shows systemic illiquidity as a function of both liquid asset buffers and network size. When banks have liquid asset buffers equal to end-2013 values, systemic illiquidity is not present even with increased network size, because few banks default due to individual liquidity mismatches and trigger contagion. By contrast, when banks' liquid asset buffers are below two-thirds of their end-2013 holdings, the number of additional and early defaults, and the proportion of the banking system in default, owing to systemic illiquidity is substantial.

4.3 Illiquidity with exogenous default

Next, we examine the impact of the failure of a major bank in the network. For this, we assume that a bank exogenously defaults on day 1. This is a plausible antecedent scenario in the sense that the default of a major bank could trigger widespread stress in the wholesale funding market. Analytically, it enables us to assess each bank's contribution to systemic illiquidity. The results of these simulations are shown in Figure 6 for individual exogenous defaults by the 19 banks with the highest impact scores. The figure shows that individual contributions to systemic illiquidity vary greatly across banks, and remain significant for a small subset of banks even when liquid asset buffers are equal to end-

2013 levels. Again, systemic illiquidity is not monotonically decreasing in the size of liquid asset buffers because vulnerable banks are more likely to be individually illiquid when liquid asset buffers are small.

4.4 Contributions to systemic illiquidity

What drives cross-sectional variation in the impact of the exogenous failure of individual banks? To explore this question, we estimate a linear regression model in which the proportion of the banking system in endogenous default (i.e. excluding the exogenously defaulted bank) is regressed on several potential determinants, the summary statistics of which are presented in Table 2. Most obviously, this includes *Ownimpact*, which is the impact score of bank i, i.e. the bank assumed to default at the beginning of the stress scenario. The failure of a bank with a greater impact score is expected to generate more systemic illiquidity, and indeed we estimate positive coefficients, significant at the 1% confidence level, in all specifications reported in Table 3. The magnitude of the estimated coefficient is such that, in the specification reported in column 6, a two standard deviation increase in *Ownimpact* is associated with a 1.4 standard deviation increase in the proportion of the banking system in endogenous default. The economic magnitude is therefore very large. This is intuitive: a bank's impact score is precisely intended to capture the importance of that bank vis-à-vis the rest of the banking system.

Interestingly, however, we find that *Ownimpact* is not the only variable of significance in explaining cross-sectional variation in the dependent variable. In particular, we find that $WeightedinstrengthLAB$ is statistically significant at the 1% confidence level. This variable measures the size of bank i's borrowing from bank j, scaled by bank j's liquid asset holdings and impact score.[8] $WeightedinstrengthLAB$ therefore captures the importance of bank i in the interbank funding network: if bank i is a large borrower from other banks relative to their holdings of liquid assets, and the banks from which bank i borrows have large impact scores, then bank i's effect on systemic illiquidity will be correspondingly greater. In the regression reported in column 6 of Table 3, a two standard deviation increase in $WeightedinstrengthLAB$ is associated with a 0.4 standard deviation increase

[8] Formally, $WeightedinstrengthLAB$ is defined as $\sum_j \frac{B_{ij}}{L_j} S_j$, where B_{ij} is the borrowing of bank i from bank j, L_j is the liquid asset holdings (after deducting net flows in the stress scenario) of bank j, and S_j is the impact score of bank j.

in the proportion of the banking system in endogenous default.[9] When bank i defaults, it fails to honour its obligations to other banks, for whom the non-payment is large relative to liquid asset holdings. This intuition helps to explain why we estimate a statistically and economically strong effect of $WeightedinstrengthLAB$ in explaining cross-sectional variation in the proportion of the banking system in endogenous default following bank i's failure, in addition to the explanatory role played by *Ownimpact*.

Unweighted network measures are statistically and economically less important. The coefficient of *Betweenness*, which measures the extent to which bank i lies "between" links among other banks, is estimated to be statistically insignificant is columns 4-6 of Table 3, and only mildly significant at the 5% confidence level in a univariate regression in column 2. Although betweenness captures a bank's centrality, it does not reflect the (relative) size of its activity in the interbank funding network, unlike *Ownimpact* and $WeightedinstrengthLAB$.

Finally, we turn to *Loginstrength*, which measures the total borrowing of bank i from other banks in the interbank funding network. As expected, we estimate a statistically significant positive coefficient of *Loginstrength* in the univariate regression reported in column 3 of Table 3. However, we estimate negative coefficients in columns 4-6. This reflects the positive correlation of *Loginstrength* with respect to variables such as *Ownimpact* and $WeightedinstrengthLAB$. Once the latter variables are controlled for, the estimated coefficient of *Loginstrength* switches sign. Although this may appear surprising, the economic magnitude of the predicted effect is negligible. The intuition is that the default of a particular bank would only generate significant systemic illiquidity if it borrows from banks that are vulnerable. As such, the size of interbank borrowing is a poor metric for banks' contribution to systemic illiquidity; more important is the fraction of "contagious links", which is better proxied by $WeightedinstrengthLAB$. This finding provides empirical support for the theoretical predictions of Amini, Cont & Minca (2016).

Wrapping up, the regression estimates reported in Table 3 suggest that it is how much a bank borrows *from whom*, rather than the size of interbank borrowing per se, that

[9] Comparison of columns 5 and 6 of Table 3 reveals that it is important to scale $WeightedinstrengthLAB$ by bank j's impact score. The estimated coefficient of a similar variable which does not scale by bank j's impact score, namely $instrengthLAB$, is statistically insignificant. Intuitively, bank i's borrowings from other banks might be large relative to their liquid asset holdings, but if those banks have low impact scores, the effect on the dependent variable will be limited.

determines a bank's contribution to systemic illiquidity, in addition to its own impact score. This insight motivates the next section, in which we conduct a policy experiment based on the joint distribution of interbank borrowing and liquid asset holdings.

5 Policy experiment

In this section we conduct a policy experiment. We ask: how should macroprudential liquidity requirements be designed so as to minimise systemic illiquidity? To motivate this question, we begin by recapping the rationale for liquidity regulation. Given that stringent aggregate liquidity requirements increase the cost of liquidity transformation for banks, the distribution of those requirements in the cross-section of banks is an important object of study. Through the lens of our multi-period simulation model, we show that a macroprudential distribution of liquidity requirements, in which contagion effects in the form of systemic illiquidity are taken into account, can improve upon the microprudential (uniform) distribution on which current regulation is based.

5.1 Rationale for liquidity regulation

The business model of banks entails substantial liquidity and maturity mismatch. Banks therefore have an inherent exposure to funding liquidity risk. While the presence of an interbank market allows banks to share idiosyncratic liquidity risks, the results presented in Section 4 indicate that the interbank market can also propagate liquidity shortfalls under extreme but plausible conditions. This externality, which we refer to as systemic illiquidity, can justify policy intervention with respect to banks' liquidity and maturity mismatches and their interbank market activities in particular.

Policymakers have a range of tools at their disposal to mitigate banks' liquidity risk. In the classical banking literature, common deposit insurance can prevent idiosyncratic bank runs (Diamond & Dybvig, 1983). However, deposit insurance reduces incentives for depositors to monitor banks' activities and for banks to self-insure against liquidity risk. In recognition of this moral hazard, most real-world deposit insurance schemes only cover retail deposits up to a certain threshold. This maintains the incentive of sophisticated wholesale depositors to exert discipline on banks' funding liquidity risk. The downside, however, is that a retail-only deposit insurance scheme cannot offer full protection against runs when banks are substantially funded by wholesale deposits.

The ultimate backstop against runs is the central bank, which has the capacity to act as lender of last resort (LOLR) by providing funding liquidity to illiquid-but-solvent institutions. LOLR facilities are effective in reducing systemic illiquidity, but like fully-fledged deposit insurance they distort incentives. Ex-ante, LOLR-eligible institutions anticipate that they will receive funding from the LOLR in the event of a liquidity shortfall, so they have a private incentive to take socially excessive funding liquidity risks (Acharya, Drechsler & Schnabl, 2014). Ex-post, LOLR-eligible institutions with low franchise value have an incentive to extract rent from the subsidy implicit in under-collateralised lending facilities by shifting downside credit risk onto the LOLR (Drechsler, Drechsel, Marques-Ibanez & Schnabl, 2016).

In theory, incentive problems associated with LOLR facilities could be mitigated by setting eligibility requirements or by providing liquidity only at a penalty rate (which can be calibrated to banks' credit risk or the market risk of securities that they provide as collateral). But penalty and risk-adjusted rates are not time consistent: given their mandate, central banks cannot credibly commit to a policy of benign neglect, with a restricted provision of liquidity, in the event of a systemic crisis. Even eligibility requirements can be loosened to provide quasi-banks with access to LOLR facilities during crises.

5.2 Microprudential liquidity requirements

To limit moral hazard, LOLR facilities should be economised upon. Enter liquidity regulation. New liquidity requirements—the LCR and NSFR—aim to decrease banks' reliance on LOLR facilities by decreasing banks' liquidity risks (Stein, 2013). These predominantly microprudential liquidity requirements help to offset the moral hazard generated by LOLR facilities as well as retail deposit insurance (Ratnovski, 2009; Cao & Illing, 2011; Farhi & Tirole, 2012). Moreover, universally applicable liquidity requirements overcome a free-rider problem, whereby an individual bank could otherwise avoid holding liquid assets thanks to the positive externality of systemic stability afforded by other banks' holdings (Ahnert, 2013).

Policymakers could require banks to hold as many liquid assets as necessary to negate banks' liquidity risks. However, holding excess quantities of liquid assets is likely to increase the cost of liquidity and maturity transformation for banks. Liquidity requirements should therefore be calibrated efficiently, such that banks' liquidity risks are minimised without unduly increasing the cost of liquidity and maturity transformation. In its current formulation, the liquidity coverage requirement requires banks to hold enough unencumbered high-quality liquid assets (with differential weights) to meet stressed outflows over one month (Basel Committee on Banking Supervision, 2013a). The net stable funding requirement requires banks to hold a certain quantity of stable funding (defined as customer deposits, long-term wholesale funding and equity) relative to long-dated assets (with differential weights by asset and maturity classes) (Basel Committee on Banking Supervision, 2014).

Both the LCR and the NSFR are essentially microprudential in nature, since they are calibrated according to individual institutions' liquidity risk, rather than institutions' contribution to systemic illiquidity. From a macroprudential perspective, however, the heterogeneous distribution of contributions to systemic illiquidity across banks could have implications for the optimal cross-sectional distribution of liquid assets. For example, it may be optimal ex-ante to require banks that are systemically important in the interbank network to hold relatively more liquid assets in order to minimise their contribution to systemic illiquidity. This cross-sectional macroprudential approach to calibrating liquidity requirements has been shown to be effective in theoretical settings by Perotti & Suarez (2011) and Aldasoro & Faia (2016) and has been considered for implementation by policymakers (Bank for International Settlements, 2010; European Systemic Risk Board, 2014; Clerc, Giovannini, Langfield, Peltonen, Portes & Scheicher, 2016; European Central Bank, 2018). We are the first to examine the benefits of a macroprudential approach to liquidity requirements in the framework of a contagion model applied to a real interbank funding network.

The LCR requires that total cash inflows are subject to an aggregate cap of 75% of total expected cash outflows. This requirement may mitigate systemic illiquidity, but only to a limited extent. For example, if a bank has wholesale inflows and outflows of £100 each, it would need to have a liquid asset buffer of at least £25. This means that the bank will have an adequate liquid asset buffer unless more than 25% of its inflows are defaulted upon. However, when the 75% cap does not bind, as is the case for most major UK banks, the bank only needs to have an adequate liquid asset buffer to cover net outflows, and may become illiquid if some of its expected inflows are not paid on time.

5.3 Macroprudential liquidity requirements

To evaluate the usefulness of macroprudential liquidity requirements in reducing systemic illiquidity for a given level of aggregate requirements, we consider the case in which the total amount of liquid asset buffers held by all banks in the system is constrained. Such a hard constraint does not exist in practice, but can be interpreted as a threshold above which it would be prohibitively costly for banks to hold additional liquid asset buffers. In this setting, the policy objective is to minimise the proportion of the banking system in default, with the constraint that total holdings of liquid assets are less than or equal to a given quantity.

Formally, in the context of our multi-period model with T business days, we find the constrained optimal ex-ante distribution of liquid asset buffers that is consistent with the six rules described in Section 2. In such a framework, the constrained-optimal liquid asset buffer $\boldsymbol{e}^*$ can be calculated by optimising:

$$\begin{aligned}
&\underset{\boldsymbol{e}^*}{\operatorname{argmin}} \sum_{t=1}^{T} 1 * \boldsymbol{s}^{*t}(\boldsymbol{e}) \qquad (5.1)\\
&\text{subject to}\\
&\sum_{t=1}^{T} 1 * \boldsymbol{e}^{*t} \leq D, \qquad \forall j, \forall t,\\
&p_j^T \leq e_j^T + \sum_{t=1}^{T} \sum_{i \neq j} p_i^t \Pi_{ij}^t, \qquad \forall j, \forall T,\\
&0 \leq p_j^{*t} \leq p_j^t, \qquad \forall j \forall t.
\end{aligned}$$

where $\boldsymbol{s}(\boldsymbol{e})$ represents the impact score vector calculated in function of the interbank payments in the system, and D is the constraint on total holdings of liquid assets. This model optimises the ex-ante distribution of liquid asset buffers in a multi-period framework by minimising the proportion of the banking system in default ex-post. For example, the optimal solution might indicate that giving more liquidity to a specific institution (at the expense of others) would be beneficial for overall network stability—as in the stylised case of a bank whose survival is imperative for the overall flow of liquidity within the system.

To see the intuition behind macroprudential liquidity requirements, consider the following. Suppose there are four banks in a network: Bank A, B, C and D. In this toy example, each bank would need a minimum liquid asset buffer of £100 to cover its individual liquidity mismatch over the 22-day period. In aggregate, a minimum of £400 of liquid assets, distributed evenly over the four banks, would be required to avoid illiquidity ex-post. For illustration, now suppose that the aggregate quantity of liquid assets is constrained at D=£320. At least one of the four banks must fail. When systemic illiquidity is not considered, it is optimal to let the bank with the lowest potential impact score (say Bank B) fail, given that the authority's objective is to minimise the total potential impact score of banks that would fail in the stress scenario. Because Bank B would fail anyway, it is optimal to set Bank B's liquid asset holdings at zero. The other banks would each be required to hold liquid asset buffers of £100, with the £20 surplus distributed arbitrarily. However, when systemic illiquidity is taken into consideration, the risk of liquidity contagion triggered by bank failures becomes important. Suppose Bank B is a substantial borrower in the interbank network, such that its failure would cause other banks to become illiquid. Then the authority may find it optimal to require Bank B to hold at least £100 ex-ante, and let another bank risk failure (say Bank C). The constrained-optimal distribution of liquid asset buffers thus depends on whether systemic illiquidity is taken into account.

In the context of our model, we use numerical methods to determine the optimal distribution of liquid asset buffers across banks that minimises the proportion of the banking system that would default in the 22-day liquidity stress scenario. That is, we iterate through all possible combinations of liquid asset distributions across banks subject to the constraint D, and calculate the total potential impact score of defaulted banks in each case. This procedure is computationally expensive but nevertheless feasible, since the number of possible combinations is finite: in the optimal liquid asset buffer distribution, a bank should hold just enough liquid assets to cover all potential outflows, or no liquid assets at all.[10] Under the macroprudential approach, the constrained-optimal

[10] Any excess liquid assets can then be distributed across banks according to some arbitrary rule (for example, in proportion to banks' constrained-optimal liquid asset buffers). The distribution of this surplus is irrelevant in the context of our simulation model.

liquid asset buffer distribution is that which minimises the total potential impact score of defaulted banks, taking into account any network effects which generate systemic illiquidity. To evaluate the social usefulness of this macroprudential approach, we compare it to a microprudential regime which is subject to the same aggregate constraint but ignores systemic illiquidity in defining the distribution of liquid assets.

When the interbank network is 300% of its size at the end of 2013 and aggregate liquid assets are constrained at £320bn, the optimal microprudential solution is to require one large UK bank to hold zero liquid assets. In this case, 28.8% of the banking system ends up in default. But when systemic illiquidity is taken into account in the macroprudential approach, the optimal solution is to require that bank to hold a positive liquid asset buffer, such that it survives the stress scenario, and instead require two other banks to hold zero liquid assets. Consequently, the fraction of the banking system in default reduces to 15.6%. The improvement from 28.8% to 15.6% reflects the benefit of the macroprudential approach, in which the contagion effects of systemic illiquidity are taken into account when calculating optimal liquid asset buffers in the cross-section of banks. Figure 7 plots this benefit for a range of liquid asset constraints. The macroprudential benefit is substantial (around 10% on average) when the constraint is between £40bn and £320bn. Beyond £398bn, the constraint is non-binding since the total stock of liquid assets is sufficient for all banks to survive the stress scenario. In this parameter region, there is no material difference between the two policy regimes.

In summary, a macroprudential calibration of cross-sectional liquidity requirements, taking account of systemic illiquidity, can unambiguously improve on microprudential requirements. This finding can motivate a re-calibration of existing liquidity requirements so that liquid asset holdings are skewed towards systemically important banks.

6 Conclusion

This paper studies UK banks' systemic illiquidity using a unique dataset on banks' liquid asset holdings, daily cash flows and bilateral payments in the short-term interbank funding network. We do so through the lens of a model that extends the single-period framework originally proposed by Eisenberg & Noe (2001) to a flexible multi-period payment system.

At the end of 2013, UK banks held historically high levels of liquid assets, and the interbank network had shrunk relative to its pre-crisis size. In this context, our multi-period model suggests that systemic illiquidity would be absent even in an extreme stress scenario that persists for one month. However, when we scale end-2013 liquid assets holdings and the interbank network to reflect their pre-crisis proportions, systemic illiquidity does emerge. These findings underscore the importance of post-crisis liquidity requirements in providing systemic stability.

In a further exercise, we identify systemically important banks whose failure would have a significant impact on other banks through liquidity contagion. Banks' systemic importance is weakly correlated with the size of their interbank lending or borrowing. Instead, banks' impact score and position in the interbank funding network is a more important determinant of systemic importance. This finding can inform optimal ex-post intervention. In crisis times, regulators are interested in protecting the financial system as a whole to avoid spillovers to the real economy. By identifying banks with the largest contributions to systemic illiquidity, our methodology allows policymakers to find optimal strategies for bail-out or targeted liquidity provision that minimise the cost of ex-post intervention.

Finally, we use our model and data to experiment with ex-ante policy design. In particular, we compare the extent of systemic illiquidity under two policy regimes: a microprudential regime, which does not take network effects into account when calculating banks' constrained-optimal liquidity requirements; and a macroprudential regime, which takes network effects into account. We find that the macroprudential regime delivers strictly superior results: systemic illiquidity is lower, and a smaller proportion of the banking system consequently fails, for all intermediate constraints on aggregate liquid asset holdings. This finding has immediate policy relevance as it supports the notion that skewing liquidity requirements towards systemically important banks can achieve greater systemic stability.

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Table 1: Summary statistics of the interbank funding network

	Funding from unsecured loans				Funding from low-quality repos			
	Degree		Strength (£mn)		Degree		Strength (£mn)	
Day	Mean	StDev	Mean	StDev	Mean	StDev	Mean	StDev
1	2.7	6.3	25.5	161.3	1.7	4.5	52.4	229.0
2	2.1	5.1	3.2	15.1	1.4	4.0	19.6	101.2
3	2.0	5.0	8.3	81.0	0.8	3.2	11.4	83.8
4	2.0	5.1	2.5	10.4	1.3	3.8	28.0	200.9
5	2.1	5.1	1.8	7.0	1.3	3.8	19.5	133.0
6	2.0	5.1	3.1	13.0	1.1	3.5	14.1	85.0
7	2.0	5.0	2.3	8.2	1.3	3.9	14.2	70.9
8	1.9	4.7	2.0	8.2	1.2	3.7	13.1	84.3
9	1.9	4.7	1.9	7.2	1.1	3.6	7.2	37.7
10	2.0	5.1	1.9	7.9	1.2	3.8	14.4	82.5
11	2.2	5.2	1.8	6.8	1.4	4.0	7.6	34.2
12	1.9	4.9	1.6	6.6	1.0	3.5	6.0	32.8
13	2.2	5.2	1.9	7.2	1.2	3.7	6.6	39.0
14	2.0	5.1	1.6	6.7	1.1	3.6	4.7	23.4
15	2.0	5.1	1.7	6.8	1.0	3.5	4.1	24.0
16	2.1	5.1	1.7	6.9	1.1	3.6	5.2	32.1
17	1.7	4.5	1.6	6.6	1.0	3.2	8.0	45.2
18	1.9	4.6	2.3	9.7	0.9	3.3	3.8	22.6
19	2.0	5.1	2.1	8.2	1.2	3.6	5.2	31.2
20	1.8	4.6	1.7	7.1	1.1	3.4	4.9	42.2
21	1.9	4.7	1.9	7.5	1.1	3.4	9.7	52.7
22	2.1	5.1	2.2	8.3	0.9	3.3	2.0	16.2

Note: Table shows summary statistics of standard network metrics for the interbank funding network over the 22 days of a liquidity stress scenario. The interbank funding network comprises funding from unsecured loans and funding from repos backed by low-quality assets (i.e. assets not eligible for inclusion in the liquid asset buffer under LCR requirements). Degree is defined as the number of other banks to which a bank has a payment obligation (i.e. the number of outward links connected to each node). Strength is defined as the total value of a bank's funding obligations to all other banks (i.e. the sum of outward link weights). The total number of banks in the network is 182.

Table 2: Summary statistics of regressors

	Mean	StDev	Min	Median	Max
$Impact$	0.000547	0.003175	0	0	0.021529
$Ownimpact$	0.004105	0.015004	0.0	0.000039	0.124050
$Betweenness$	152.2473	408.7897	0	0	2117
$Loginstrength$	0.000000005	0.00000003	0	0	0.0000002
$InstrengthLAB$	0.006528	0.025667	0	0	0.270344
$WeightedinstrengthLAB$	0.006874	0.031893	0	0	0.341717

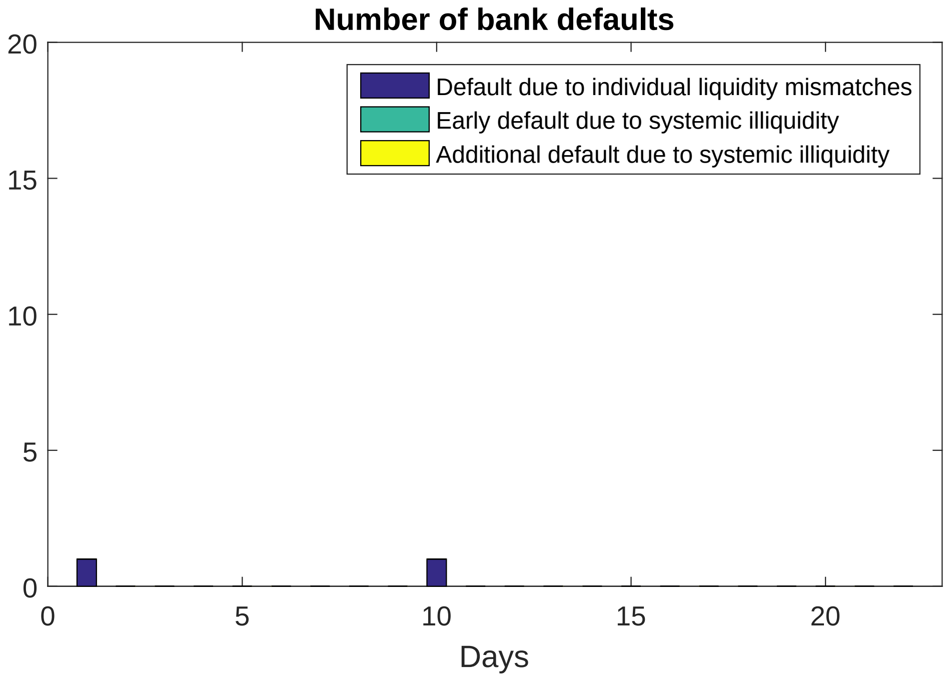
Note: Table shows summary statistics of regressors in Table 3. $Impact$ is the dependent variable in Table 3, namely the proportion of the banking system in endogenous default following the exogenous default of each bank i. $Ownimpact$ is the impact score of bank i. $Betweenness$ is the betweenness centrality of bank i. $Loginstrength$ is the logarithm of the total borrowing of bank i from other banks in the network. $InstrengthLAB$ for bank i is defined as $\sum_j \frac{B_{ij}}{L_j}$, where B_{ij} is the borrowing of bank i from bank j, and L_j is the liquid asset holdings (after deducting net flows in the stress scenario) of bank j. $WeightedinstrengthLAB$ for bank i is defined as $\sum_j \frac{B_{ij}}{L_j} S_j$, where S_j is the impact score of bank j.

Table 3: OLS regression estimation to explain the proportion of the banking system in default following the exogenous default of each bank i

	(1)	(2)	(3)	(4)	(5)	(6)
$Ownimpact$	0.183***			0.180***	0.175***	0.152***
	(0.026)			(0.023)	(0.022)	(0.021)
$Betweenness$		39.404**		8.062	7.776	5.570
		(16.004)		(8.474)	(8.226)	(8.280)
$Loginstrength$			10.962**	-3.335**	-4.326***	-3.730***
			(4.397)	(1.439)	(1.581)	(1.422)
$InstrengthLAB$					0.011	
					(0.007)	
$WeightedinstrengthLAB$						0.021***
						(0.008)
R^2	0.751	0.257	0.089	0.761	0.768	0.781
N	182	182	182	182	182	182

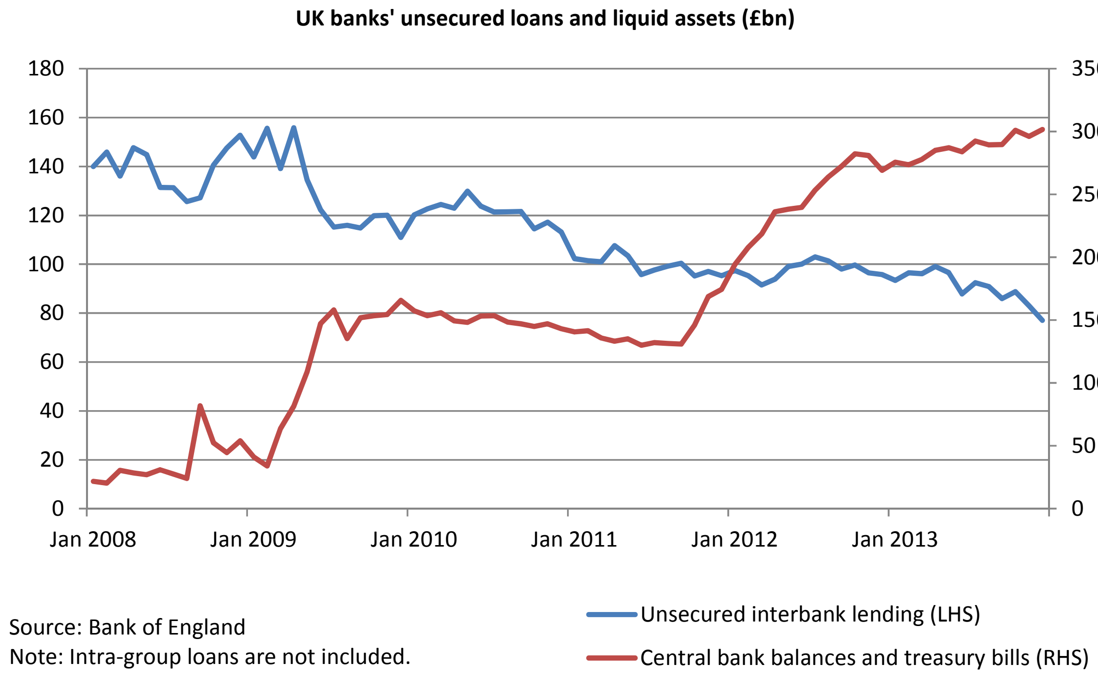
Note: The dependent variable is the additional proportion of the banking system in default following the exogenous default of each bank i. *Ownimpact* is the impact score of bank i. *Betweenness* is the betweenness centrality of bank i. *Loginstrength* is the logarithm of the total borrowing of bank i from other banks in the network. $InstrengthLAB$ for bank i is defined as $\sum_j \frac{B_{ij}}{L_j}$, where B_{ij} is the borrowing of bank i from bank j, and L_j is the liquid asset holdings (after deducting net flows in the stress scenario) of bank j. $WeightedinstrengthLAB$ for bank i is defined as $\sum_j \frac{B_{ij}}{L_j} S_j$, where S_j is the impact score of bank j. Network size is assumed to be 300% of that which prevailed at the end of 2013, and liquid asset holdings are 70% of banks' end-2013 holdings. Standard errors are shown in brackets. *, ** and *** indicate statistical significance at the 10%, 5% and 1% levels respectively. Constants are estimated but not reported.

Figure 1: Number of bank defaults with end-2013 liquid asset buffers



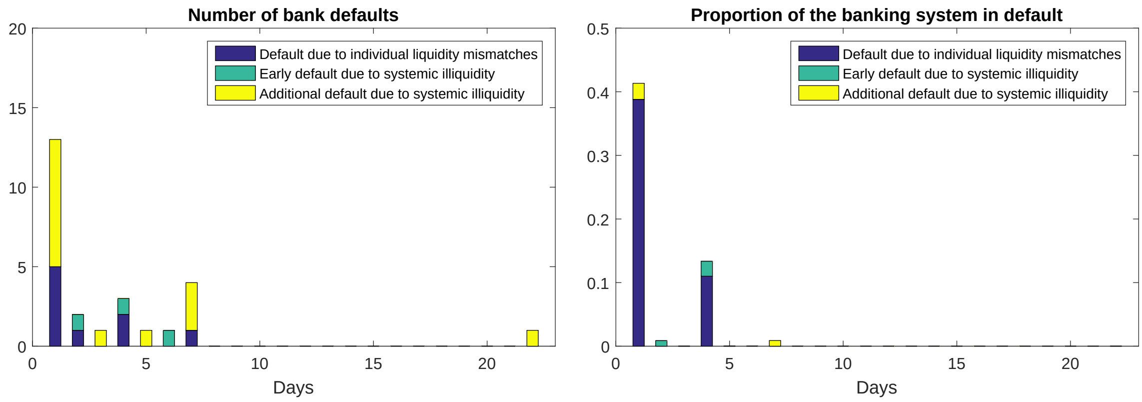
Note: Figure shows the number of bank defaults due to illiquidity in a hypothetical stress scenario in which short-term wholesale funding is not rolled over for a period of 22 business days. In this scenario, and with liquid asset buffers as of end-2013, two banks would default due to illiquidity: one after the first day, and the second after the tenth day of stress. These banks would default due to their own liquidity mismatch, without generating any systemic illiquidity for other banks.

Figure 2: UK banks' unsecured interbank loans and holdings of central bank balances and treasury bills



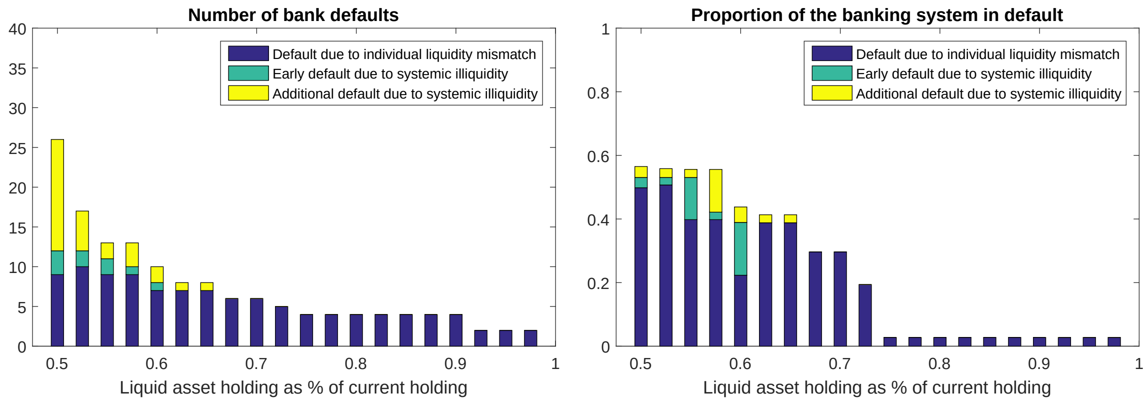
Note: Figure shows UK banks' outstanding unsecured loans (blue line, left axis) and holdings of central bank balances and treasury bills (red line, right axis) as of end-2013. In the figure, unsecured interbank loans include all maturity buckets; this category is therefore larger than the subset of banks' unsecured interbank loans with a horizon of 22 working days. The simulations in the paper focus only on the latter category (plus repos). Holdings of central bank balance and treasury bills represents a narrower definition of high-quality liquid assets than that which is used for regulatory purposes. Aggregate central bank balance and treasury bill holdings of approximately £300bn as of end-2013 therefore represents a fraction of UK banks' total liquid asset buffers, which amounted to £724bn as of end-2013.

Figure 3: Bank defaults by day with 50% liquid asset buffers and 300% network size



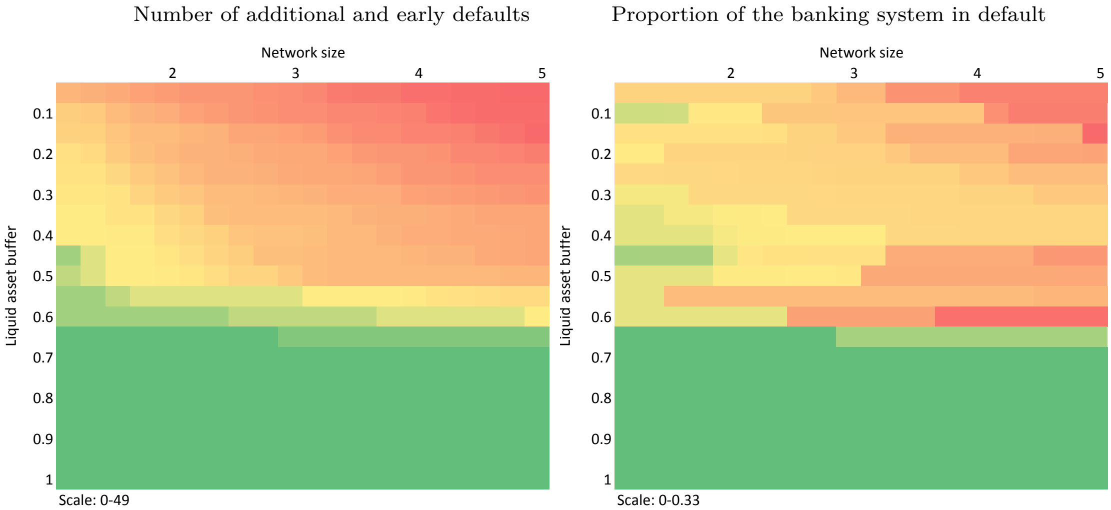
Note: Figure shows the number of bank defaults due to illiquidity (left panel) and the proportion of the banking system in default (right panel) in a hypothetical stress scenario in which short-term wholesale funding is not rolled over for a period of 22 business days. In this scenario, and with liquid asset buffers at 50% of their end-2013 levels and short-term interbank lending at 300% of its end-2013 level, 25 of the 182 sample banks would default. Of these 25 defaulting banks, 9 would default due to individual liquidity mismatch, 3 would default earlier than otherwise due to systemic illiquidity, and 13 would default due to systemic illiquidity (even though they would be individually liquid if other banks had not defaulted). The picture looks different when measuring the proportion of the banking system, rather than the number of banks, in default, as we do in the right panel. Here, we see that the five individually illiquid banks on day one represent nearly 40% of the banking system, and the two individually illiquid banks on day four represent just over 10% of the banking system. These defaults cause an additional c.5% of the banking system to default, or default earlier than otherwise, due to systemic illiquidity.

Figure 4: Total bank defaults by liquid asset buffers with 300% network size



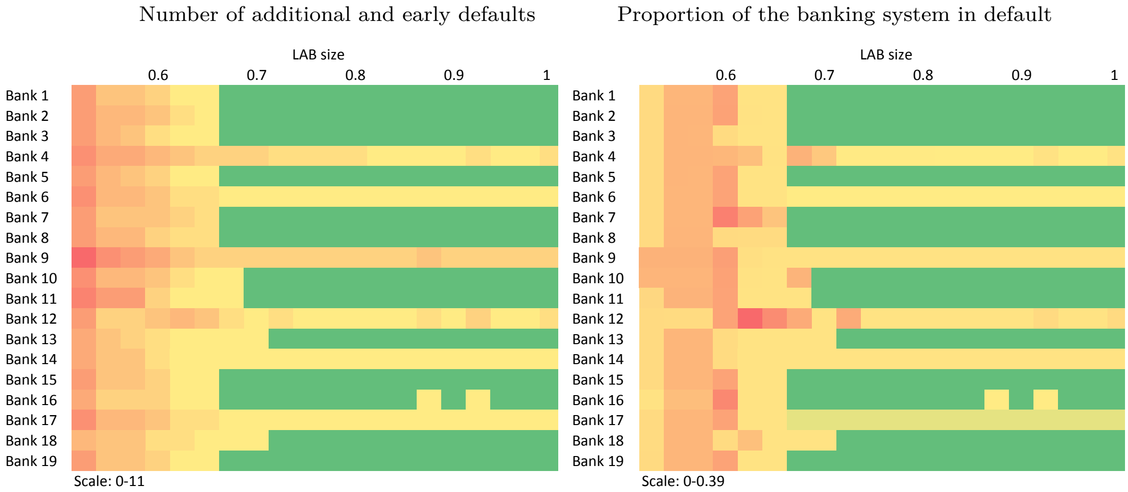
Note: Figure shows the number of bank defaults due to illiquidity (left panel) and the proportion of the banking system in default (right panel) in a hypothetical stress scenario in which short-term wholesale funding is not rolled over for a period of 22 business days. In this scenario, and with short-term interbank lending at 300% of its end-2013 level, the figure plots the number of bank defaults and proportion of the banking system in default as a function of liquid asset holdings. 50% of liquid asset holdings corresponds to the sum of defaults over the 22-day stress horizon depicted in Figure 3.

Figure 5: Systemic illiquidity as a function of liquid asset buffers and network size



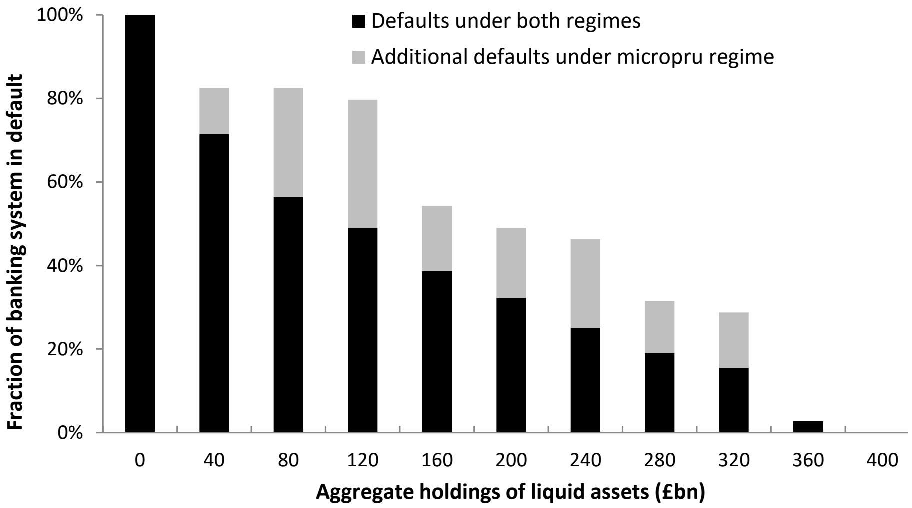
Note: Figure shows the number of bank defaults due to systemic illiquidity (left panel) and the proportion of the banking system in default (right panel) in a hypothetical stress scenario in which short-term wholesale funding is not rolled over for a period of 22 business days. In this scenario, the figure plots the number of bank defaults and proportion of the banking system in default as a function of liquid asset holdings (vertical axis) and size of the network of interbank lending as a multiple of end-2013 network size (horizontal axis). Red/green squares indicate a higher/lower number of early and additional defaults (in the left-hand panel) and proportion of the banking system in default (in the right-hand panel) owing to systemic illiquidity, according to the scale at the bottom of the matrix. For example, a scale of 0-0.33 means that the greenest square has a value of 0 and the reddest square has a value of 0.33.

Figure 6: Systemic illiquidity from exogenous defaults by size of liquid asset buffers



Note: Figure shows the number of bank defaults due to systemic illiquidity (left panel) and the proportion of the banking system in default (right panel) in a hypothetical stress scenario in which short-term wholesale funding is not rolled over for a period of 22 business days, and a given bank exogenously defaults on the first day. In this scenario, the figure plots the number of bank defaults and proportion of the banking system in default as a function of liquid asset holdings (horizontal axis). Red/green squares indicate a higher/lower number of early and additional defaults (in the left-hand panel) and proportion of the banking system in default (in the right-hand panel) owing to systemic illiquidity, according to the scale at the bottom of the matrix. For example, a scale of 0-0.39 means that the greenest square has a value of 0 and the reddest square has a value of 0.39. In this figure, network size is kept at 300% of that which prevailed at the end of 2013.

Figure 7: Proportion of banking system in default with microprudential vs macroprudential liquidity requirements



Note: Figure shows the proportion of the banking system in default with microprudential vs macroprudential liquidity requirements as a function of aggregate holdings of liquid assets. The black bar refers to defaults that occur under both microprudential and macroprudential policy regimes. The grey bar refers to additional defaults that occur under the microprudential regime, but which do not occur when liquidity requirements are calibrated macroprudentially. The grey bar can therefore be interpreted as the incremental benefit of the macroprudential approach, which takes systemic illiquidity into account when defining cross-sectional liquidity requirements. In this simulation, the interbank funding network is set at 300% of its end-2013 size.

Appendix

A Framework

As stated in Eisenberg & Noe (2001), a clearing payment vector should satisfy three conditions. First, it should be consistent with *limited liability*, which requires that the total payments made by a node at time t must never exceed its available resources at time t. Second, it should respect *absolute priority*, which requires that, if a node i does not default at time t, then it pays its liabilities in full. Third, *proportionality* requires that if default occurs, all claimant nodes are paid by the defaulting node in proportion to the size of their nominal claims. In a context in which one firm is indebted to another firm or firms, these rules always clearly specify a unique division of value between the debtor and creditor nodes.

Definition 1. At each time t a clearing payment vector p_t^* for a dynamic financial network $(\Pi_t, e_t, \bar{p}_t, t)$ is a vector $p_t^* \in [\mathbf{0}\,, \bar{p}_t]$ representing each bank's payment at time t satisfying the following criteria:

(a) *Limited liability:* At each $i \in \mathbb{N}$,

$$p^* \leq \Pi_t\, p_t + e_t \,.$$

(b) *Absolute priority:* At each $i \in \mathbb{N}$, if a node i does not default at time t, then obligations are paid in full. If it defaults before time t, it does not make any payment to its creditors.

(c) *Proportionality:* At each $i \in \mathbb{N}$, if a node i does default at time t, all debtors are paid in proportion to the size of their nominal claim.

The network is considered deterministic, where all debt claims have equal priority.

A.1 Network architecture

There is a fixed and finite set of nodes $N = \{1, 2, 3, ..., n\}$, with $n > 3$. We can consider all subsets S of N such that $S \subseteq N$ and $S = 0$. The direct liabilities of each node are to the nodes to which the entity has contractual obligations.

The *absolute priority* boundary also states that defaults should be avoided if possible. Clearly, no default can be absorbed by a bankrupt node of the system. Instead, this shock will be transferred to other entities in the network. The ability of a node to fulfil its creditor obligations depends on its available resources. In particular, the realised payments by the node to its creditors depend not only on its liabilities, but also on the realised value of payments by the node's debtors.

More formally, let $p_{i,j,1}$ denote the payment by node i on its debt to node j at t. By definition, $p_{i,j,t} \in [0, \bar{p}_{i,j,t}]$. The total payment of node i at time t is then equal to

$$p_{i,t} = \Pi_t^T p_t + e_{i,t}, \qquad \forall i \in N, \tag{A.1}$$

where $e_{i,t}$ is the initial liquid asset buffer owned by node i at time t.

If $e_{i,t}$ is larger than node i's total liabilities at time t then it is capable of meeting its obligations in full. On the contrary, if $e_{i,t}$ is less than total liabilities, node's i creditors are repaid less than $\bar{p}_{i,t}$. Note that whenever the node is unable to meet its obligations in full, the *proportionality* criterion will be applied to calculate the clearing vectors.

A.2 Existence and uniqueness of clearing vectors (Equation 2.3)

The criteria that must be satisfied in our problem are *limited liability*, *absolute priority* and *proportionality*. To establish the existence of a clearing vector, a fixed-point characterisation $\boldsymbol{p^*}$ of clearing vectors is required. The *limited liability* criterion implies that $\boldsymbol{p^*}$ is a clearing vector if and only if the following condition holds:

$$p_i^* = min\Big[0,\ \Pi_t^T\, p_t + e_t\Big], \qquad \forall i \in N, \tag{A.2}$$

where the second term in the minimum expression represents the total liquid asset buffer held by the i-th node at time t. Furthermore, a payment vector cannot be negative. To prove the existence and uniqueness of clearing vectors, we need to introduce a few more notions.

Definition 2. Let $\Upsilon : [\mathbf{0}, \bar{p}] \rightarrow [\mathbf{0}, \bar{p}]$ be a function that maps $[\mathbf{0}, \bar{p}]$ onto itself. The clearing vector is a fixed point of $p_i^* \in P$, on the map Υ, defined by

$$\Upsilon(p_t;\, \Pi_t,\, \bar{p}_t, t) = (\Pi_t^T p_t + e_t) \wedge \bar{p}_t \,. \tag{A.3}$$

From the above definition, it can be seen that the clearing vector is a fixed point, $p_{i,t}^*$, of the function $\Upsilon(\,\cdot\,;\, \Pi_t,\, \bar{p}_t) : [\mathbf{0}, \bar{p}_t] \rightarrow [\mathbf{0}, \bar{p}_t]$. Economically, $\Upsilon(\,\cdot\,;\, \Pi_t,\, \bar{p}_t)$ can be seen as the total amount of money that should be paid in the system $(\,\cdot\,;\, \Pi_t,\, \bar{p}_t)$, assuming that all participants receive inflows specified by $\bar{p}_t$ from their debt claims on other nodes. Using the above definition, one can write the following theorem:

Theorem 1. There exists at least one fixed clearing vector p_i^* for each t.

Proof. To prove this theorem, it should be noticed that Equation A.3 is monotonically increasing because Υ is the composition of two monotonic increasing functions. Let $A = \{x : x \leq \Upsilon(x)\}$. If $x \in A$, then $f(x) \in A$, because if $x \leq f(x)$ then $f(x) \leq \Upsilon(\Upsilon(x))$ due to Υ being order preserving. Now suppose that $y = sup\, A$. Then if $x \in A$ we have $x \leq y$ and so $\Upsilon(x) \leq \Upsilon(y)$. But $x \leq \Upsilon(x)$ by hypothesis of $x \in A$, so we have $x \leq \Upsilon(x) \leq \Upsilon(y)$. Therefore $\Upsilon(y)$ is also an upper bound of A and thus, because y is the least upper bound, $y \leq \Upsilon(y)$ and thus $y \in A$. But then we must also have $\Upsilon(y) \in A$, and thus $\Upsilon(y) \leq y$. Therefore $\Upsilon(y) = y$. $\square$

The problem of non-uniqueness of clearing vectors in a financial network was first pointed out by Eisenberg & Noe (2001). They showed that cases exist in which clearing vectors are not unique. Suppose the system contains two nodes, 1 and 2, both without any liquid asset buffer, i.e. $\mathbf{e} = (0, 0)$. Moreover, each node has nominal liabilities of 4.00 to the other node, $\bar{\mathbf{p}} = (4.00, 4.00)$. In this example, the flow of payments that goes from node 1 to node 2 depends only on the payments that node 1 receives from node 2 and vice versa. Therefore, they can reimburse each other with any payment between zero and 4.00.

The origin of this non-uniqueness problem lies in the joint and simultaneous determination of the losses. If a set of defaulting nodes is cyclically[11] connected, the losses that these nodes pass to one another are cyclically interdependent, and their insolvency

[11] A cycle in a directed graph is a path such that its start node and end node coincide.

functions are simultaneously determined, like in the above example of two institutions. This simultaneity can generate the problem of non-uniqueness. Such a simultaneity does not arise at all if the propagation unfolds only along directed paths,[12] as is the case in our framework.

Lemma 1. *A propagation in N is uniquely defined if it does not embed any cycle flow.*

Proof. Given a fixed and finite set of nodes $N = \{1, 2, 3, ..., n\}$, with $n > 3$, we can consider all subsets S such that there exists a directed path from any node to any other. Let $\bigcup S$ be the union of such subsets. In the absence of cycles of defaulting agents, $p_{i,t}$ is obtained through the non-recursive iteration of uniquely defined functions. Thus, they are uniquely defined as well. □

The next theorem provides conditions under which the fixed point of the mapping is unique for the case in which all nodes in the system have positive resources to distribute.

Theorem 2. If all nodes have an operating liquid asset buffer greater than their liabilities at any time t, then the clearing vector is unique.

Proof. This theorem follows directly from the definition of our optimisation problem. □

Theorem 2 characterises a sufficient but not necessary condition for uniqueness. For large exogenous income in which $e_t > \bar{p}_t$, then $\bar{p}_t$ is the only possible clearing vector.

Now consider a system with two nodes, 1 and 2, with an operating liquid asset buffer vector $\mathbf{e} = (0.1, 0)$ and time horizon $T = 1$. Each node has nominal liabilities of 4.00 to the other node, i.e. $\bar{\mathbf{p}} = (4.00, 4.00)$. In this example, the flow of payments that goes from node 1 to node 2 will be 4.00, otherwise node 1 would not satisfy the *absolute priority* criterion because node 1 would keep a positive balance greater than 0.1 without satisfying its nominal obligation to node 2. For this reason, under the assumption of limited liability of equity and absolute priority of debt, Eisenberg & Noe (2001) demonstrate that, for a clearing payment vector to be uniquely defined, it is sufficient that all sets of descendants for each node are *surplus sets*.

Another important factor in establishing uniqueness is the structure of our network. In the previous paragraph, we have seen that the cyclical interdependence of obligations

[12] A directed path is a sequence of nodes, with a start node and an end node, such that for any two consecutive nodes, i and $i + 1$, there is a link (only a link) going from i to $i + 1$.

may render indeterminate the flow of losses that is passed among such nodes. Conversely, a directed graph does not generate any indeterminacy. The next theorem shows another path to establish the existence of a unique clearing vector.

Theorem 3. Let the financial network be such that the subsets S composed by the non-defaulted nodes with $e_i > 0$ have a directed path at all times. Then the sequence p_t is uniquely determined.

Proof. The proof of this result can be found in Eisenberg & Noe (2001) (Proof of Theorem 2), since we simply need to verify the uniqueness for each period of time separately. □

A.3 Existence and uniqueness of clearing vectors (Equation 5.1)

To derive the clearing vector, we iteratively apply a script similar to the algorithm introduced by Eisenberg & Noe (2001). Given a dynamic network $(\Pi_t, e_t, \bar{p}_t, t)$, a time sequence of payments is a clearing vector if it satisfies Definition 1. The clearing payment vectors can be identified by solving the programming problem that places weight on maximising the expected total sum of payments by all nodes over the time horizon subject to our criteria. Therefore, we maximise the volume of repaid network liabilities. This formulation also ensures proportional payments for every feasible solution. Priority of debts results trivially from the maximisation. As above, limited liability is ensured by the last constraint. Thus, any clearing vector must be a feasible solution to the linear programme above, and vice versa. The uniqueness of the solution to the linear programme is provided by the following theorem.

Theorem 4. The linear programme has a unique optimal solution.

Proof. Using a fixed point argument, Eisenberg & Noe (2001) establish the uniqueness of the clearing vector under a restrictive assumption, namely the absence of connected subgraphs of defaulting agents with null liquid asset buffer that honour their obligations. We have seen this case with the example of a financial network discussed in Lemma 1.

To show that there exists a unique solution p_t^* to the optimisation problem, first note that all nodes are assumed to have a positive operating liquid asset buffer at any time t. Then we have already shown in Theorem 3 that the linear programme has a unique solution. □

However, since we are maximising the total volume of repaid network liabilities over time (and not the total payment in each period of time separately), the algorithm needs to start from a generic candidate payment vector p_T (using backward induction) to produce a sequence of plausible payment vectors, $p(t)$, until convergence is reached in fewer than $N*t$ iterations (i.e. the number of nodes in the network times the number of periods). Solving the multi-period problem requires solving a set of single-period problems at different points in time. Therefore, it requires us to keep track of how the decision situation evolves over time. For example, to decide how much to liquidity is necessary at each point in time, the authority needs to know the impact of their decision across time since future payments could depend on it. More generally, the next period is affected by the obligations of the institutions and the available liquidity. For example, in the simplest two-period case, today's payment and liquidity might exactly determine tomorrow's available liquid asset buffer. Our programming approach identifies the optimal solution by finding a rule that defines the liquid asset buffer given any possible value of the impact resulting from different choices. Thus, each period's ex ante liquidity is made by explicitly acknowledging that all future payments will be optimally made by using all available resources.

B Maximum entropy

We use the maximum entropy algorithm to estimate the networks day-by-day. The algorithm estimates all elements of a matrix from the vectors of column-sum and row-sum. The degree of freedom when the algorithm estimates a $N \times N$ square matrix is $N \times N - 3N$, since the diagonal elements of the matrix are known to be zero, and the number of elements to be estimated is $N \times N - N$. Therefore the algorithm spreads the elements as evenly as possible to the whole matrix as long as the elements are consistent with the column-sums and the row-sums. Since we use the algorithm to estimate a (number of lenders) $\times$ (maturities up to a calendar month) matrix, the algorithm tries to spread the liabilities evenly throughout the maturity buckets, as long as these are consistent with the borrowing bank's cash flow schedule.

Günlük-Senesen & Bates (1988) propose an entropy-based bi-proportional technique for estimating a matrix consistent with its exogenously specified row and column totals. In related work, Temurshoev (2012) and Temurshoev, Miller & Bouwmeester (2013) show

how to improve this technique for balancing matrices with both positive and negative elements. Let A and X be, respectively, the prior and the posterior matrices with their typical elements a_{ij} and x_{ij}. Then we define the ratio $z_{ij} = x_{ij}/a_{ij}$ which should be equal to 1 if $a_{ij} = 0$. The optimisation problem is the following:

$$\min_{z_{ij}} \sum_{ij} |a_{ij}| z_{ij}\ ln\left(\frac{z_{ij}}{2.71828}\right) \tag{A.1}$$

subject to

$$\sum_j a_{ij} z_{ij} = u_i, \qquad \forall i, \tag{A.2}$$

$$\sum_i a_{ij} z_{ij} = v_j, \qquad \forall j. \tag{A.3}$$

for a given row and column totals u_i and v_j.

Then, we decompose the prior matrix as:

$$A = P - N$$

where P contains the positive elements of A and N contains the absolute values of the negative elements of A. Then the solution of our optimisation problem is:

$$x_{i,j} = r_i a_{ij} s_j, \qquad \forall a_{i,j} \geq 0 \tag{A.4}$$

$$x_{i,j} = r_i^{-1} a_{ij} s_j^{-1}, \qquad \forall a_{i,j} \leq 0 \tag{A.5}$$

where $r_i > 0$ and $s_j > 0$. These multipliers are derived from the following quadratic equations, which are obtained by plugging (A.4) and (A.5) into the constraints (A.2) and (A.3):

$$p_i(s) r_i^2 - u_i r_i - n_i(s) = 0, \tag{A.6}$$

$$p_j(r) s_j^2 - v_j s_j - n_j(r) = 0, \tag{A.7}$$

where the coefficients are defined as:

$$p_i(s) = \sum_j p_{ij} s_j \tag{A.8}$$

$$p_j(r) = \sum_i p_{ij} r_i \tag{A.9}$$

$$n_i(s) = \sum_j \frac{n_{ij}}{z_j} \tag{A.10}$$

$$n_j(r) = \sum_i \frac{n_{ij}}{r_i}. \tag{A.11}$$

Assuming that $p_i(s)$ and $p_j(r)$ are both equal to *0*, one may derive the following multipliers:

$$r_i = \begin{cases} \frac{u_i + \sqrt{u_i^2 + 4p_i(s)n_i(s)}}{2p_i(s)} & \text{for } p_i(s) > 0 \\ -\frac{n_i(s)}{u_i} & \text{for } p_i(s) = 0 \end{cases} \tag{A.12}$$

$$s_j = \begin{cases} \frac{v_j + \sqrt{v_j^2 + 4p_j(r)n_j(r)}}{2p_j(r)} & \text{for } p_j(s) > 0 \\ -\frac{n_j(s)}{v_j} & \text{for } p_j(s) = 0 \end{cases} \tag{A.13}$$

One of the attractive features of the maximum entropy algorithm is the availability of its analytical solution that can be easily used in an iterative procedure. Therefore, there is no need to use high-performance computers to estimate the matrix of liabilities for each bank day-by-day represented by X.

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