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Exposure to international crises: trade vs. financial contagion

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Abstract

I identify new patterns in countries' economic performance over the 2007-2014 period based on proximity through distance, trade, and finance to the US subprime mortgage and Eurozone debt crisis areas. To understand the causes of the cross-country variation, I develop an open economy model with two transmission channels that can be shocked separately: international trade and finance. The model is the first to include a government and heterogeneous firms that can default independently of one another and has a novel endogenous cost of sovereign default. I calibrate the model to the average experiences of countries near to and far from the crisis areas. Using these calibrations, disturbances on the order of those observed during the late 2000s are separately applied to each channel to study transmission. The results suggest credit disruption as the primary contagion driver, rather than the trade channel. Given the substantial degree of financial contagion, I run a series of counterfactuals studying the efficacy of capital controls and find that they would be a useful tool for preventing similarly severe contagion in the future, so long as there is not capital immobility to the degree that the local sovereign can default without suffering capital flight.

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1 Introduction

Economic crises often spread across international borders, with a crisis in one country leading to significant economic disruptions for its trade and finance partners. In recent decades, emerging economies have experienced many such periods of crisis contagion, from the 1982 Latin American debt crisis to the 1994 Tequila Crisis, the 1997 Asian Flu, and the 1998 Russian Virus. However, none of these periods of contagion matched the magnitude of the late 2000s global crisis period. During that period, issues originating in the US sub-prime mortgage and Eurozone debt markets spread globally with world real GDP growth slowing from 5.4% in 2007 to 2.8% in 2008 before contracting 0.6% in 2009.[1]

The late 2000s highlighted the importance of understanding how economic crises propagate between nations, given the greater impact that these crises had when they originated in the world's two largest economies and then spread globally. This paper identifies novel patterns over the 2008-2014 period where countries that were more remote from the two crisis areas — when measured by distance, trade, or financial ties — experienced significant recoveries above pre-crisis trend output levels, while countries relatively closer continued to fall farther below trend. Motivated by these findings, the goal of this paper is to study the cross-country variation of the crisis contagion using a model that allows for the decomposition of the two primary economic transmission channels: cross-country trade and financial linkages. The results of separating these two channels indicate that: the trade channel was responsible for a minor portion of the crisis transmission, with similar magnitudes for near and far countries; financial transmission was minimal — and positive — for far countries; and the finance channel was the primary contagion channel for near countries. Finally, given the importance of the finance channel for those countries most affected, I model implementing capital controls and find that they would have been a useful policy for near countries to mitigate contagion.

There exists a large empirical literature on the factors driving cross-country contagion, and in particular the importance of bilateral trade and financial relationships for transmission. Kaminsky and Reinhart (2000)'s contagion definition is the one generally used in the empirical literature: "a case where knowing that there is a crisis elsewhere increases the probability of a crisis at home," with the incidence of a crisis found to increase with greater trade and financial relationships with a partner in crisis. Using this definition, their paper — along with others such as Kaminsky and Reinhart (1999), Hernandez and Valdes (2001), De Gregorio and Valdes (2001), Kali and Reyes (2010), Forbes (2012), and Forbes and Warnock

[1] Data from the International Monetary Fund's *World Economic Outlook Database April 2013*.

(2012) — finds empirical evidence of cross-country trade and finance as significant contagion channels for several types of economic crises during the 1980s-2000s.

Given the results of this literature, it was a puzzle when Rose and Spiegel (2010) performed similar analysis for the 2008 Global Financial Crisis and found that greater exposure to the United States did not increase the crisis incidence of other countries. In fact, they found that, if anything, greater exposure to the United States corresponded with greater growth.

I extend the focus of this research beyond the initial 2008 crisis year to find that, while the experiences of countries near to and far from the United States and Eurozone aggregate crisis zone were similarly severe in that year, there was significant variation across countries as the crises and subsequent recoveries developed from 2009-2014.[2] As shown in Section 2, proximity to the crisis areas was negatively related to countries' economic performance over this period, with closeness measured by any of pre-crisis trade, financial linkages, or distance with the United States and Eurozone. From 2010-2014, countries that were near to the crisis zone had real GDP per-capita that was on average below trend by a statistically significant amount, whereas countries farther from the crisis zone did not. In fact, countries that were farther from the crisis zone had on average risen above their trend real GDP per-capita levels by 2012, while those nearer to the crisis areas continued to experience declines versus their previous trends. These results imply that after the initial crisis years, cross-country trade and financial relationships regained their significance as crisis transmission channels.

Given the high correlations between trade and finance in the data, the roles of the two in crisis transmission cannot be reliably linearly decomposed using empirical methods.[3] Because of this fact, I study the importance of contagion through these two transmission channels using a model of international trade and finance that augments the standard small open economy setup and allows for separate disturbances to the trade and finance channels. In standard models, the trade and finance channels are normally closely tied together. For example, a negative production shock adversely affects trade while also decreasing asset prices and balances sheets, causing an adverse financial shock. Thus, the ability to separate the trade and financial transmission channels is a valuable contribution of the model for the purpose of evaluating their relative importance in transmitting the crises and affecting

[2] The crisis zone is defined as the US and Eurozone entering 2007 (Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain). Given the overlap between the US sub-prime and Eurozone debt crises, and the lack of a clear delineation between the two's global effects, I model them as one large late 2000s crisis period that is passed on to the rest of the world from the aggregate crisis zone.

[3] See Kaminsky and Reinhart (2000) for a discussion of this issue.

countries' recoveries over the 2008-2014 period.

The separation of the parameters governing shocks to these channels does not mean that there is no interaction between these two aspects of the economy. The important feature is that the model parameters for each channel can be independently matched to the data and adjusted to see the effects from a lone shock to each. The ability to individually include these shocks enables the study of whether the late 2000s global crisis period and subsequent years of varied recoveries were primarily a crisis zone output and demand shock that transferred globally through the trade channel, a financial shock arising from the worldwide tightening of credit, or a precise combination of the two.

My model is closely related to those in two strands of literature: papers such as Paasche (2001) that extend the small open economy setup to examine transmission theoretically through both trade and credit; and research into international borrowing following Eaton and Gersovitz (1981) analyzing the costs and motivation for country default and the effects of financial disruptions on the real economy. My model builds on the work of Mendoza and Yue (2012) and Sosa-Padilla (2015) in particular.[4]

Rather than a standard small home economy trading a single good with a rest of the world that has infinite demand for that good at a fixed price, there is a foreign endowment economy with endogenously determined demand for a set of heterogeneous home goods, market prices, and terms of trade. The home country is affected through the trade channel when changes in the foreign endowment alter these quantities. The degree of these effects will vary across countries, with increased potency as trade costs with the foreign economy decline and the countries become more integrated through trade.

With regard to finance, instead of the home economy facing a single world interest rate permitting infinite borrowing or investment at that same rate, there is a local banking sector facilitating lending and investment with the rest of the world at endogenously determined rates. Varying levels of international bank capital inflows each period transmit the degree of credit tightness abroad to the home economy; further, the capital inflows will be greater for countries that are closer to the foreign economy financially. The home government and firms borrow and make default decisions independently of one another, with their default probabilities reflected in the prices of their debt. Further, these agents indirectly interact with one another through the banking sector, as greater borrowing or a default will increase

[4] Sosa-Padilla (2015)'s model is similar to mine in that bank balance sheet issues lead to an externality for firm borrowing when there is government default. The model of Mendoza and Yue (2012) is also similar, with firms relying on working capital to pay for a portion of production, access to which is tied to government default decisions through financing.

credit tightness for all others. This dynamic can produce endogenous crowding out and output declines during default periods, and is the deterrent for sovereign default. With this structure the model replicates the sovereign-banking sector Diabolic Loop from Brunnermeier et al. (2016) and the related coincidence of sovereign debt and banking crises as described in Correa and Sapriza (2014).[5] The state of the foreign financial system is transferred to the home economy by both changes in international securities' interest rates and bank capital inflows faced by the domestic banking sector.[6]

The model is calibrated for two representative countries, one near to and one far from the crisis zone. The representative countries are based on average data for corresponding groups of countries near and far from the crisis zone, with different distances modeled using varying levels of iceberg trade costs and bank capital inflows from abroad. To model the trade channel, I match the changes in the foreign endowment to the declines in the aggregate crisis zone detrended GDP, which reduces the foreign demand for home goods. To model the financial channel, the interest rates and bank capital inflows in my model are matched to US dollar 1-year real treasury and LIBOR rates, and the Bank for International Settlements' (BIS) international banking claims data. The sizes of the shocks to the transmission channels are replicated by taking these averages separately over historical boom, recession, and crisis periods. Doing so produces contagion for both the representative near and far countries with GDP changes similar in magnitude to those observed, without explicitly targeting the output effects.

To gauge the relative importance of the trade and financial transmission channels, I run four counterfactual experiments wherein I separately include the disturbances to each channel as experienced during the late 2000s for the representative near and far countries. For the representative near country, employing both the financial and trade shocks produces a GDP decline of 12.2%, compared with the actual 12.1% average decline. Including only the credit shock results in an 11.4% output reduction and the trade shock alone in a 2.1% drop. Likewise, for the representative far country, replicating both shocks produces a 0.2% output increase — compared with the actual 0.9% average — while separate credit and trade shocks produce +2.2% and -1.8% changes, respectively.

These results indicate the decline in capital from abroad caused a credit crunch in the near economies, which drove the contagion there. The far countries had lower initial levels

[5] The works of Bocola (2016) and Broner et al. (2014) further investigate the macroeconomic effects of sovereign debt and credit risk on the real economy during the Eurozone Debt Crisis, with the latter also finding that these effects appeared to be transmitted internationally.

[6] Cetorelli and Goldberg (2012) and Guerrieri et al. (2012) provide evidence of the importance of bank capital flows for the international propagation of shocks during the late 2000s crisis period.

of international banking claims and saw smaller declines in them during the crisis period, making these countries less susceptible to such a credit crunch. In fact, with the decline in global interest rates, the financial channel was a net positive for the far countries. These results match the observation of greater contagion for countries closer to the crisis areas. Further, they suggest credit disruption as the primary contagion driver for the near countries, those most affected by crisis transmission.

Finally, I study the effectiveness of various levels of capital controls for the near countries due to the importance of the financial transmission channel for them. The results of counterfactual analyses across a spectrum from complete capital openness to capital immobility suggest that the limited use of capital controls would effectively mitigate financial contagion in both the short and long runs. On the other hand, very tight capital controls — or total banking capital immobility — will induce government default by easing the credit tightness experienced during default periods, as capital is unable to flee the country. This policy leads to less long run financial contagion, but at the cost of greater short run contagion and decreased output in non-crisis periods as the priced in risk of government default causes greater crowding out. These results are in agreement with much of the recent literature on capital controls — such as Korinek (2011), Korinek and Sandri (2016), Magud et al. (2011), and Ostry et al. (2012) — in finding that capital controls can help reduce crisis transmission, though the particulars of their implementation and the circumstances of an economy may alter their efficacy.

2 Crisis Zone Proximity & Contagion

The 2008 United States financial crisis and 2010-2013 European sovereign debt crisis triggered a period of weak global economic activity.[7] The global economy experienced a worldwide recession from 2008-2009 followed by several years of anemic growth. Looking beyond the aggregate global impact of these crises, there were large differences in country level outcomes for nations outside of the US and Eurozone crisis areas. In particular, those countries nearer to the aggregate crisis zone performed worse on average over the subsequent half decade than those that were farther from it.

To add more precision to this observation, I collected population weighted bilateral country distance data from CEPII and real GDP per-capita (Constant 2005 USD) data from the

[7] The timing of the European sovereign debt crisis is defined as the continuous period when 10-year government bond yields for at least one of the 2007 Eurozone countries other than Greece were above 5%. Greece is excluded because of its material fiscal issues contributing to high yields over a longer time period. The bond yield data are from the European Central Bank.

World Bank. To remove extremely small, outlier countries, only countries with real GDP per-capita of at least $5,000 or total GDP of at least $5 billion were retained, leaving 156 countries. The average annual growth by country for 2001-2007 was calculated and used to forecast trend levels of real GDP per-capita for the 2008-2014 period. To normalize across countries, the levels relative to trend were calculated by taking each year's actual real GDP per-capita as a percentage of the country's trend forecast. Individual country average annual growth was used because countries farther from the crisis zone — especially those in eastern Asia — had high levels of growth entering the crisis period that could artificially amplify the growth-distance relationship. Not detrending the data by individual country growth, or following the Kehoe and Prescott (2002) practice of detrending all countries by 2% — or any other single rate — would strengthen the findings below that countries farther from the crisis zone performed better than those nearer it.[8]

The first column of Table I presents the results of regressing 2013 detrended real GDP per-capita on each country's minimum distance to the nearest crisis zone country using the CEPII data. As expected, there is a positive and statistically signficant relationship between distance from the crisis zone and output. The regression suggests that a thousand miles of extra distance from the crisis zone is associated with 2.2% greater 2013 detrended real GDP per-capita.

While this result is interesting in and of itself, it likely holds because inter-country economic linkages through trade and finance have been shown to adhere to gravity relationships, with greater bilateral linkages for country pairs that are physically closer to one another. To examine how international trade and financial connections were correlated with crisis propagation, I collected bilateral country data on each of these channels. The trade channel is measured as the sum of a nation's bilateral imports and exports across all crisis zone countries from the UN Comtrade database. The finance data are measured as total banking claims of the crisis areas on a country. The banking claims were gathered from the Bank for International Settlements' international banking statistics database.[9] To evaluate each country's closeness to the crisis areas, the 2007 levels of these measures as a percentage of GDP were used; the levels during the crisis period itself were not used to avoid endogeneity issues.

The second and third columns of Table I show the results of regressing 2013 detrended real

[8] The results are also robust to: not detrending the data; using different windows to calculate average country growth rates; using total local currency or USD GDP; several redefinitions of the crisis zone and the sample of non-crisis countries; using 2005 or 2006 as the base year; and looking at GDP for any year from 2009 to 2014.

[9] Note that banking data for Luxembourg were not available so it was excluded.

GDP per-capita on these variables individually, similar to what was done for distance. These regression results should be interpreted as describing the relationships present in the data and are not meant to be taken as reflecting strict causality. The fourth column then presents the results when controlling for both of these measures of economic proximity together. In all cases, the signs of the estimated coefficients match the contagion theory with greater crisis zone proximity associated with poorer economic performance, and in all cases the results are statistically significant. The final regression's estimates in particular suggest the importance of both the trade and finance channels in transmitting the crises, as the estimated coefficient for each channel is negative and statistically significant even accounting for the effects of the other one. From columns (2) and (3), one standard deviation lower 2007 trade and finance with the crisis zone are consistent with 6.0% and 2.5% increases in detrended growth, respectively.

An obvious follow up question to the results of Table I is how much exports to China might have propped up the economies of its trading partners given its large size and relatively high growth over this period. In other words, do these proximity relationships hold because countries farther from the crisis zone are often closer to China? Table II repeats the analysis of Table I with 2007 exports to China as a percentage of GDP included in the regressions. The results of these regressions suggest pretty convincingly that this theory of Chinese driven growth divergence does not hold. While exports to China were generally positively related with country growth outcomes, they were not so at a statistically significant level. Additionally, when including Chinese exports the coefficients on the distance, trade and finance variables remain at similar levels and retain their statistical significance.

As another form of robustness, Table III depicts average and median 2013 detrended real GDP per-capita broken out by distance, trade, and finance proximity quartiles. These are arranged so that lower ranked quartiles are always more connected to the crisis areas by the given measure. The generally increasing average and median output levels as the quartile numbers increase supports the idea that countries closer to the crisis zone by these measures performed worse and experienced contagion.

To get a clearer understanding of the magnitudes and economic significance of these relationships — and to act as the basis for later modeling countries near to and far from the crisis zone — I use each of these measures of crisis zone proximity to categorize countries into those within, near to, and far from the crisis zone. To classify countries into the near and far groups, I apply a κ-clustering algorithm to each measure of crisis zone proximity for the non-crisis area countries.[10] To get the average annual detrended real GDP per-capita for each country group, the following regression was run separately for the crisis zone and each group of near and far countries:

$$\hat{y}_{ct} = \alpha + \sum_t \beta_t * YearDV_t + \epsilon_{ct}$$

where $\hat{y}_{ct}$ is country c's real GDP per-capita in year t as a percentage of the trend level as described above, and the $YearDV_t$ are dummy variables equal to one in year t. The regression is estimated with errors clustered at the country level. The estimates by year for each country group along with 95% confidence intervals were gathered to get the average annual real GDP per-capita relative to trend by group. As Figure I shows, countries that were geographically closer to the United States and Eurozone crisis areas performed significantly worse than those farther away as the crises and initial recoveries unfolded. On average, countries in the near group fell 12.1% below trend by 2013 and continued to experience declines in real GDP per-capita relative to trend. Countries in the far group had on average risen above trend levels by 2012. In fact, there was only one year for the far country group from 2008 to 2014 where the trend level was not within the 95% confidence interval.

These results were not solely driven by countries in the immediate proximity of the Eurozone crisis area. Figure II plots these same three sets of countries with a fourth group separated out: countries within 1,000 miles of the Eurozone crisis area. Two observations stand out from this figure. The first is that the relationships in Figure I are only marginally changed. The second is how terrible the outcomes were for countries in the immediate Eurozone periphery.

Similar relationships hold when closeness to the crisis zone is measured by trade or financial linkages with the crisis zone rather than raw distance from it. To isolate the differences between countries near to and far from the crisis zone by distance, trade, and finance, Figure III plots the average yearly differences between each set of country groups closer to and farther from the crisis zone by these three measures, along with 95% confidence intervals. Lower values mean the group that was more connected to the crisis zone did on average worse than the less connected group. While the differences are small during the peak of the global financial crisis in 2008, through the later years the average in each group nearer the crisis zone is worse than the associated far group by 7.8-13.8%, levels that are

[10] Clustering was performed using the STATA "cluster kmedian" command on the pertinent variable with two groups. Each observation is assigned to the group whose median is closest according to L2 (Euclidian) distance. Based on that categorization, new group medians are determined. These steps continue until no observations change groups. Maps of the resulting country groups can be found in the Online Appendix.

economically and statistically significant.

In addition to the differences in average real GDP per-capita for countries near to and far from the crisis zone, there were also differences in how trade, international finance, and government bond yields evolved. Figure IV plots average aggregate trade with the rest of the world by country distance groups. To normalize the values across countries they are divided by each country's trend GDP. The figure shows that both the near and far country averages had large 2009 declines; however, the far group quickly returned to its initial level while the near group continued to decline. Figure V plots similar results for annual average banking claims of the rest of the world on a nation divided by trend GDP. The far country group's average banking claims were pretty static, while the near countries experienced average declines of 33.2% of trend GDP through 2014.

Figure VI plots average government bond credit spreads over 10-year US Treasury bonds. Initially, the average near country credit spread was below the average for far countries, 1.12% to 1.82%. The average credit spread for both country groups increased through 2009, though the far country average declined thereafter to near pre-crisis levels. By 2012, the credit risk priced into the near countries' bonds increased enough to bring the average spread for that group to 4.08%, which was 1.40% above the far country average. These results suggest that investors perceived significantly different credit risks for sovereign bonds in the two country groups, with the far country ones seen as relatively safer investments.

These regressions and plots show how distance, trade, and financial connections were related to country outcomes during the late 2000s crisis period and immediate recovery. These results suggest contagion from the crisis zone to the rest of the world, with differential economic performance based on proximity. Further, both the trade and finance channels were significantly related to output in the data. In the following sections, I intend to replicate and explain with my model the divergence in growth across countries and what drove that variation, using the near and far distance groups as the bases for calibrating the model.

3 Model

In the model there are five sets of agents across two countries, home and foreign, acting in discrete time. The agents in home include: households; firms; the home government; and international bankers. The foreign economy contains a continuum of identical consumers who receive an exogenous endowment each period; it can be thought of as the crisis zone.[11]

[11] Similar results to what I find can be obtained with a model of two countries fully specified as the home economy is by adjusting the foreign country's model parameters to match what occurred in the real world

In the following sections we will review the problems facing each set of agents. For reference, Figure VII presents the interactions between agents within and between time periods.

3.1 Home and Foreign Households

In each country there is a unit continuum of identical households that must decide on a quantity of labor to supply to domestic firms at the start of a period and make consumption decisions at the end of the period, after the production stage during which exogenous shocks to home and foreign are realized. A household's utility in each period is a function of labor supplied (n_j), as well as the consumption of a set of home goods ($\{c_{ji}\}$) and a single foreign good (f_j) where i indexes the unit continuum of home firms and their products, and $j \in \{H, F\}$ is the country of the household.

In addition to being a consumption good, the foreign good also acts as the unit of account for both home goods and borrowing.[12] To remove the wealth effect on labor supply I follow Greenwood et al. (1988) by stating period utility as a function of consumption net of the disutility of labor. Removing the wealth effect on labor supply is important to prevent increases in labor supply in states with low consumption, the reverse of what is seen in the data. The period utility function for a household has a standard constant relative risk aversion kernel:

$$U_j(\{c_{ji}\}, f_j, n_j) \equiv \frac{\left(C_j(\{c_{ji}\}, f_j) - \frac{n_j^{\omega}}{\omega}\right)^{1-\rho}}{1-\rho}; \rho > 0, \omega > 0$$

where

$$C_j(\{c_{ji}\}, f_j) = \left(\int_0^1 (c_{ji})^{\frac{\sigma-1}{\sigma}} di + \alpha_j f_j^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}; \sigma > 1, \alpha_j > 0.$$

The household relative risk aversion is ρ, ω is the curvature of labor disutility, σ is the elasticity of substitution across goods, and α_j is a weighting factor between the home and foreign goods' benefits.[13,14] The period utility functions for home and foreign households are

crisis zone; however, the current setup better allows for focusing on how aggregate conditions in the foreign economy are transmitted to the home country in a more parsimonious model.

[12] Foreign currency borrowing is consistent with the liability dollarization (Krugman (1999)) of many countries.

[13] The preferences of the $C_j(\{c_{ji}\}, f_j)$ function are similar to what one would find if the foreign good were replaced by a continuum of goods that entered a CES function along with the home goods; however, in order to abstract from the foreign wage determination, firm problems, government, and banking sector the foreign market is assumed to produce one good exogenously.

[14] The Frisch elasticity of labor supply is then $\frac{1}{\omega-1}$

identical except for the α_j term.

Each household maximizes the present discounted value of its period utility function:

$$\max_{\{\{c_{jits}\},\{f_{jts}\},n_{jts}\}_0^\infty} E_0 \sum_{t=0}^{\infty} \beta^t U_j(\{c_{jits}\}, f_{jts}, n_{jts}), \tag{1}$$

where $0 < \beta < 1$ is the household discount factor, t the time period, and s the state of the world. Consumption is limited in each period by a household's budget constraint, which is different for home and foreign households.

The expenditures of a home household are constrained by the amount it receives in after-tax wages, a transfer it receives from the government, and its share of the profits of home firms. Every home household is paid at the competitive market wage rate, w_t, which it takes as given, and must choose the amount of labor that it will supply before production occurs. The government taxes home households a share of their wages, T_t, that it selects at the beginning of each period and is known to the home households, and transfers the fixed amount G to the households. In addition, consumers own equal amounts of all home firms so that the aggregate profits from firms, Π_{ts}, are shared — and taken as given — by home consumers. The home households save using the domestic equity market; however, since the households are all identical and the initial allocation is assumed equal across them, there will not be any net trade in equities. Therefore, for simplicity I exclude the purchase and sale of domestic equities from the household problem. Let $s \equiv \{Z, W\}$ be the post-production state of the exogenous home and foreign shocks, and Ω be the set of all such possible states. The home consumers' budget constraint in period t given a final state s is:

$$B_{Hts} \equiv (1 - T_t)n_{Ht}w_t + G + \Pi_{ts} = \int_0^1 c_{Hits}p_{its}di + f_{Hts}, \tag{2}$$

where p_{its} is the price of the product produced by firm i.

Foreign households share their endowment equally and have the following budget constraint:

$$W_{ts} = \int_0^1 c_{Fits}\tau p_{its}di + f_{Fts}, \tag{3}$$

where W_{ts} is the endowment of the foreign good in state s and τ is the iceberg transaction cost associated with international trade.[15] The size of the endowment follows an exogenous

[15] For one unit of a good to arrive abroad $\tau > 1$ units must be sent. Since the foreign good is the unit of payment it is not logical to send it abroad unless it is in exchange for home goods, so there is balanced trade, save for the loss of home goods during transit. The iceberg costs are only on home goods, but adding

random process, with the realization revealed during the production stage. By modeling the foreign country as an endowment economy I create an export destination for home goods with endogenous terms of trade and product demands, while abstracting from the unnecessary details of the foreign economy. This setup enables me to focus on how the foreign country's output level passes through to the home country as the trade channel.

3.2 Home Firms

Home has a unit continuum of firms, indexed by i, producing differentiated goods under monopolistic competition. The variation in firms allows me to model sectors with differing foreign market exposures, including tradable versus non-tradable sectors. This is important given the variation in firm outcomes during crises.[16]

Each period before production occurs, home firms choose the amount of labor to employ, how much working capital to borrow, and whether to export to the foreign country. After production, each firm individually decides whether to default on its debt, the price it will charge for its good, and the allocation of sales across the home and foreign countries.

Firms utilize labor as the single input into their production process, and the production function for firm i is:

$$Q_{its} \leq \frac{Z_{ts}}{a_i} N_{it}^{\alpha}$$

where N_{it} is the labor employed, Z_{ts} is an economy-wide productivity shock realized during the production process in period t, and a_i is the productivity factor of firm i.[17] There is no entry or exit for firms and firms are not able to draw new a_i, so there is a fixed distribution of firm productivities. The economy-wide productivity shock follows an exogenous random process.

While labor is the sole input to production, following Fuerst (1992)'s pay in advance setup, firms require within period working capital because workers demand their wages in advance of production. Home firms face a fixed cost to export and iceberg trade costs similar to the Melitz (2003) model.[18] The reliance of international trade on financing is modeled by requiring that firms pay the fixed export cost (γ) upfront before production

trade costs on the foreign good would not materially alter the solution or model dynamics given that there is balanced trade.

[16] See for example Durbin and Ng (2005), Chor and Manova (2012) and Manova (2013) for studies of variation in firm outcomes during crises.

[17] Physical capital dynamics did not drive the crises, so it is left to be captured in the productivity terms.

[18] The fixed cost is in terms of the foreign good; however, the solution and model dynamics would not materially change if it were instead a fixed labor cost or in terms of the home goods.

occurs to gain access to the foreign market if they plan to export in the current period.[19] Additionally, across the continuum of heterogeneous firms which endogenously choose each period whether to export or not, changes in both the intensive and extensive margins of trade can be modeled, the importance of which have been demonstrated in papers such as Melitz (2003) and Ruhl (2008).

All profit or loss is transferred to the households at the end of each period in the form of a dividend, π_{its}. Since firms then do not have any resources on hand at the start of a period, they must take out intra-period working capital loans to pay their pre-production costs. Firm i sells zero coupon commercial paper to the bank at a price of h_{its}, and the amount of commercial paper that it sells is κ_{its}. Since firms independently choose whether to default or not there will be firm specific commercial paper price schedules dependent on the amount a firm wishes to borrow, its a_i, and the aggregate state of the world. A firm's working capital constraint is:

$$h_{its}\kappa_{its} \geq \begin{cases} w_{ts}N_{its} & \text{if do not choose to export} \\ w_{ts}N_{its} + \gamma & \text{if choose to export} \end{cases} .$$

Firms are expected to pay back their borrowing after production is completed; however, each firm has the option to default and be penalized the fraction μ of its output.[20,21]

The above facets of the firm problem can be combined to yield the profit function. Let:

$$d_{its} = \begin{cases} 0 & \text{if pay off debt} \\ 1 & \text{if default} \end{cases}$$

and

$$x_{its} = \begin{cases} 0 & \text{if do not choose to export} \\ 1 & \text{if choose to export} \end{cases} .$$

[19] For analysis of the dependence of firm exports on external financing and trade credit during crises see Chor and Manova (2012) and Manova (2013).

[20] Because firms' commercial paper prices will be less than one — due to default risk and the return of the short-term risk free asset being > 1 — and firms have no need for working capital beyond their up-front costs, the working capital constraint will hold with equality.

[21] One could think of this lost fraction of output as litigation costs associated with default or a reputation cost that diminishes the perceived quality of a firm's goods. See Mendoza and Yue (2012) for an alternate approach wherein default leads to restricted access to productive capital, reducing firm productivity.

Each firm maximizes expected current period profit taking the market aggregates as given:

$$E_s\left(\pi_{its}\right) = \max_{\{\Theta_{its}\}} E_s\left[p_{its}Q_{its}(1-\mu d_{its}) - w_t N_{it} + h_{it}\kappa_{it} - \kappa_{it}(1-d_{its}) - \gamma x_{it}\right]$$

$$\begin{aligned} s.t. \quad & \tfrac{Z_{ts}}{a_i} N_{it}^{\alpha} \geq Q_{its} \\ & Q_{its}(1-\mu d_{its}) \geq m_{Hits} + \tau m_{Fits} \\ & \text{On consumer demand curves: } \begin{matrix} m_{Hits} \leq c_{Hits} \\ m_{Fits} \leq c_{Fits} \end{matrix}, \\ & h_{it}\kappa_{it} \geq w_t N_{it} + \gamma x_{it} \\ & m_{Fits} \begin{cases} = 0 & \text{if } x_{it} = 0 \\ \geq 0 & \text{if } x_{it} = 1 \end{cases} \end{aligned} \tag{4}$$

where m_{Hit} and m_{Fit} are the quantities allocated for home and foreign sales and $\Theta_{its} \equiv \{N_{it}, \kappa_{it}, x_{it}, \{Q_{its}\}, \{m_{Hits}\}, \{m_{Fits}\}, \{p_{its}\}, \{d_{its}\}\}$ is the choice set.[22] The firms' aggregate payment to home households is then:

$$\Pi_{its} \equiv \int_0^1 \pi_{its} di.$$

3.3 Banking Sector

The competitive home banking sector is modeled using a representative banker. The goal of the banker is to maximize the net present value of his end of period cash flows, I_{ts}. With these preferences the banker is risk neutral as he is maximizing a linear function of cash flows. Entering each period, the banker's only assets are government bond holdings from the previous period. At the start of the period the government pays the banker or defaults, and the banker receives a capital inflow of A_t. This inflow can be thought of as an allocation from a foreign parent bank, or revenue from other ongoing business activities. With the former interpretation, the home economy is assumed to not be small in the sense that it can borrow or invest an infinite amount at a single international rate. It is, however, small in the sense that regardless of the amount that the banking sector profits today, the capital inflows to the home banking sector in future periods will not be affected, as they are determined externally. The size of the A_t can vary between countries, with greater levels of the capital inflows corresponding to greater international financial integration.

The banker intermediates the home economy's access to international capital markets,

[22] Agents can arbitrage home goods if the price of a good in foreign does not equal τ times its price at home, preventing firms from price discriminating between the two markets.

with government and firm borrowing occurring through him.[23] Credit instruments available to the banker are in one of two tenures: short-term instruments that are repaid in the same period after production occurs; and long-term securities that are repaid at the start of the following period. The firms take part in the intra-period lending market, though the government is assumed not to do so.[24] The government, however, issues one period bonds to the banker that will be repaid at the start of the following period. The prices for these securities are determined endogenously by their respective default risks, with individual firms and the government able to independently default on their debts.

In addition to the commercial paper from firms and government bonds, there are three international investment securities that the banker can use. He can invest an unlimited quantity in a risk free international security, borrow by using government bond holdings as collateral in a repurchase agreement, or borrow an unlimited amount with uncollateralized debt. All three of these investment options are short-term, which means that they will be agreed to at the start of a period pre-production and paid at the end of the same period post-production. The interest rates on these international investments are taken as given by the banker, as they are determined exogenously in the rest of the world based on the state entering the period. K_t is the amount that the bank invests in the international risk free asset returning r^f, and L_t is the amount of uncollateralized bank borrowing at a rate of r^H. R_t is the amount that the bank borrows via bond repurchase agreements at a rate of r^R. Given the nature of the repurchase agreements, the quantity used must be weakly positive and is limited by the market value of the banker's bond holdings, $p_B B'$. Also, it is assumed that $r^f < r^R < r^H$, with the risk free asset requiring the lowest return, and the collateralized repurchase agreement borrowing rate being lower than the uncollateralized one, as participation in the uncollateralized lending market is a signal that the bank is leveraged beyond its assets on hand.

This capital market structure has the benefit of mirroring the institutional structure of real world banks. Further, modeling the banking sector in this manner fits the interest rate dynamics that occurred during the late 2000s global crisis period better than a single international rate would. Figure VIII shows the 1-year US Treasury bill and LIBOR rates,

[23] Having international borrowing funneled through the banking system fits the experiences of many countries. As an example, in a sample of 3,312 international corporate bonds from Euromoney, Durbin and Ng (2005) finds that half of them are from the Banking and Finance sector. Further, most corporate bonds are ultimately held by banks, pension funds, insurance companies, or other financial institutions.

[24] The banker would not loan short-term to the government because post-production the government does not have any cash on hand to repay such debts. Therefore, this assumption comes down to assuming that the government would not lever itself to seek short-term profit from investments in the banking sector.

and the spread between the two. The treasury and LIBOR rates align with the risk free rate that the model's banking sector can invest at and the collaterlized borrowing rate, respectively. As can be seen in the diagram, the credit spread — the difference between the treasury rate and bank borrowing rate — is significant in magnitude and countercyclical. For low levels of the spread, it would be reasonable to use a standard model with one world interest rate that the banking sector can both borrow and invest at; however, that is not what is reflected in the data. During the global crisis period, the 1-year US Treasury rate was near zero while many credit spreads were at or near their all time highs.

The pre-production lending constraint for the banker at the beginning of the period is:

$$A_t + B_t + L_t + R_t \geq \int_0^1 h_{it}\kappa_{it} di + K_t + p_{Bt}B_{t+1}, \tag{5}$$

where p_{Bt} is the price of one period zero coupon government bonds. The pre-production constraint specifies that net investment in government bonds, firm borrowing and the risk free asset must be less than or equal to a bank's capital plus the amount it borrows short-term.

The post-production investment constraint for the banker in final state s is:

$$r^f K_t + \int_0^1 \kappa_{it}(1 - d_{its})di \geq I_{ts} + r^H L_t + r^R R_t. \tag{6}$$

The post-production constraint states that the banker's end of period investment cash flow, I_{ts}, is less than or equal to his net profit, inclusive of defaults.

There are effectively two cases for the banker, depending on the default choice of the government. If the government defaults then it will be excluded from borrowing in the bond market for the given period and enter the following period with only ϕB_t of its debt remaining, where $0 < \phi < 1$. Therefore, depending on the government's default behavior there either will or will not be bonds available for the banker to invest in, and if the government defaults the bank's funds today will be only A_t rather than $A_t + B_t$. Further, a government default will cause an interruption that closes the bond repurchase agreement market for the current period.

If the government does not default in the current period then the banker's problem is:

$$
\begin{aligned}
\Psi(B, Z_{-1}, W_{-1}) \mid_{D=0} = \max_{\{B', K, L, R, \{I_s\}, \{\kappa_i\}\}} & E_s \left\{ I_s + \delta \Psi(B', Z_s, W_s) \right\} \\
s.t.\ & A + B + L + R \geq \int_0^1 h_i \kappa_i di + K + p_B B' \\
& r^f K + \int_0^1 \kappa_i (1 - d_{is}) di \geq I_s + r^H L + r^R R \ \ \forall s \\
& p_B B' \geq R \geq 0 \\
& K \geq 0; \ \ L \geq 0.
\end{aligned}
$$

The first constraint is the pre-production lending constraint, and the second is the post-production investment constraint. The next constraint requires that the money borrowed in a repurchase agreement be between zero and the market value of the new held bond position. The final two constraints restrict the banker to borrow and invest risk free at different interest rates. With government default this period, the banker's problem is:

$$
\begin{aligned}
\Psi(B, Z_{-1}, W_{-1}) \mid_{D=1} = \max_{\{K, L, \{I_s\}, \{\kappa_i\}\}} & E \left\{ I_s + \delta \Psi(\phi B, Z_s, W_s) \right\} \\
s.t.\ & A + L \geq \int_0^1 h_i \kappa_i di + K \\
& r^f K + \int_0^1 \kappa_i (1 - d_{is}) di \geq I_s + r^H L \ \ \forall s \\
& K \geq 0; \ \ L \geq 0.
\end{aligned}
$$

3.4 Home Government

The Home government seeks to maximize the discounted present value of its households' utilities; however, to support government debt levels nearer those in the data I follow Grossman and Van Huyck (1988) and Rieth (2014) with a myopic government. The government has the same period utility function as its households but a lower discount factor, seeking to maximize:

$$
\max_{\{\{c_{Hits}\}, \{f_{Hts}\}, n_{Hts}\}_0^\infty} E_0 \sum_{t=0}^{\infty} (\xi \beta)^t U_H(\{c_{Hits}\}, f_{Hts}, n_{Hts})
$$

where $0 < \xi \leq 1$ is the degree of government myopia.[25] Note that other than that constant this is the same as the households' target utility given in Equation 1.

The government makes three choices each period before production occurs to achieve its goal: whether to default or repay the banker at the start of the period (D_t); the tax rate on wages (T_t); and the amount to borrow into the next day via bonds (B_{t+1}).[26] Greater values

[25] The term $h = \frac{1}{1-\xi}$ can be interpreted as the expected planning horizon of the government in years.

[26] The government being the actor to borrow from the banking sector to smooth household consump-

of B_{t+1} mean greater government indebtedness. If the government chooses to default then it is excluded from the debt market for the current period and reduces its debt by a factor of $(1-\phi)$. Government spending, G, is exogenously fixed. The government budget constraint is:

$$T_t n_{Ht} w_t + p_{Bt} B_{t+1}(1-D_t) \geq G + B_t(1-D_t)$$

where

$$D_t = \begin{cases} 0 & \text{if pay off debt} \\ 1 & \text{if default} \end{cases}.$$

4 Model Equilibrium

In this section I outline the solution to the model, how contagion occurs in the model and define the equilibrium. The problems of the households, firms, and banker within a period given the government's choices will be examined first, followed by a determination of the government's optimal policies knowing the effects it will have on the other agents.

4.1 Solution to the Home and Foreign Households' Problems

The households make their labor supply decisions before production occurs, and their consumption decisions after production taking the pre-production labor choices, the shocks, taxes, firm profits, firm default choices, firm export choices, and prices as given. To write their equilibrium conditions governing these choices it is helpful to provide some notation:

1) Price index for home goods in home $P_{Hts} \equiv \left(\int_0^1 (p_{its})^{1-\sigma} di\right)^{\frac{1}{1-\sigma}}$

2) Price index for all goods in home $\Phi_{Hts} \equiv \left(P_{Hts}^{1-\sigma} + \alpha_H^{\sigma}\right)^{\frac{1}{1-\sigma}}$

3) Price index for home goods in foreign $P_{Fts} \equiv \tau\left(\int_{\chi_t} (p_{its})^{1-\sigma} di\right)^{\frac{1}{1-\sigma}}$

4) Price index for all goods in foreign $\Phi_{Fts} \equiv \left(P_{Fts}^{1-\sigma} + \alpha_F^{\sigma}\right)^{\frac{1}{1-\sigma}}$

where χ_t is the set of home goods exported to foreign. The first order conditions for the home households are then:

$$f_{Hts}\ FOC:\ f_{Hts} = \left(\frac{B_{Hts}}{\Phi_{Hts}}\right)(\alpha_H \Phi_{Hts})^{\sigma} = C_{Hts}\,(\alpha_H \Phi_{Hts})^{\sigma} = B_{Hts}\Phi_{Hts}^{\sigma-1}\alpha_H^{\sigma}, \tag{7}$$

tion matches extant programs by which the government does much of this smoothing (e.g., social security, unemployment insurance).

$$c_{Hits}\ FOC:\ p_{its}c_{Hits} = B_{Hts}\Phi_{Hts}^{\sigma-1}p_{its}^{1-\sigma}\ \ \forall\, i, \tag{8}$$

$$n_{Ht}\ FOC:\ n_{Ht}^{\omega-1} = \frac{(1-T_t)w_tE_s\left(U_{Hts}^{\frac{\rho}{\rho-1}}\Phi_{Hts}^{-1}\right)}{E_s\left(U_{Hts}^{\frac{\rho}{\rho-1}}\right)}. \tag{9}$$

Similarly, the first order conditions for the foreign households are:

$$f_{Fts}\ FOC:\ f_{Fts} = \left(\frac{B_{Fts}}{\Phi_{Fts}}\right)(\alpha_F\Phi_{Fts})^{\sigma} = C_{Fts}\left(\alpha_F\Phi_{Fts}\right)^{\sigma} = B_{Fts}\Phi_{Fts}^{\sigma-1}\alpha_F^{\sigma}, \tag{10}$$

$$c_{Fits}\ FOC:\ \tau p_{its}c_{Fits} = B_{Fts}\Phi_{Fts}^{\sigma-1}\tau^{1-\sigma}p_{its}^{1-\sigma}\ \ \forall \text{ exported } i, \tag{11}$$

$$n_{Ft}\ FOC:\ n_{Ft} = 0. \tag{12}$$

Since households in foreign receive their endowment regardless of the labor effort they exert, the households there will choose to not work. Additionally, it can be found that — as in the standard Dixit and Stiglitz (1977) model that consumer preferences in my model are based on — both the home and foreign households' budget constraints will hold strictly, and that given prices household consumption is a linear function of wealth:

$$C_{jts} = \frac{B_{jts}}{\Phi_{jts}}.$$

4.2 Solution to the Home Firms' Problem

The firms can calculate the optimal demand choices of the consumers given prices and will utilize these reaction functions when selecting their optimal choices. Let

$$\psi_{sExporter} \equiv B_{Hts}\Phi_{Hts}^{\sigma-1} + B_{Fts}\Phi_{Fts}^{\sigma-1}\tau^{1-\sigma}$$

and

$$\psi_{sNonexporter} \equiv B_{Hts}\Phi_{Hts}^{\sigma-1}$$

be the pricing indices for firms that do and do not export to foreign, respectively. The consumers' problems yield the following pricing rule for all firms:

$$\psi_{sx_{it}} = Q_{its}(1-\mu d_{its})p_{its}^{\sigma}.$$

It is shown in the Online Appendix that the price indices have an unique positive solution given the state, pre-production labor choices, export and tax decisions, and current firm default choices that are the solution to the following two equation system:

$$\psi_{sX}\alpha_H^{\sigma} = (1-T_t)w_t n_{Ht} + G - \int_0^1 (1-d_{its})\kappa_{it}di + (\psi_{sX}-\psi_{sH})\left(\psi_{sX}^{\frac{1-\sigma}{\sigma}}k_{ts} + \alpha_H^{\sigma}\right)$$

$$\frac{W_{ts}}{\psi_{sX}^{\frac{1-\sigma}{\sigma}}k_{ts} + \tau^{\sigma-1}\alpha_F^{\sigma}} = (\psi_{sX}-\psi_{sH})$$

where

$$k_{ts} = \int_{\chi t}\left(\frac{Z_{ts}}{a_i}N_{it}^{\alpha}(1-\mu d_{its})\right)^{\frac{\sigma-1}{\sigma}}di.$$

Upon substitution of the pricing rule and all strictly binding constraints into the problem for firm i the problem becomes:

$$E_s(\pi_{its}) = \max_{\{N_{it},x_{it},\{d_{its}\}\}} E_s\left[\psi_{sx_{it}}^{\frac{1}{\sigma}}\left(\frac{Z_{ts}}{a_i}N_{it}^{\alpha}(1-\mu d_{its})\right)^{\frac{\sigma-1}{\sigma}} - (1-d_{its})\frac{(w_tN_{it}+\gamma x_{it})}{h_{it}}\right].$$

The first order condition for labor demand is

$$w_t\frac{E_s(1-d_{its})}{h_{it}} = N_{it}^{\frac{\alpha(\sigma-1)}{\sigma}-1}a_i^{\frac{1-\sigma}{\sigma}}\frac{\alpha(\sigma-1)}{\sigma}E_s\left[\psi_{sx_{it}}^{\frac{1}{\sigma}}\left(Z_{ts}(1-\mu d_{its})\right)^{\frac{\sigma-1}{\sigma}}\right], \tag{13}$$

where the left hand side can be shown from the banker's problem to be constant for all producing firms (i.e., all firms that do not default in all post-production states and therefore are able to acquire funds in the short-term capital market). Firms will maximize their expected profits over this choice and the discrete choices of whether to export and default. Note that the default threshold for a firm is based on its output and the aggregate price index that will prevail after production. Specifically, a firm will repay its debt if and only if the extra revenue received when not defaulting is greater than the amount of short-term commercial paper due:

$$\psi_{sx_{it}}^{\frac{1}{\sigma}}Q_{its}^{\frac{\sigma-1}{\sigma}}\left(1-(1-\mu)^{\frac{\sigma-1}{\sigma}}\right) \geq \kappa_{it}.$$

The export choice is based on whether moving to the greater price index facing exporters is worth the fixed cost of exporting.

4.3 Solutions to the Banker's Problem

The solutions to the banker's problem in the cases where the home government does and does not default produce similar equilibrium conditions (see the Online Appendix for detailed solutions). The primary difference is that when the government defaults it shuts down the bond and repurchase markets, such that the equilibrium conditions for the problem under default are a subset of those that exist with repayment.

These conditions state that within a period the short-term required returns of all investments are the same. Γ is used to denote this required short-term return. The specific pricing formulas for firm commercial paper and government bonds are then:

$$h_i = \frac{E_s(1 - d_{is})}{\Gamma} \;\; \forall \;\; i, \tag{14}$$

and

$$p_B = \frac{\delta E_s \left(\Gamma'_s(1 - D'_s)\right)}{\Gamma - \Lambda}, \tag{15}$$

with the second pricing equation not applying if the government defaulted in the current period, closing the bond market. The Λ in the denominator of the latter equation is the shadow cost of the constraint limiting the amount borrowed via repurchase agreements to the value of the held bond position. In both cases the price of debt is discounted by the short-term required return in the current period and the chance of default. For government debt, which spans two periods, there are also price adjustments for the banker's inter-period discount rate, and the short-term required return that will prevail in the following period. If the full bond position is not borrowed against then Λ will be zero, and the current period discount for bonds is simply Γ, the required return. On the other hand, if the full bond position is borrowed against then Λ will adjust such that $\Gamma - \Lambda = r^R$, the bond financing cost.

Under both home government default and repayment, it can be shown that the pre-production lending and post-production investment constraints will hold strictly. This means the banker's end of period investment decision is trivially to select I_{ts} to make up the shortfall of his short-term investments or to set aside his short-term profits.

The level of Γ will be influenced by the banker's opportunity cost of capital. This opportunity cost comes from the banker's outside investment opportunity in K_t, or his marginal cost of funds through either bond repurchase agreements or uncollateralized borrowing. I define Home Borrowing as being the total borrowing by home firms and the government (if the home government defaults then this is just the total home firm borrowing):

$$\text{Home Borrowing } \equiv \int_0^1 h_{it}\kappa_{it}di + p_{Bt}B_{t+1}.$$

I also define the Bank Cash as $A_t + (1 - D_t)B_t$, the capital inflow from abroad added to the amount that the government repays on its bonds. There are then five potential cases for the required return, Γ, when the government honors its debt and three when it does not based on the total desired amount of Home Borrowing as demonstrated in Figure IX. To be clear, the B bond positions in the figure are those that the bank holds entering the period, and the B' bond positions are those that the bank invests in pre-production that pay off in the following period.

The aggregate interest rate schedule for Home Borrowing when the government repays is given in the left panel of the figure. If the amount of Home Borrowing is less than the banker's cash on hand, $A_t + B_t$, then the opportunity cost of funds for the banker will be the return on the risk free investment of r^f, so $\Gamma = r^f$. In the case where the amount of Home Borrowing is greater than the banker's cash assets on hand by an amount less than the value of his new bond position, $p_B B'$, then the short-term required rate is the repurchase agreement rate that he will borrow marginal funds at, so $\Gamma = r^R$. Similarly, if the aggregate amount of Home Borrowing is greater than the banker's total assets on hand of $A_t + B_t + p_B B'$ then the opportunity cost of funds is the rate of r^H at which the banker borrows the marginal dollar he is investing, so $\Gamma = r^H$. The other two cases are the edge cases between these three zones. If the level of Home Borrowing equals the banker's cash on hand then the equilibrium conditions stipulate that $\Gamma \in [r^f, r^R]$, and if it is equal to $A_t + B_t + p_B B'$ then $\Gamma \in [r^R, r^H]$.

In the right panel of Figure IX is the rate schedule if there is a government default. With a default the government bond and repurchase markets are closed and the banker does not receive the payment B_t at the start of the period, but firms will benefit by not being crowded out by government borrowing. This reduces the banker's cash on hand and shifts the threshold for the highest rate down to A_t. Below that level the required return is r^f, at the edge case where total firm borrowing is A_t then $\Gamma \in [r^f, r^H]$, and above that $\Gamma = r^H$.

This framework for the domestic banker has important consequences for the indirect interaction of home borrowers. As the bank leverages itself to support greater loans to firms and the government, the required return for lending increases endogenously from the risk free investment rate to the higher rate at which the bank itself borrows. Therefore, when there is significant borrowing in the home economy the cost of borrowing increases. Also, firms and the government indirectly affect each other with their default risks altering asset pricing and credit tightness.

The collapse of the interest rate schedule as shown between the two panels of Figure IX provides the endogenous cost of government default. The potential tightening of the credit schedule if the bank's government bond assets are not repayed provides the incentive for the government to repay its debt, without making the common assumptions that there is a set amount of production lost during default, a strict collateral requirement facing the banking sector that cuts off borrowing when exceeded, or an infinite amount of borrowing at a fixed rate before an exogenous cessation of all lending to agents within a country when there is a government default.[27] This form of default penalty is similar to the dynamics that many feared in the wake of a potential Greek default during the Eurozone debt crisis and to the sovereign-banking sector Diabolic Loop of Brunnermeier et al. (2016).[28] Additionally, the interaction between firms and the government created by this setup better fits many crisis experiences. Specifically, the combination of firm reliance on working capital and the government borrowing through the banking sector can produce twin sovereign default and Barro style GDP crises, as is often seen in the data.[29]

4.4 Solution to the Government's Problem

For the solution to the home government's problem I focus on Markov perfect equilibria where the government's policies are functions of the state variables entering the period, $\{B, Z_{-1}, W_{-1}\}$. The home government's default, borrowing and taxation decisions will have consequences for the other agents in the model. Therefore, we can think of home period utility for a given post-production state s, U_{Hts}, as being a function of the government's decisions and existing debt, specifically $U_{Hts}(T, D, B)$. If the government chooses to default then its tax revenue must cover its fixed expenditure and its level of bonds tomorrow are set

[27] These alternate assumptions are, for example, often used in the sudden stop literature as in Calvo (1998). Korinek and Mendoza (2014) provide a good summary of the sudden stop literature.

[28] For example, see this representative article from the time of the Eurozone debt crisis for a discussion of the perceived threat of a collapse in the European banking system due to losses on Greek bonds: http://www.theguardian.com/world/2011/jun/17/greece-bailout-germany-private-banks.

[29] Barro GDP disasters are defined as large declines in real GDP per-capita, normally of at least 10%. See Barro and Jin (2011) for further discussion of GDP crises, and Levy Yeyati and Panizza (2011) for a historical discussion of this relationship.

at $B' = \phi B$. These requirements yield the following value function in the case of default:

$$V(B, Z_{-1}, W_{-1})\mid_{D=1} = E_s\left\{U_{Hs}\left(T_G, 1, 0\right) + (\xi\beta)V(\phi B, Z_s, W_s)\right\}, \tag{16}$$

where T_G is the level of taxation such that the government's tax revenue equals G. The general form of the government problem can then be written as:

$$V(B, Z_{-1}, W_{-1}) = \max_{\{D\}} \begin{cases} \max_{\{T,B'\}} E_s\left\{U_{Hs}(T, 0, B) + (\xi\beta)V(B', Z_s, W_s)\right\} & \text{if pay off debt} \\ \qquad s.t.\ T_t n_{Ht} w_t + p_B B' \geq G + B & \\ V(B, Z_{-1}, W_{-1})\mid_{D=1} & \text{if default.} \end{cases} \tag{17}$$

4.5 Modeling Contagion

This section describes how the trade and financial channels transmit the state abroad to the home economy. The model allows separation of shocks via the trade and financial channels, as each of these two channels is governed by a separate set of model parameters; however, this does not mean that there is no interaction between the two. In fact, it is quite the opposite, with changes in the credit market substantially affecting trade, and vice-versa. The important feature is that the model parameters for each channel can be independently adjusted to see the effects from a lone shock to each (e.g., the foreign endowment state space can be changed without altering the financial parameters).

4.5.1 Contagion through Trade

There are three specific parameters that affect the trade channel: the fixed export cost; the foreign endowment; and the iceberg trade cost. The fixed export cost will govern the fraction of home firms that choose to export; however, this cost is assumed to be constant for both representative countries in all states, so it is not significant for the dynamics of trade channel contagion in my analysis. The other two parameters are important for transmission through the trade channel and cross-country variation.

The foreign endowment is the driver of trade shocks. A trade shock occurs when changes in the foreign endowment alter the export demand facing home firms and the terms of trade. For example, a negative foreign endowment draw will have a number of consequences for the home economy. First, there is the direct effect of diminished demand from abroad for home goods, reducing the prices at which exporting firms can sell their goods and the terms of trade. Second, the reduced profitability of home exporters will decrease income to the home households. There will then be a negative demand shock for all home goods, not just the exported ones. Third, if the reduction in a firm's profitability from these former effects is great enough, they can lead the firm to default on its debt at a cost of a fraction of its output. Through this mechanism negative shocks in the foreign economy can produce large real decreases in home output and financial distress.

The above effects are the result of a realized poor draw of the foreign endowment; however, lower endowment expectations can also be harmful for the home economy, even if the realized endowment level does not change. A lower expected endowment decreases the demand for labor of exporters, and with that household income and output. Additionally, the export decisions of home firms may change resulting in less trade profit entering the home economy.

The iceberg transaction cost influences the degree of trade integration between the home economy and the rest of the world, and with that home's susceptibility to trade contagion. The variation in the degree of trade contagion with the level of trade costs can be seen in Figure X for a representative set of model parameters. The figure plots aggregate home firm profit on the y-axis — which is used as a measure of the health of the home economy — against alternate iceberg trade costs, τ, along the x-axis. The four upper solid and dashed lines are for each of four post-production states, the cross product of good and bad draws of the home productivity factor and the foreign endowment. The lines with the same shade are for the same home productivity draw, and the dashed lines are for the bad foreign draw. As the iceberg trade costs decline to the left, the world becomes more integrated through trade and the profits of the home firms increase for a given final state.

As the lower two dotted lines in the plot show, though, there is a cost to greater integration. The dotted lines present the differences in aggregate firm profit between good and bad foreign endowment draws, taking home productivity as given. As the variable trade cost decreases and trade increases, the amount of contagion from abroad rises as evidenced by larger differences in home profits between the good and bad foreign states.

4.5.2 Contagion through Finance

Financial channel transmission from the rest of the world to home occurs through borrowing by firms and the government from the banking sector. The capital inflows, A_t, vary by country, though the bankers in both representative countries will face the same three international interest rates. The levels of these factors will depend on the state abroad entering a period, and the transmission of a financial shock occurs as a response to changes in these quantities.

As the capital inflows decrease, or the interest rates increase, credit in the domestic economy will tighten, which can endogenously increase the required return on borrowing. The amount of cash on hand that the banker has is very important for the response of the home economy. For example, if r^f decreases and r^R increases as in 2009, then countries with low levels of bank cash relative to their domestic financing demands would be forced to borrow from abroad at a higher rate, increasing the required return and tightening credit. On the other hand, a well-funded economy with excess bank cash relative to domestic borrowing demands would actually benefit, as the opportunity cost for funds would decrease with the risk free investment rate. Changes in the bank capital inflows also affect credit tightness. Similar to what Forbes and Warnock (2012) find relating booms and busts with capital inflow increases and declines, capital flows to a country are positively related to output due to easing credit conditions and vice-versa.

An increased required return on borrowing would adversely affect firms, by making it more costly to finance the wages paid up-front and the fixed cost of exporting. The former leads to decreased output for firms as the marginal cost of labor increases, and the latter makes firms less likely to choose to export.

On the government side, increased government borrowing costs make issuing debt less attractive. Higher levels of government borrowing will crowd out home firms, making it more costly for firms to produce and lower output. The interaction between these two sides, however, goes beyond this simple crowding out effect. Since both sets of agents are competing for the same funds, each group's risk of default will affect the other through banks, with the potential to endogenously alter the required return for all borrowers. Further, when selling more bonds to the banker, the government increases the leeway for collateralized borrowing this period, and the cash on hand that the banker will have in the following period, assuming that the government does not default. In other words, by increasing the amount of safe bonds issued, the government is able to increase the assets retained in the home banking sector, at the cost of increased crowding out and repaying more in the following period.

When the government defaults, there is a significant reduction in bank balance sheets and a closing of the bond repurchase agreement market, with credit tightening effects analogous to a reduction in the capital inflows. A government default can increase credit tightness to such a degree that it induces firms to default, with real output costs. These dynamics produce endogenous output declines during default periods and are the deterrent for sovereign default.

4.6 Definitions of Model Equilibria

In this section I provide definitions for the model's equilibria. The first is for the competitive equilibrium given the home government's actions, and the second is for the government's policies knowing the effects it will have on the other agents in the model.

Definition 1. *Recursive Competitive Equilibrium Given Government Policies*
A competitive equilibrium given government policies is sequences of:

- *Allocations* $\{\{c_{jits}\},\{f_{jts}\},K_t,L_t,R_t,\{I_{ts}\},B^{supply}_{t+1},\{\kappa^{supply}_{it}\},$ $\{m_{jits}\},\{Q_{its}\},\{\kappa^{demand}_{it}\},\{N_{it}\},\{n_{jt}\}\}^{\infty}_{t=0}$
- *Firm default* $\{\{d_{its}\}\}^{\infty}_{t=0}$ *and export* $\{\{x_{it}\}\}^{\infty}_{t=0}$ *choices*
- *And prices* $\{\{p_{its}\},\{h_{it}\},w_t,\{\pi_{its}\},\{\Pi_{ts}\}\}^{\infty}_{t=0}$

such that given sovereign bond prices $\{p_{Bt}\}^{\infty}_{t=0}$*, government policies* $\left\{D_t,T_t,B^{demand}_t\right\}^{\infty}_{t=0}$*, and shocks* $\{Z_{t-1},W_{t-1}\}^{\infty}_{t=0}$ *the following hold:*

- $\{\{c_{jits}\},\{f_{jts}\},\{n_{jt}\}\}^{\infty}_{t=0}$ *solve the households' problems in 7-12 subject to the budget constraints 2-3*
- $\{\{m_{jits}\},\{Q_{its}\},\{\kappa^{demand}_{it}\},\{N_{it}\},\{p_{its}\},\{d_{its}\},\{x_{it}\}\}^{\infty}_{t=0}$ *solve the firms' problems in 13 and discrete export and default decision rules, subject to the constraints described in its full problem in 4*
- $\{K_t,L_t,R_t,\{I_{ts}\},B^{supply}_{t+1},\{\kappa^{supply}_{it}\}\}^{\infty}_{t=0}$ *solve the banker's problem 14-15, meet its budget constraints 5-6, and its* Γ *rule given leverage.*
- *And markets clear:*

$$B^{supply}_{t+1}=B^{demand}_{t+1};\qquad \kappa^{supply}_{tt}=\kappa^{demand}_{tt};\qquad n_{Ht}=\int_0^1 N_{it}di;$$

$$n_{Ft}=0;\qquad m_{jits}=c_{jits};\qquad \&\quad \Pi_{ts}=\int_0^1 \pi_{it}di$$

for all $j\in\{H,F\}$*,* $i\in[0,1]$ *and states* $s\equiv\{Z,W\}\in\Omega$*.*

Definition 2. *Markov Perfect Equilibrium*
The Markov perfect equilibrium for this economy is government borrowing, default and taxation rules $(B'(B,Z_{-1},W_{-1}),D(B,Z_{-1},W_{-1}),T(B,Z_{-1},W_{-1}))$ *with associated value functions* $V(B,Z_{-1},W_{-1})$ *and* $V(B,Z_{-1},W_{-1})\mid_{D=1}$*, consumption and labor plans* $\{\{c_{His}(B,Z_{-1},W_{-1})\},$

$\{f_{Hs}(B, Z_{-1}, W_{-1})\}, \{n_H(B, Z_{-1}, W_{-1})\}\}$, *and an equilibrium government bond pricing function* $p_B(B'; B, Z_{-1}, W_{-1})$ *such that:*

- *Given the price* $p_B(B'; B, Z_{-1}, W_{-1})$*, the borrowing and default rules solve the government's maximization problem 16-17*
- *Given the price* $p_B(B'; B, Z_{-1}, W_{-1})$ *and the borrowing and default rules, the consumption and labor plans* $\{\{c_{His}(B, Z_{-1}, W_{-1})\}, \{f_{Hs}(B, Z_{-1}, W_{-1})\}, \ \{n_H(B, Z_{-1}, W_{-1})\}\}$ *are consistent with the competitive equilibrium defined above*
- *Given the prices* $p_B(B'; B, Z_{-1}, W_{-1})$ *and the borrowing and default rules, the taxation rule* $T(B, Z_{-1}, W_{-1})$ *satisfies the government's budget constraint*
- *The government bond price equilibrium function* $p_B(B'; B, Z_{-1}, W_{-1})$ *satisfies the banker's bond pricing condition in equation 15*

for all $i \in [0, 1]$ *and states* $s \equiv \{Z, W\} \in \Omega$.

5 Model Calibration

Since the model is run for several different scenarios — countries both near to and far from the crisis zone in both non-crisis and crisis times — there are multiple levels of parameters that must be set.[30] First, Table IV summarizes parameters that are the same across all scenarios and are taken from standard values in the literature or individually matched to specific moments in the data. For the distribution of firms, I assume there are two types based on their productivity factors. This distribution is chosen to simulate tradable and non-tradable sectors of the economy. A fraction $\mathcal{H}$ of firms are high cost and the remainder low cost, with the difference being in the productivity factor of firm i where $a_i \in \{a_L, a_H\}; \ \ a_L < a_H$. The fraction of firms that have a high cost parameter is selected to match the portion of US manufacturing firms that do not export from Bernard et al. (2012), 0.82. The low cost parameter (a_L) is given a normalized value of 1.00, and the high cost parameter (a_H) is selected to match the average productivity ratio of the top 18% of firms to the bottom 82% using estimates of the distribution of firm productivities from Del Gatto et al. (2007).

Different distances to the crisis zone across the near and far countries are modeled using varying levels of iceberg trade costs and bank capital inflows, A_t. The fixed costs of trade are assumed constant, and the financial inflows are assumed to exogenously shift with the

[30] Further details on the calibration and parameter setting can be found in the Online Appendix.

foreign state. The target total trade costs for the near and far countries are derived from previous studies of international trade costs, as detailed in the Online Appendix. The bank capital inflow levels are chosen to match the BIS banking claims data. The average banking claims of the crisis areas as a percentage of trend GDP on countries in the near and far groups for 2007 were used for the non-crisis times levels, and for 2008-2013 for the crisis levels. These levels — along with the changes between non-crisis and crisis times — are provided in Table V. As can be seen, the near countries had greater banking claims before the crisis occurred, but then had their average banking claims decline by over 10% as much as did the level for the far countries.

There are several other parameters that define the states of the world: interest rates; Z levels; W levels; and transition probabilities between these. The interest rates are assumed to be determined in the foreign country so their values depend on the foreign state. This setup is inspired by risk tolerances in the major financial centers of the US and Eurozone connecting the states of their economies with global interest rates. The real interest rates from 1-year US Treasury bills are used for r^f and real LIBOR rates are used for r^R. The levered borrowing rate is set to 30% to match data on yields observed during defaults.[31] Table VI provides the rates used.

The Z and W production shocks follow two and four state random Markov processes independent of one another. The Z states represent high and low productivity shocks for the home economy. The mid Z value is selected to normalize the expected non-crisis home steady-state production to 100. I use the Barro and Ursua (2008) real GDP per-capita data from 1870-2007 for the available countries in my two non-crisis area regions to determine the amount by which to adjust Z for good and bad draws based on historical outcomes.[32] The mid-value for the foreign endowment is set to 100*(2007 aggregate crisis zone GDP) / (Average 2007 non-crisis zone country GDP) to get the proper relative sizes for the home and foreign economies. The W_t levels are similarly taken by looking at historical levels of output in the crisis zone during booms, recessions and crises.

The total state transition matrix (provided in the Online Appendix) is determined using the foreign state probabilities, the Z transition probabilities, and that the home and foreign

[31] Durbin and Ng (2005) and Neumeyer and Perri (2005) find bond yields spiking to 25-40% during government defaults. To support my results I only need a levered borrowing rate of 19%, well below this range.

[32] The available countries far from the crisis areas were: Argentina, Australia, Brazil, Chile, China, India, Indonesia, Japan, Korea, Malaysia, New Zealand, Peru, Philippines, South Africa, Singapore, Sri Lanka, and Uruguay. The available countries near to the crisis areas were: Canada, Colombia, Egypt, Greece, Iceland, Mexico, Norway, Russia, Sweden, Switzerland, Turkey, and Venezuela.

parameters follow independent Markov processes. Additionally, it is assumed that there is a 2% chance of entering a crisis state from a non-crisis one — reflecting the two major global crises of the Great Depression and late 2000s global crisis period that were entered in the last hundred years — and crises are expected to continue into another crisis period with an 80% chance — to give a five year expected duration to match the two actual crises. This means that on average there is a crisis abroad 10% of the time.

Finally, there are three sets of parameters that are simultaneously determined to match specific moments in the data. The iceberg trade costs and Z levels for the representative near and far countries were already mentioned above. Additionally, there are the home and foreign consumers' utility coefficients on the foreign good, α_H and α_F. For symmetry, the home and foreign coefficients on the foreign good are assumed to be multiplicative inverses. They then are chosen to match the levels of home exports seen in the data. The values selected for these parameters and the model moments versus their targets can be seen in Table VII.

6 Results

This section examines the results of the model analysis in three steps. First, the results of the model calibrated to fit the average experiences of countries near to and far from the crisis zone are described. Second, a series of counterfactual scenarios are reviewed wherein only the trade or finance channel shock occurs for representative near and far countries. Finally, I examine the effectiveness of capital controls for the near countries to mitigate the high degree of financial contagion they experienced.

6.1 Modeling Near & Far Countries during 2000s Crisis Period

This section describes the results of applying the late 2000s trade and financial shocks to the representative near and far countries. To model the trade channel, the changes in the foreign endowment are matched to the changes in aggregate crisis zone detrended real GDP. To model the financial channel, the interest rates and foreign banking claims are matched to those observed in the data for each country group.

To evaluate the results of these scenarios, a comparison of the steady-state values for the non-crisis and crisis periods can be found in Table VIII. The non-crisis times columns provide information on the steady-state when there is a high productivity draw at home and the best draw of the foreign endowment, and the crisis period columns are for the steady-state for a good home draw and the late 2000s equivalent crisis state abroad.[33] Choosing this set of states as the point of comparison isolates the effects of the crisis, without conflating them with the consequences of a negative home productivity draw. The corresponding real world values are provided for several fields in italics in the margins next to each of the columns, with explicitly targeted values underlined.

The first row contains information on the expected levels of home production and is the benchmark for measuring contagion.[34] Crisis period output levels were not explicitly targeted, but the model is able to produce changes in output similar to what was measured in the data for both the near and far countries. For the representative near country, the model's contagion from abroad results in a 12.2% decline in output against a 12.1% decline in the data. For the far country, production increases 0.2%, which is similar to the actual 0.9% increase in average output for the far country group by the end of the crisis period in 2013 as shown in Figure I. These results reflect greater contagion for those countries closer to the crisis zone, a prominent feature of the data.

While the aggregate output reductions are replicated well by the model, the declines in trade for the two countries are too small. For the near country, the model has trade falling from 14.1 to 13.0, as opposed to a 14.0 to 10.2 decline in the data. For the far country, trade drops from 7.8 to 7.4 as opposed to the 7.9 to 7.0 decline in the data. The model does, however, have near countries' trade declining by about twice the rate of far ones, as actually occurred.

Another shortcoming is that the government debt levels produced by my model are significantly lower than the actual levels. This is an issue endemic to models such as this in the literature.[35] The differences between the debt levels in the near and far countries do, however, align well with two features of the data: the nearly static levels of near country debt; and greater borrowing by far countries during both non-crisis and crisis periods.

The differences in crisis output between the two representative countries are closely tied with the governments' decisions. For the near country, the government chooses to leave the level of debt largely unaltered, while the far country government implements an austerity policy, paying off a fraction of its debt to reduce its future financing burden and crowding out.

To help explain these decisions and study the dynamics around a crisis period, Figures

[33] This was selected as the basis of comparison because 86% of non-crisis countries had above trend growth in 2007, as did the region that became the crisis zone.

[34] The home steady-state production levels are expectations over the home productivity draw conditional on remaining in the same foreign state at the end of the current period.

[35] See, for example, Aguiar and Gopinath (2006) and Mendoza and Yue (2012).

XI and XII plot the initial transitions from the non-crisis times steady-states in Table VIII to the crisis ones. Each of the panels in Figures XI and XII plots twelve periods: from five years before through six years after a crisis that commences during the production stage of T=0, corresponding with 2008.[36] The conditional expected values over the level of home productivity and associated government debt levels are plotted in solid black, and the dashed lines are two standard deviation confidence intervals.

The plots in the first row of Figure XI show the expected values of aggregate home production in the model, along with the real-life country group averages. The near country output initially declines towards the crisis steady-state of 87.9, then levels off close to that point and remains there. The far country output also declines initially and — similar to 2008-2009 — there is little difference between the near and far country averages in the first years of the crisis. Further, as in the data, in the year the crisis period commences there is virtually no decline in output for the non-crisis countries, and only during the second full year of the crisis does far output significantly decline before recovering toward the pre-crisis level. The dip for the far group is greater in the model than in the data as the model government chooses an austerity policy, leading its economy to grow beyond the actual far countries in the subsequent years. Except in one case, the actual levels are within the model's two standard deviation confidence intervals, though the model economies react faster and approach the steady-state output levels in fewer periods than what was observed in the data.

The output declines are closely associated with decreases in the after-tax real wages. Between the non-crisis and crisis period steady-states, hours worked decline 17.3% in the near economy because of lower wages and an increased tax burden. As can be seen in the bottom row of the figure, the tax rate for the near country increased 4.2%, driving a 6.1% decline in the after-tax real wage. There was also a shift of labor into the low cost, exporting firms as home income declined.

Alternately, wages declined by 3.7% in the far country, though this was offset by a decrease in the tax rate after an initial spike due to the government's austerity measures. For the far country, there was also a shift between labor utilization in the high and low cost industries; however, this time the shift was into the domestic, non-tradable sector as home

[36] The state entering period zero is assumed to be high output draws of both the home productivity factor and foreign state, at the corresponding steady-state government debt level. For the five preceding periods, the foreign state draw is assumed to be a high one, and the average value across the home productivity states and associated government debt levels is taken conditional on there being a high home productivity realization entering period zero. Beginning during production in period zero and continuing through the subsequent six periods, it is assumed that there is a draw of the foreign state corresponding to the late 2000s crisis period.

income increased while the foreign endowment decreased.

Figure XII presents some graphs to help explain the choice of the far country to follow an austerity policy of paying down its debt, while the near country marginally increased its tax rate to keep its debt level unchanged. The first row shows the debt levels for the two countries, and the second row the levels of bank cash at the start of each period. In both the near and the far countries, the bank capital inflows fall once the crisis begins. The level of bank cash in the near economy falls one for one with the 13.9% decrease in its capital inflows because the level of government debt changes little, while the bank cash for the far country falls by more than the 3.3% decline in its bank capital inflows. The third row provides some insight into why this is. The two plots in that row show the level of bank cash minus the aggregate amount of firm borrowing in each economy. In both economies, prior to the crisis the amount that the bank had to borrow to support the home economy's financing was less than the amount of government debt, placing the economies on the central, flat portion of Figure IX. This position corresponds to a required return on borrowing equal to the government bond repurchase agreement rate, which was 3.52% before the crisis.

After the beginning of a crisis at T=0, credit conditions in the near economy tightened, shifting it to the right hand vertical edge of the aggregate interest rate schedule between the government bond repurchase and uncollateralized borrowing rates. This situation can be recognized by the fact that aggregate firm borrowing is equal to the level of bank cash for the latter few periods as the plot line reaches zero. The degree of credit tightness pushed rates up to 10.79%, whereas in the far economy the required return actually declined to the new, lower collateralized borrowing rate of 0.95%. It remained below 1% for five periods, before gradually increasing to 5.52% after nine periods in a crisis, a level that is still roughly half that of the near economy.

A lower A_t level left the near economy in a position where it did not want to deleverage, which would lead its banking sector to do so as well. Conversely, the far country was able to undertake austerity measures without causing significant deleveraging. In neither case did the government wish to default and accept the resultant destruction of bank wealth, which would have caused a greater contraction in the real economy by further increasing the required return on firm borrowing and potentially inducing the firms themselves to default with real output losses.

6.2 Separating Trade & Financial Contagion: Counterfactuals

To determine the relative importance of the trade and financial channels for transmission from the crisis zone, four counterfactual simulations were run. In these simulations, shocks to the trade and financial channels were included separately. For the case with only a trade channel shock, the bank capital inflows and interest rates on international investments remained at the non-crisis period levels and only the foreign endowment changed during a crisis. In the finance only case, the reverse was done, matching the crisis period's finance channel changes while leaving the foreign endowment at a non-crisis level.

The results from the counterfactual analysis are given in Table IX. There are six columns of data presented in the table, those being pairs of data for representative near and far countries when exposed to contagion through: both transmission channels; only the trade channel; and only the financial channel. Each column lists the changes between the non-crisis and crisis period steady-states. The first two columns — containing information on contagion through both channels simultaneously — are identical to the difference columns in Table VIII and serve as baselines for the other scenarios.

Applying only the trade shock resulted in small declines in output on the order of 2%, as the decline in the foreign endowment flowed through the trade channel for the near and far countries. Additionally, the exporting, low cost firms experienced greater declines in labor utilization than did the non-trading firms, corresponding to a labor shift into the non-tradable sector. These changes are not surprising, given that the lower demand from abroad would flow to the home economy through the decreased export demand faced by the low cost firms. Government borrowing levels and tax rates remained largely unchanged. The changes in labor utilization (-1.5% for the near country and -1.0% for the far one) and wages (-0.6% and -0.8%) were minor enough that the steady-state levels of firm borrowing were relatively constant, leaving the required return on borrowing unchanged in each case.

Figure XIII provides transition plots of expected home output for each of the six cases of near and far countries crossed with the three combinations of international shocks. The first row corresponds to the two cases already discussed with both trade and financial shocks. The second row shows the results for only the trade shock. From these two plots, it is clear that the effects of a trade shock alone were relatively minor and nearly indistinguishable between the near and far countries.

The final two columns of Table IX show the results of only applying the financial shock. Doing so results in an 11.4% output decline for the near country, and a 2.2% output increase for the far one. The results and associated dynamics are similar to what occurred when both the trade and financial shocks were applied, but with the economic contraction muted absent the decline in the foreign endowment. Looking at the plots in the first column of Figure XIII, one can see the extreme similarity between the cases with only the credit shock and with both shocks for the near country. For the far country, the results towards the end of the period were similar between the two cases, but the initial decline in output was significantly less without the exacerbating effects of the trade shock.

Examining the results in Table IX, it is evident that the trade shock was less impactful than the credit shock for both the near and far countries, with similar magnitudes for each. Far countries initally had lower average international banking claims and experienced smaller declines in them during the crisis period, making these countries less susceptible to a credit crunch. The lower interest rate environment during the crisis period, combined with the small change in the average bank capital inflows among far countries, made the credit shock alone a net positive for them. The counterfactuals suggest that the worst transmission of the late 2000s crisis period — that of the near countries — was driven by financial contagion rather than the trade channel. This result has important implications for policymakers looking to combat contagion in the future.

6.3 Capital Controls & Contagion Mitigation

Given the substantial degree of financial contagion experienced by the represenative near country, an obvious follow up is to examine what policies might have attenuated it. Since the decline in interest rates during the crisis period in and of itself would have been expansionary for the near country, it must have been the reduction in banking inflows that caused the contagion. Therefore, implementing capital controls to prevent the loss of banking funds appears to be a suitable policy to consider for the near countries.

To evaluate the efficacy of capital controls, the representative near country case is repeated while varying only the level of the crisis A_t value. To model different degrees of capital controls that could be implemented, values are considered ranging from the near countries' average crisis A_t level of 26.5 up through their non-crisis average of 40.4. The former would be equivalent to no capital controls, and the latter to complete capital immobility.[37]

Figure XIV plots the percentage decline in aggregate home country output during a crisis on the y-axis versus the crisis level of A_t on the x-axis. The "SS to SS contagion" line plots the non-crisis steady-state to crisis steady-state output differences. The "1st crisis period

[37] Note that this exercise does not account for the fact that implementing capital controls might make foreign investors more or less willing to invest in the home economy depending upon their reactions to decreased liquidity versus less severe crises.

contagion" line plots the decline in output from the non-crisis steady-state through the first full period of a crisis. In both cases, one unit on the y-axis is equivalent to a 1% decline in output from the non-crisis steady-state.

From the "SS to SS contagion" line, it is evident that the degree of steady-state to steady-state contagion generally decreases as the crisis A_t increases. This relationship suggests that capital controls are an effective tool for fighting financial contagion in the long run. The "1st crisis period contagion" line, however, suggests a more complicated relationship between capital controls and immediate financial contagion. At the left of the plot with low degrees of capital controls, the short run contagion is monotonically reduced with more restrictive controls. In these cases, the greater levels of the crisis bank A_t enforced by capital controls lead to less credit tightness and improved output during a crisis.

As stricter capital controls are implemented, the short run results improve to such a degree that they better the long run level of contagion, with a peak of only 5.3% short run contagion at a crisis A_t of 36. Over this range, the government pays down its debt over time, incrementally increasing credit tightness and shifting the economy towards the greater long run contagion level.

As the crisis A_t level increases to 37 or above — approaching the non-crisis times level — the short run contagion steeply increases. Over this range the government defaults since high capital controls guarantee adequate bank reserves. Therefore, the destruction of bank assets today caused by a default is no longer enough to incite a severe banking crisis and deter the government from defaulting. This is the region over which the assumption that the non-crisis A_t remains constant must be kept in mind, since this is where it is most likely to fail, potentially leading to even worse outcomes. Additionally, as can be seen in Figure XV, the non-crisis steady-state output starts to decline when defaults are realized during crises with high A_t because of compensating higher government borrowing costs. Worsening non-crisis output is the primary reason the steady-state contagion is reduced over the right hand range of Figure XIV, not that the crisis output is improved.

Taken together, these results suggest that moderate capital controls can mitigate financial contagion in both the short and long run. However, it is critical to consider whether they restrict the flow of capital so much as to induce government default during a crisis. If so, then lesser capital controls and correspondingly more severe crises may be worth enduring in order achieve higher non-crisis output, given that foreign crises are expected only 10% of the time. Furthermore, the outcomes may even be better with low levels of capital controls than suggested here, as less severe crisis transmission might attract more capital to the country as it becomes a safer environment in which to invest.

7 Conclusion

Episodes of cross-country crisis contagion occurred numerous times during the past century. These events can have severe consequences for large blocks of countries, or even the global economy as evidenced by the late 2000s crisis period.

In this paper, I identified new patterns in country growth rates during the late 2000s based on proximity to the US subprime mortgage and Eurozone debt crises through distance, trade, and finance. Economic growth in the countries near the crisis zone remained stalled years after the crises commenced. Countries that were farther from the crisis zone still had poor short-term economic performance, but on average they rose above their trend growth levels by 2012.

Using a model of firm and sovereign default with international trade, I find that the primary cause of the differences in country outcomes was heterogeneous exposure through the finance channel. To arrive at this result, I model two transmission channels that can be separately perturbed: cross-country trade and finance. I calibrate the model to the experiences of representative countries near to and far from the crisis areas. Using these calibrations, disturbances on the order of those observed during the late 2000s are applied to each channel to study transmission through them. For the representative near country, applying both the financial and trade shocks produces a GDP decline of 12.2%, compared with the 12.1% average decline in the data. Applying only the credit shock results in an 11.4% output reduction and only the trade shock in a 2.1% drop. For the representative far country, replicating both shocks produces 0.2% growth compared with the actual 0.9% average change, while separately simulating the credit and trade shocks produce 2.2% and -1.8% changes, respectively. The results suggest credit disruption as the primary contagion driver, rather than the trade channel. Finally, I find that the contagion experienced by near countries could have been significantly diminished by the use of capital controls.

This paper offers several avenues for future research and possible extensions. In this paper I analyze the near and far countries independent of one another, and an obvious first extension would be to add more countries to the model to study the third party network effects of contagion. Perhaps the reason that the countries farther from the crisis zone performed better over this period was because of greater levels of trade and banking with other non-crisis countries, rather than only lesser ties with the crisis areas. Additionally, having multiple countries borrowing from the same banking sector would allow for a better study of the interactions between governments' default decisions, as evidenced by the apparent coordination of government defaults in Reinhart and Rogoff (2009).

Over the late 2000s, many countries utilized monetary easing to combat the global recession. Also, many governments increased their debt levels beyond what my model produces. Adding money and a monetary authority could help analyze these decisions. Having endogenous levels of government spending rather than a fixed level could help evaluate expansionary fiscal policy versus a program of government austerity during the crises. Finally, higher levels of government reserves and improved exchange rate policies have been proposed as reasons why the developing world weathered the late 2000s crises better than previous ones. Adding currencies to the model would help analyze these propositions and shed light on the optimal local/foreign currency mix for government debt.

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Tab. I: 2013 Real GDP Per-Capita Detrended by Individual Country Growth vs 2007 Crisis Zone Proximity

	(1) Distance	(2) Trade	(3) Bank	(4) Bank and Trade
Distance	0.00223***			
	(0.000478)			
Trade PctGDP		-0.371***		-0.327***
		(0.0949)		(0.0992)
Bank PctGDP			-0.0351***	-0.0179**
			(0.0123)	(0.00866)
Constant	82.25***	100.2***	92.25***	99.66***
	(2.345)	(2.862)	(1.725)	(2.910)
Observations	134	132	130	128
R-squared	0.132	0.104	0.019	0.095

Note: Robust standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Distance is population weighted bilateral country distance from the nearest crisis zone country from CEPII. Trade is measured as 2007 bilateral imports and exports from the UN Comtrade database across the entire crisis zone as a percentage of GDP. Banking is measured as 2007 BIS banking claims of the crisis zone on a country as a percentage of GDP.

Tab. II: 2013 Real GDP Per-Capita Detrended by Individual Country Growth vs 2007 Crisis Zone Proximity Controlling for Chinese Exports

	(1) CHN Exp	(2) Distance and CHN Exp	(3) Trade and CHN Exp	(4) Bank and CHN Exp	(5) Bank, Trade and CHN Exp
CHN Exports PctGDP	0.0330	-0.151	0.0599	0.0931	0.0971
	(0.121)	(0.127)	(0.124)	(0.126)	(0.136)
Distance		0.00236***			
		(0.000528)			
Trade PctGDP			-0.373***		-0.328***
			(0.0952)		(0.100)
Bank PctGDP				-0.0343***	-0.0177**
				(0.0124)	(0.00864)
Constant	90.93***	82.57***	99.92***	91.59***	99.17***
	(1.737)	(2.360)	(3.064)	(1.898)	(3.151)
Observations	131	130	131	127	127
R-squared	0.000	0.136	0.105	0.021	0.098

Note: Robust standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Distance is population weighted bilateral country distance from the nearest crisis zone country from CEPII. Trade is measured as 2007 bilateral imports and exports from the UN Comtrade database across the entire crisis zone as a percentage of GDP. Banking is measured as 2007 BIS banking claims of the crisis zone on a country as a percentage of GDP. CHN Exp are the 2007 exports to China as a percentage of GDP.

Tab. III: Crisis Zone Proximity Quartile Mean and Median 2013 Real GDP Per-Capita Detrended by Individual Country Growth

Measure	Quartile	N	Mean	Median
	1	34	81.42	81.84
Distance to	2	33	88.14	90.19
Crisis Zone	3	34	94.79	98.57
	4	33	100.60	94.12
	1	33	83.75	88.42
2007 Trade with	2	33	88.15	86.25
Crisis Zone PctGDP	3	33	95.12	97.75
	4	33	97.45	94.12
	1	33	86.93	86.04
2007 Fin with Crisis	2	32	91.98	93.14
Zone PctGDP	3	33	89.71	89.80
	4	32	96.64	95.92

Note: Quartiles arranged so lower numbers are always more connected to the US and Eurozone by the given measure.

Tab. IV: Model Calibration — Common Parameters

Type	Parameter	Description	Value	Reasoning
Bank	δ	Banker's discount rate	0.99	Standard value from literature
Utility	β	Household's discount rate	0.99	Standard value from literature
Utility	σ	Inter-variety elasticity of substitution	5	Standard value from literature
Utility	ω	Curvature of labor disutility	1.33	Macroeconomic Frisch wage elasticity of 3 (Peterman (2016); Prescott (2004))
Utility	ρ	Household risk aversion	2	Standard value from literature
Firms	$\mathcal{H}$	Fraction of high cost firms	0.82	Fraction US manufacturers not exporting from Bernard et al. (2012)
Firms	a_L	Low cost productivity parameter	1	Normalized value
Firms	a_H	High cost productivity parameter	2.65	Average productivity ratio between top 18% of firms & bottom 82% using Del Gatto et al. (2007) estimated productivity distribution
Firms	α	Labor share in output	0.64	From Hansen (1997)
Firms	γ	Fixed export cost	10	High cost firms do not export
Firms	μ	Fraction of production lost when a firm defaults	0.7	Firms repay debt in best state
Govt	G	Government spending	15	Match government spending / GDP for non-crisis countries
Govt	ξ	Government discount factor	0.8	5 year government time horizon
Govt	ϕ	Fraction of debt remaining after default	0.6	Sturzenegger and Zettelmeyer (2008) country default data

Tab. V: Model Calibration — Average BIS Banking Claims as a Percentage of GDP

Country Group	2007	2008-2013	Difference
Near to Crisis Zone	40.4%	26.5%	-13.9%
Far from Crisis Zone	31.7%	28.4%	-3.3%

Note: The Bahamas, Bermuda, and Hong Kong were excluded because, as major international banking sectors with small economies, their banking claims to GDP ratios were not representative of the larger groups.

Tab. VI: Model Calibration — Gross Interest Rates

Parameter	Description	Non-Crisis Good Draw	Non-Crisis Bad Draw	Late 2000s Crisis	Depression
r^f	Risk free investment rate	1.0312	1.0465	1.0005	1.0536
r^R	Bond repurchase agreement rate	1.0352	1.0513	1.0095	1.0844
r^H	Uncollateralized borrowing rate	1.3000	1.3000	1.3000	1.3000

Note: The real interest rates from 1-year US Treasury bills are used for r^f and real USD LIBOR rates are used for r^R.

Tab. VII: Model Calibration — Parameters Simultaneously Determined to Match Data Moments

Parameter	Description	Value	Reasoning
α_F	Foreign consumers' utility coefficient on foreign good	1.605	Match trade levels.
α_H	Home consumers' utility coefficient on foreign good	0.623	$= \frac{1}{\alpha_F}$
τ	Iceberg trade costs	1.79 near; 2.28 far	Target total trade costs
Z_G	Home high productivity draw	30.06 near; 32.99 far	Expected aggregate home output = 100.
Z_B	Home low productivity draw	27.48 near; 30.16 far	Expected aggregate home output = 100.

Targeted Quantity	Target Value	Model Value
Near country trade	14.0	14.1
Far country trade	7.9	7.8
Near country total trade costs	0.91	0.92
Far country total trade costs	1.51	1.51
Near country expected output	100	100.0
Far country expected output	100	100.1

Tab. VIII: Model Results — Near & Far Non-Crisis vs Crisis Period Steady-States

	Both Trade and Financial Shocks									
	Near					Far				
Description	Non-Crisis Period		Crisis Period		Diff	Non-Crisis Period		Crisis Period		Diff
Expected Home production	100.0	*100.0*	87.8	*87.9*	-12.2%	100.1	*100.0*	100.3	*100.9*	0.2%
Expected trade	14.1	*14.0*	13.0	*10.2*	-7.4%	7.8	*7.9*	7.4	*7.0*	-4.9%
Government Debt	15.2	*32.6*	15.2	*31.5*	0.0%	33.3	*43.4*	21.9	*56.3*	-34.3%
Tax rate	30.5%		34.7%		4.2%	31.4%		29.0%		-2.4%
Wage	4.07		4.03		-0.8%	3.77		3.63		-3.7%
After-tax wage	2.83		2.63		-6.8%	2.59		2.58		-0.3%
After-tax real wage	2.27		2.13		-6.1%	2.33		2.34		0.6%
Labor supply	12.13		10.02		-17.3%	13.09		13.35		2.0%
Low cost firm labor	38.94		32.39		-16.8%	40.64		41.31		1.6%
High cost firm labor	6.24		5.11		-18.1%	7.04		7.22		2.6%
Required effective return	3.52%		10.79%		7.27%	3.52%		5.52%		2.01%

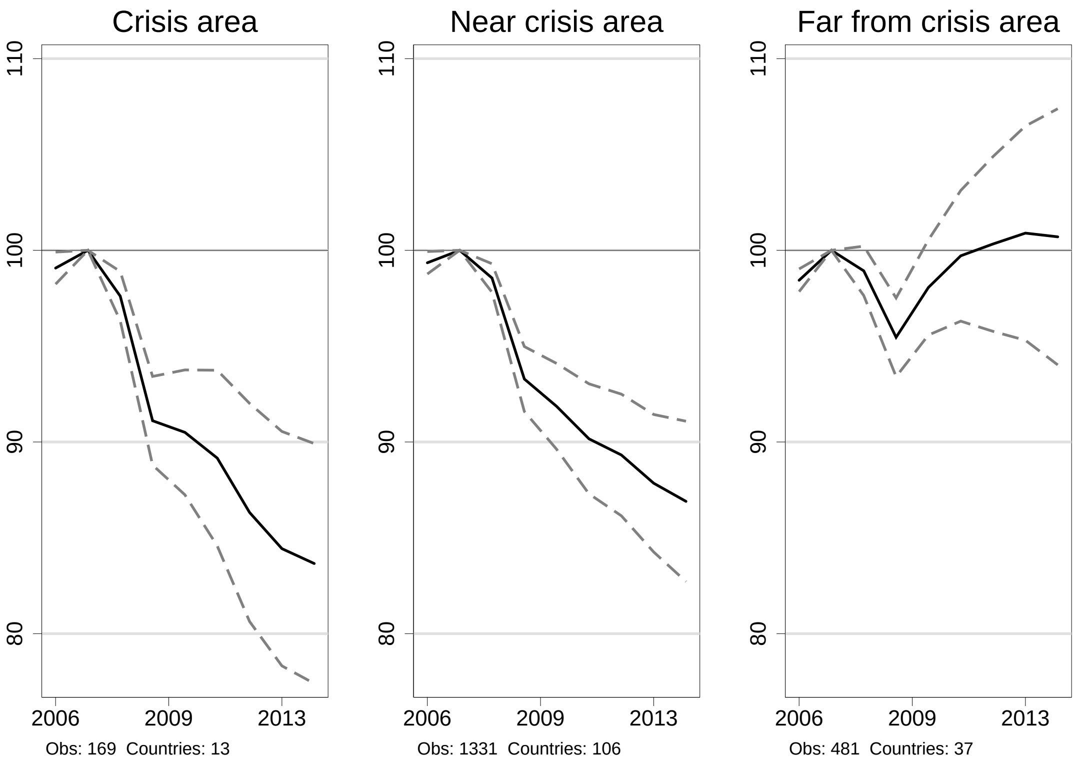
Note: Real world values provided in italics next to relevant items. Underlined real world values were explicitly targeted for model calibration. The Diff columns show percentage changes between con-crisis and crisis times for items measured in quantity terms and differences for items measured as percentages.

Tab. IX: Model Results — Non-Crisis vs Crisis Period Steady-State Scenario Analysis

	Both Shocks		Only Trade Shock		Only Credit Shock	
Description	Near	Far	Near	Far	Near	Far
Expected Home production	-12.2%	0.2%	-2.1%	-1.8%	-11.4%	2.2%
Expected trade	-7.4%	-4.9%	-4.9%	-5.0%	-3.1%	0.0%
Government Debt	0.0%	-34.3%	0.0%	0.0%	0.0%	-31.4%
Tax rate	4.2%	-2.4%	0.5%	0.4%	4.0%	-3.0%
Wage	-0.8%	-3.7%	-0.6%	-0.8%	-0.5%	-3.2%
After-tax wage	-6.8%	-0.3%	-1.4%	-1.4%	-6.1%	1.1%
After-tax real wage	-6.1%	0.6%	-0.6%	-0.4%	-6.3%	1.0%
Labor supply	-17.3%	2.0%	-1.5%	-1.0%	-17.7%	3.4%
Low cost firm labor	-16.8%	1.6%	-1.9%	-1.3%	-16.8%	3.2%
High cost firm labor	-18.1%	2.6%	-1.1%	-0.7%	-18.9%	3.6%
Required effective return	7.27%	2.01%	0.00%	0.00%	8.33%	2.09%

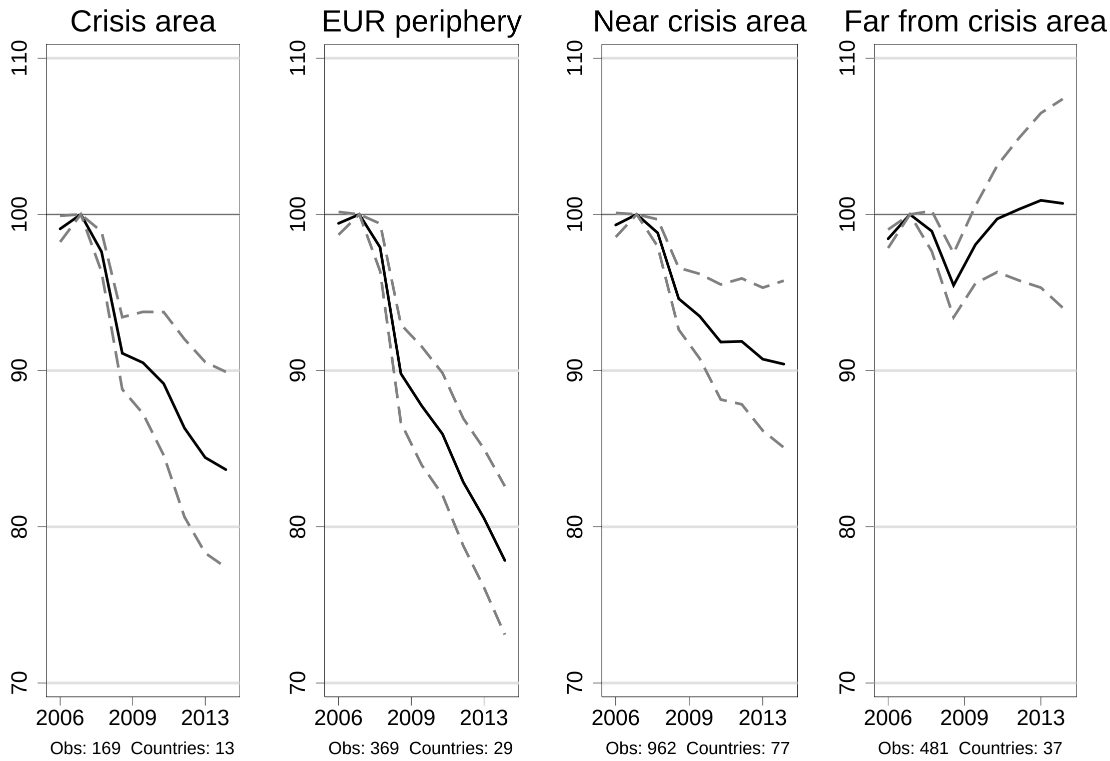
Note: The columns show changes between non-crisis and crisis times, calculated as percentage changes for items measured in quantity terms and in differences for items measured as percentages.

Fig. I: Average Detrended Real GDP Per-Capita Index by Country Distance Groups



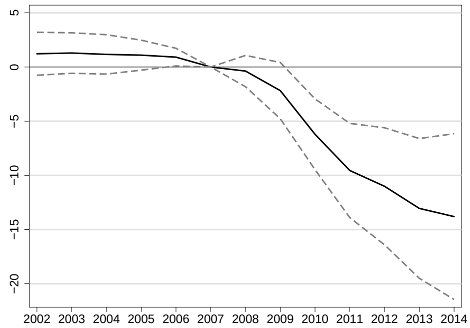
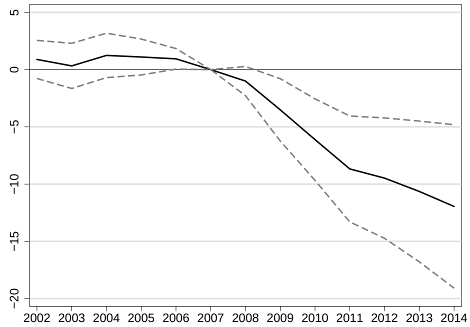
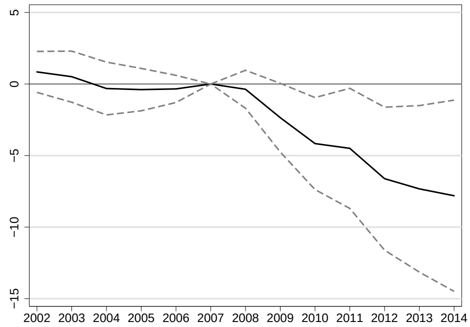
Note: Base year of 2007 is 100. Detrended by individual country growth from 2001 to 2007. The dotted lines are 95% confidence intervals. The crisis zone is the US and Eurozone entering 2007 (Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain). κ-clustering based on minimum distance to a crisis zone country was used to determine the near and far country groups.

Fig. II: Average Detrended Real GDP Per-Capita Index by Country Distance Groups, with the European Periphery Broken Out



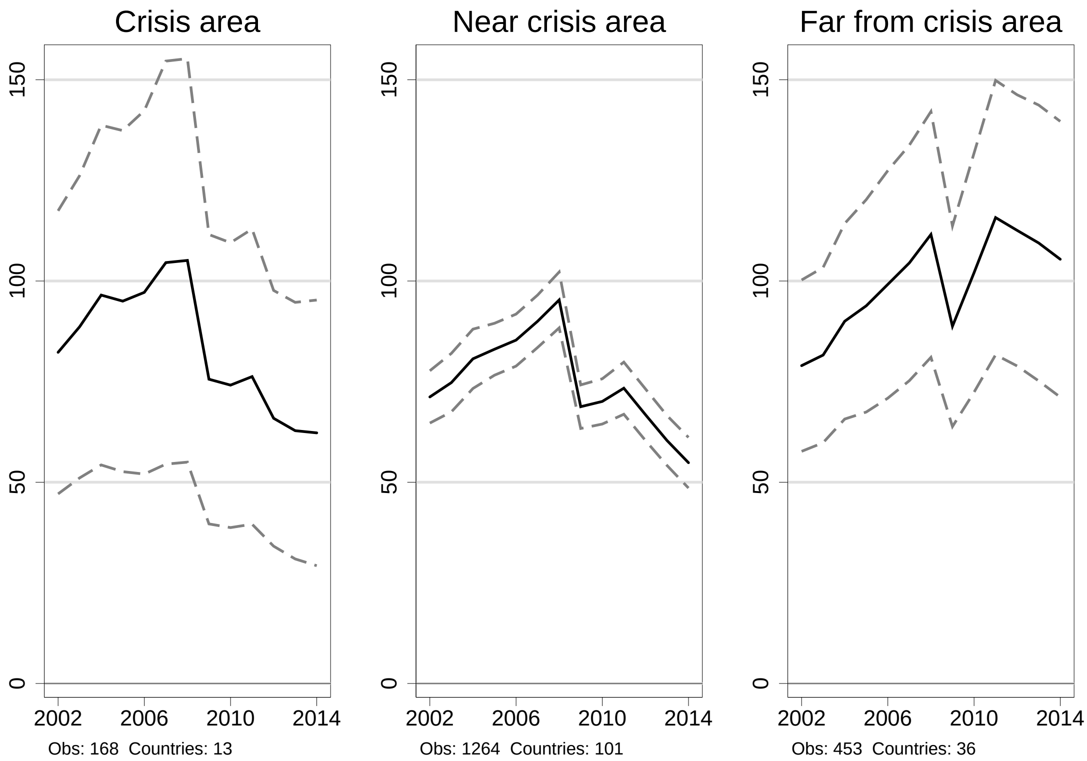
Note: Base year of 2007 is 100. Detrended by individual country growth from 2001 to 2007. Countries within 1,000 miles of the Eurozone are separated into the EUR periphery group. The dotted lines are 95% confidence intervals. The crisis zone is the US and Eurozone entering 2007 (Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain). κ-clustering based on minimum distance to a crisis zone country was used to determine the near and far country groups.

Fig. III: Average Detrended Real GDP Per-Capita Index Differences Between Close & Far Country Groups

Crisis Zone Proximity Measure	**Plot**
Distance to the crisis zone	
2007 Trade with the crisis zone / GDP	
2007 BIS banking with the crisis zone / GDP	

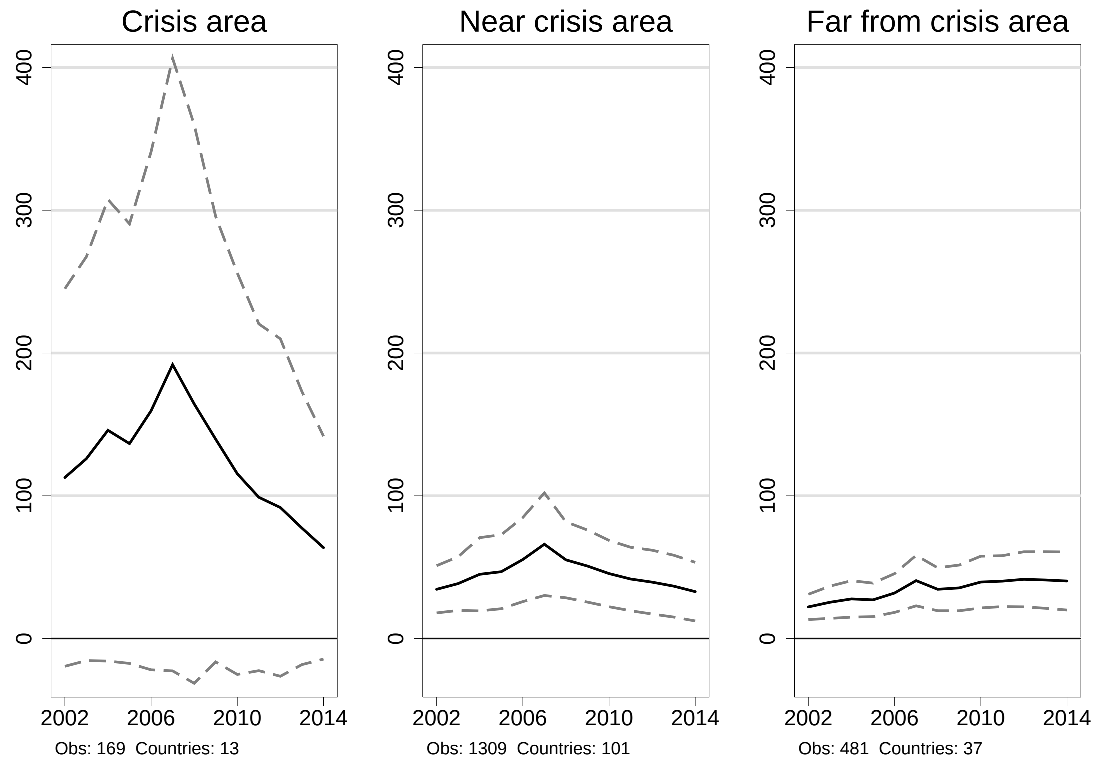
Note: Base year of 2007 is 100. Detrended by individual country growth from 2001 to 2007. Plots of yearly dummy variables of differences between the two non-crisis groups. Lower values mean the group that was more connected to the crisis zone did on average worse than the less connected group. The dotted lines are 95% confidence intervals.

Fig. IV: Average Trade with the Rest of the World by Country Distance Groups



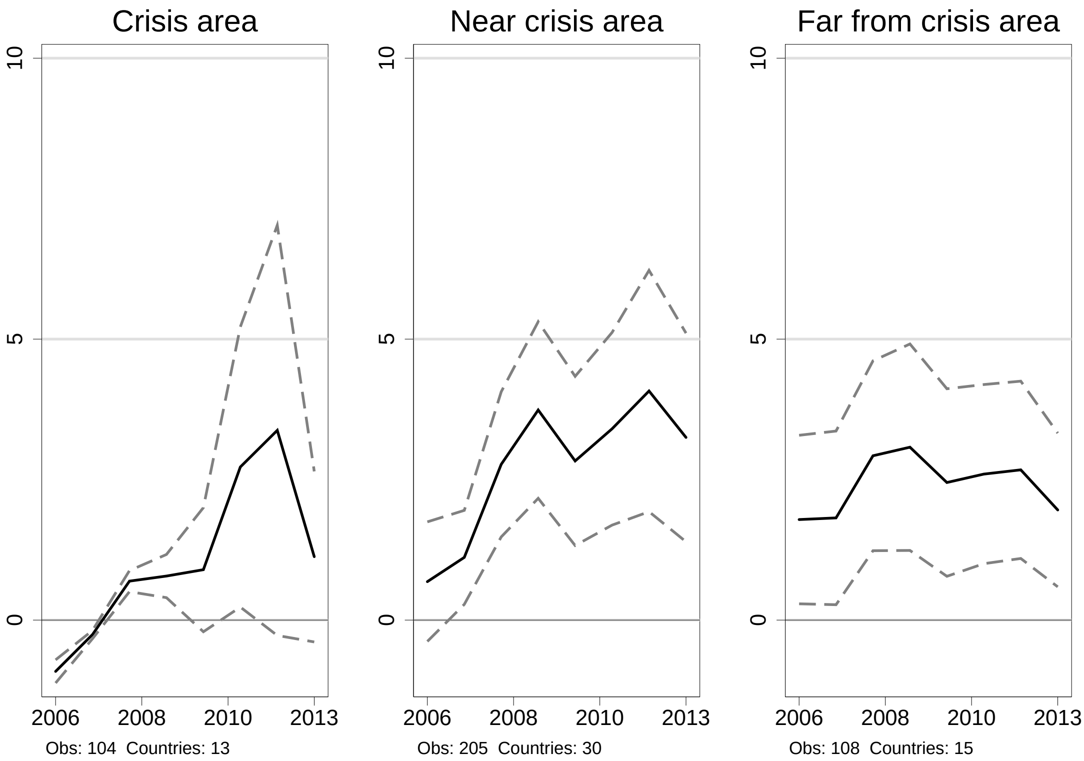
Note: The dotted lines are 95% confidence intervals. The crisis zone is the US and Eurozone entering 2007 (Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain). κ-clustering based on minimum distance to a crisis zone country was used to determine the near and far country groups. Divided by trend GDP extrapolated from 2007 using individual country growth from 2001 to 2007.

Fig. V: Average Banking Claims of the Rest of the World by Country Distance Groups



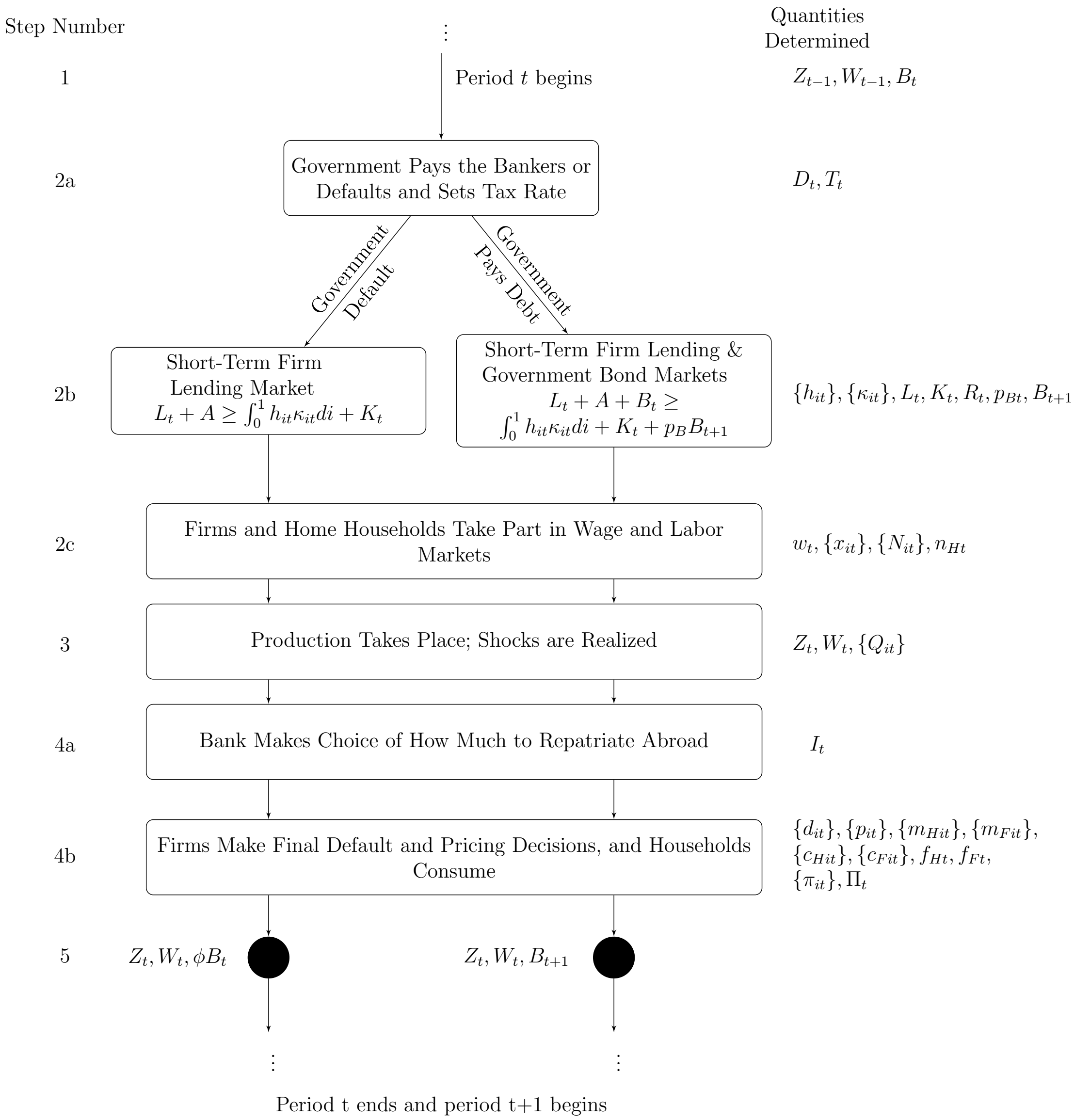
Note: The dotted lines are 95% confidence intervals. The crisis zone is the US and Eurozone entering 2007 (Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain). κ-clustering based on minimum distance to a crisis zone country was used to determine the near and far country groups. Divided by trend GDP extrapolated from 2007 using individual country growth from 2001 to 2007.

Fig. VI: Average Government Bond Spread vs US 10-Year Treasury Bonds by Country Distance Groups



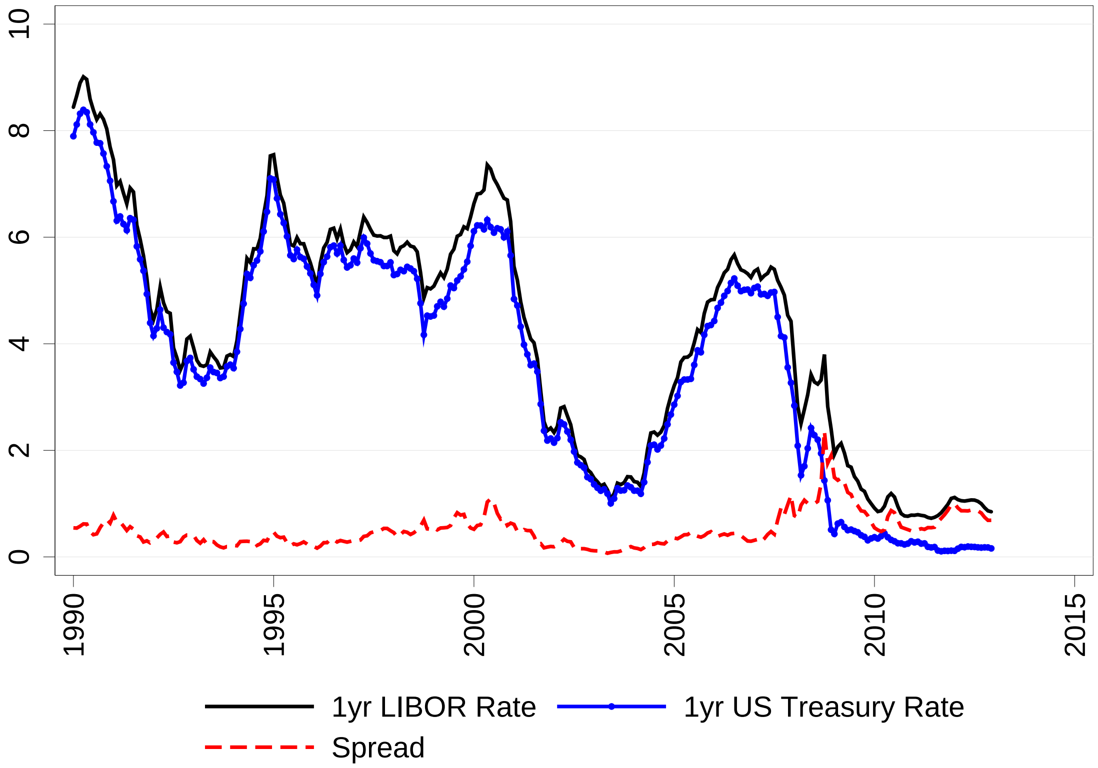
Note: The dotted lines are 95% confidence intervals. The crisis zone is the US and Eurozone entering 2007 (Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain). κ-clustering based on minimum distance to a crisis zone country was used to determine the near and far country groups. The plots show each group's annual average government bond yield spread over the US 10-year bond.

Fig. VII: Period Timeline



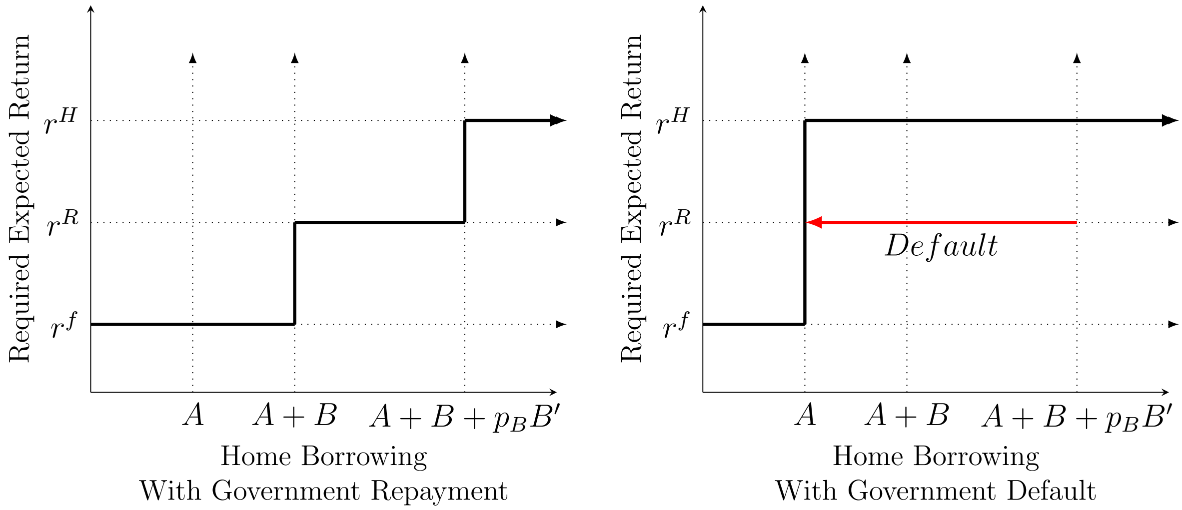
Note: All steps with the same number occur simultaneously.

Fig. VIII: US Dollar Based Interest Rates



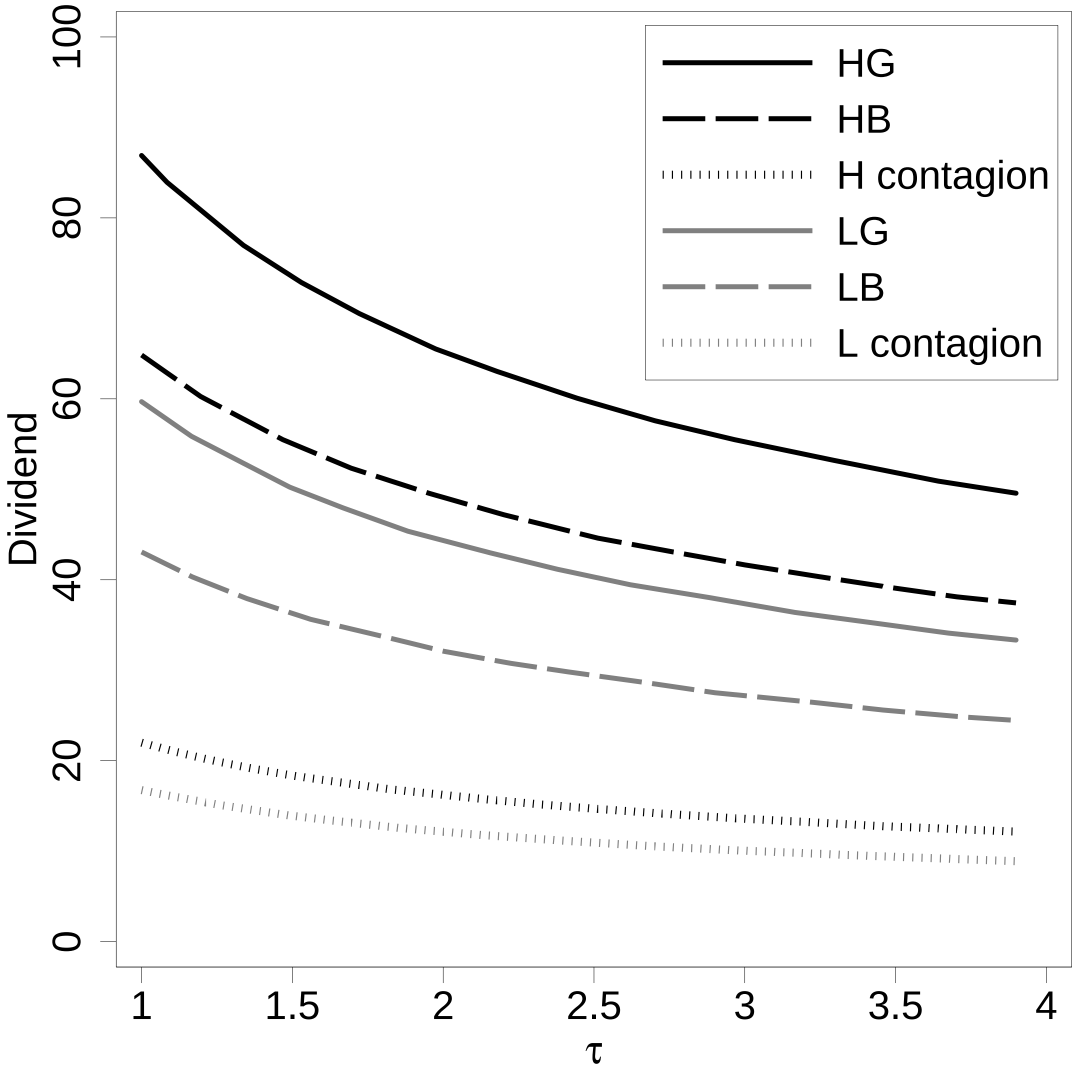
Source: Haver Analytics and author's calculations.

Fig. IX: Aggregate Interest Rate Schedules



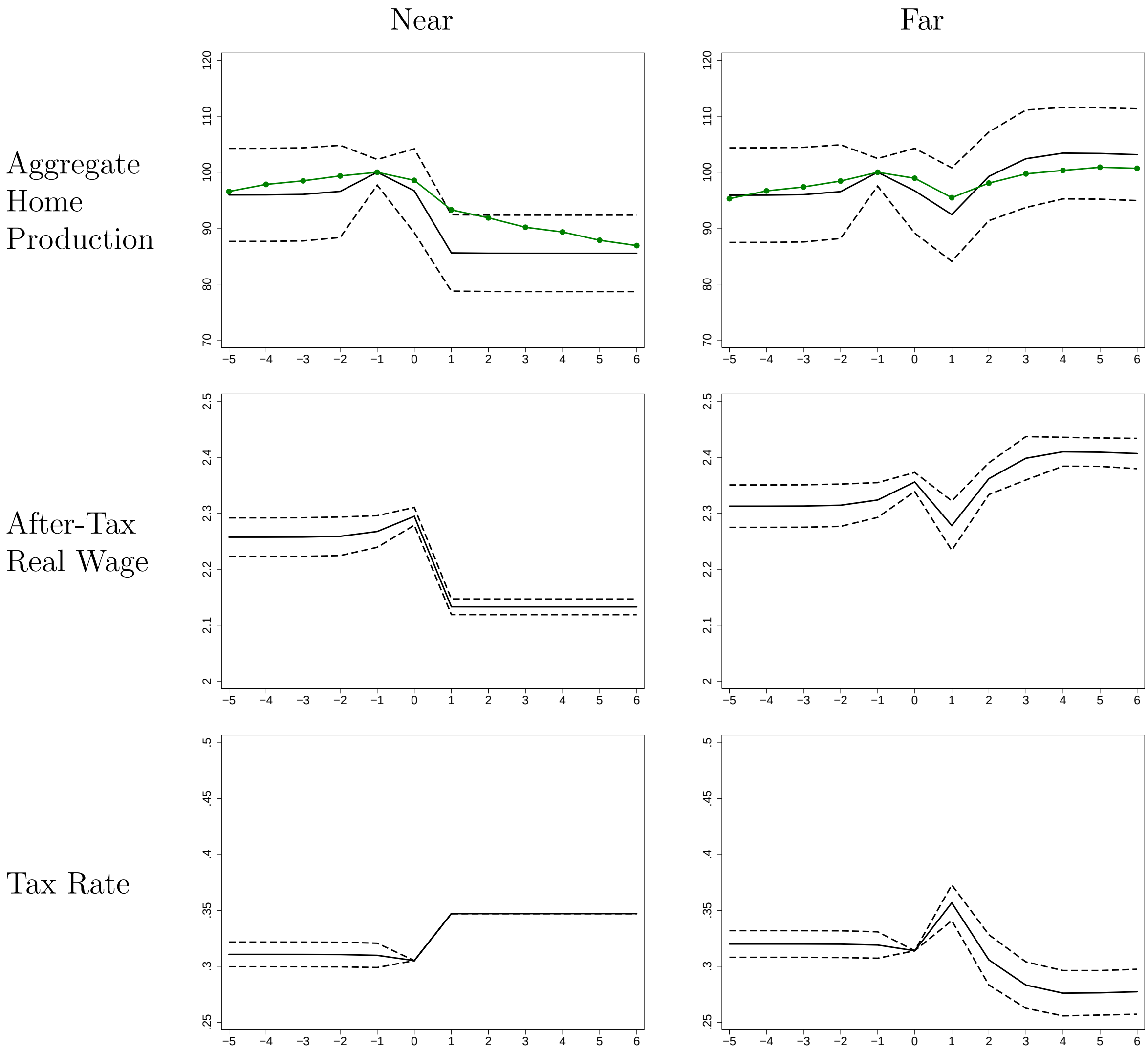
Note: Required expected return for home firm and government borrowing vs. the aggregate amount of money these two groups wish to borrow. Home Borrowing is total borrowing by home firms and the government. r^H is the rate the bank can borrow at uncollateralized, r^R is the rate the bank can borrow at with collateral, and r^f is the risk free asset return.

Fig. X: Aggregate Home Firm Dividend vs. Iceberg Trade Costs



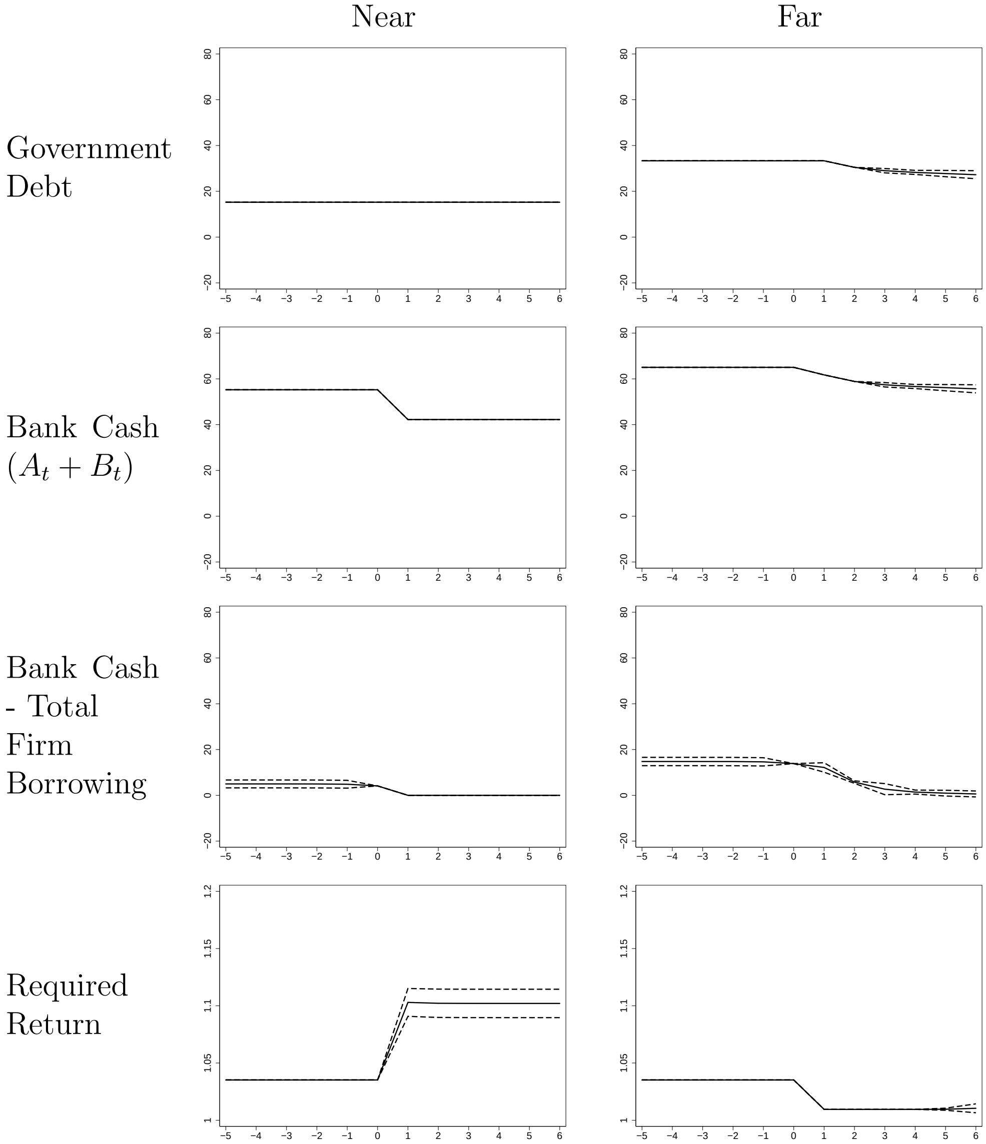
Note: Plot of the aggregate dividend across all home firms in each of four states: the cross product of good and bad Z draws at home and good and bad foreign state draws. The black lines are for high draws of Z and the grey for low draws. The solid lines are for good foreign draws and the dashed ones for bad draws. The bottom two dotted lines are the differences between the results with good and bad foreign draws keeping the home state static to isolate the contagion effect: the higher the line the greater the contagion level.

Fig. XI: Transitions Around a Crisis at T=0 — Home Production and Wages



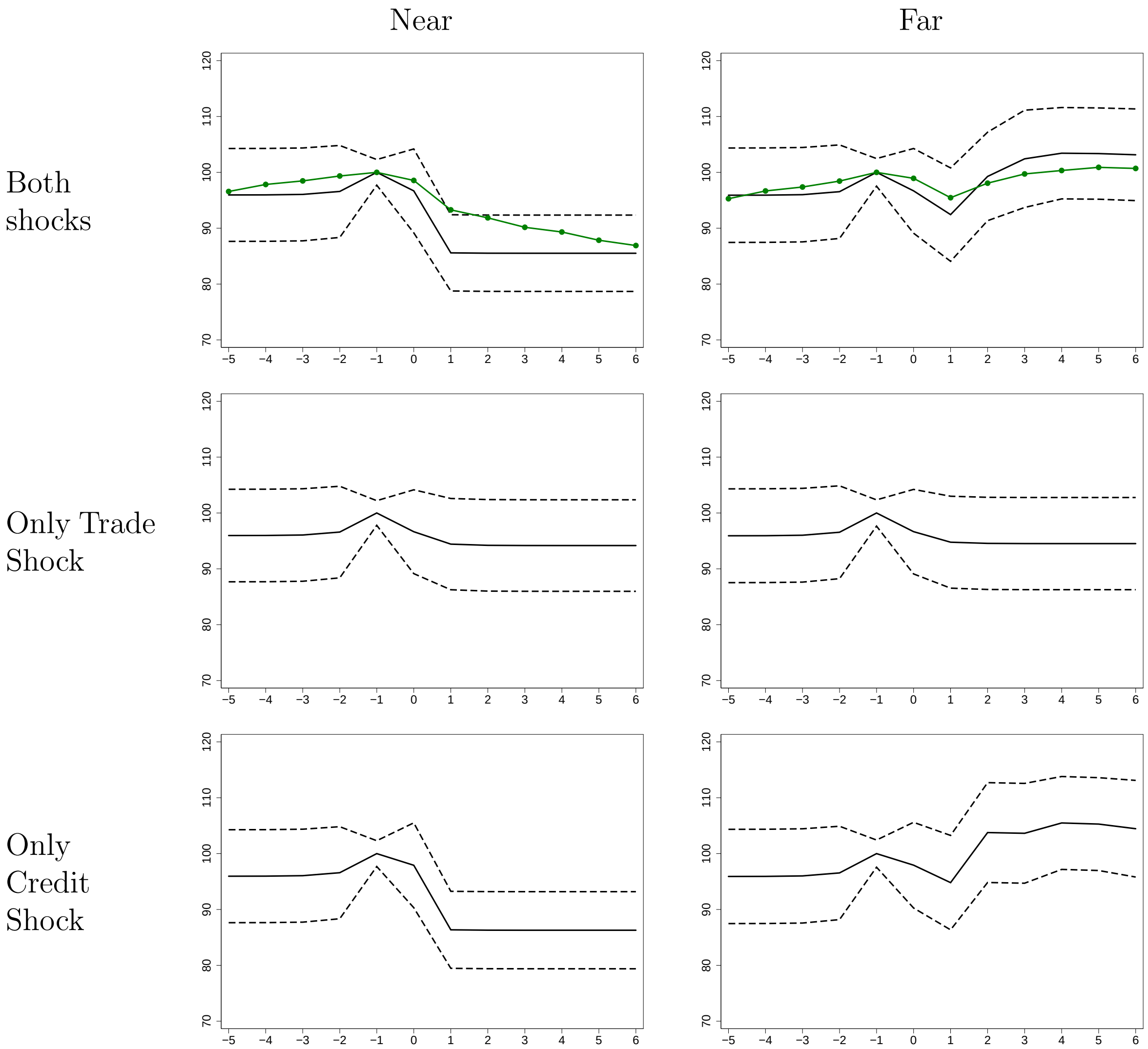
Note: Each panel plots from 5 years before a crisis that commences at T=0 through 6 years after. The conditional expected values are plotted in solid black, and the dashed lines are two standard deviation confidence intervals. The dotted lines are the actual country group averages. T=0 corresponds with 2008.

Fig. XII: Transitions Around a Crisis at T=0 — Home Assets



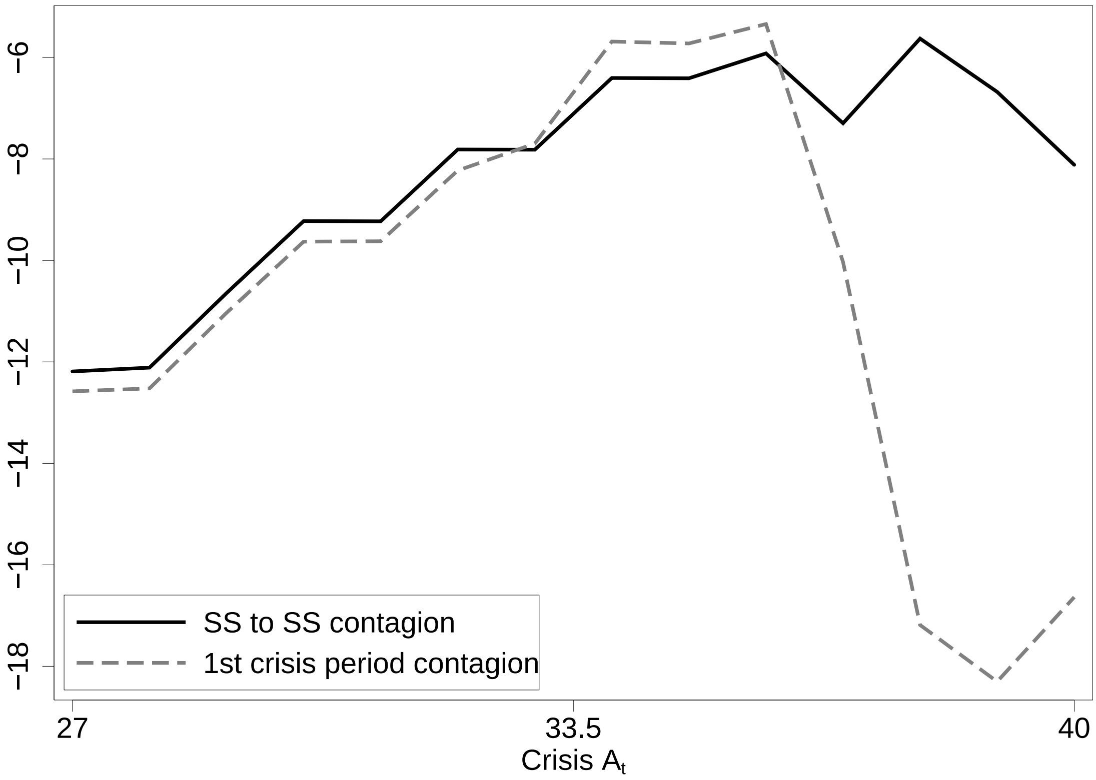
Note: Each panel plots from 5 years before a crisis that commences at T=0 through 6 years after. The conditional expected values are plotted in solid black, and the dashed lines are two standard deviation confidence intervals. T=0 corresponds with 2008.

Fig. XIII: Transitions Around a Crisis at T=0 — Home Production for Counterfactual Scenarios



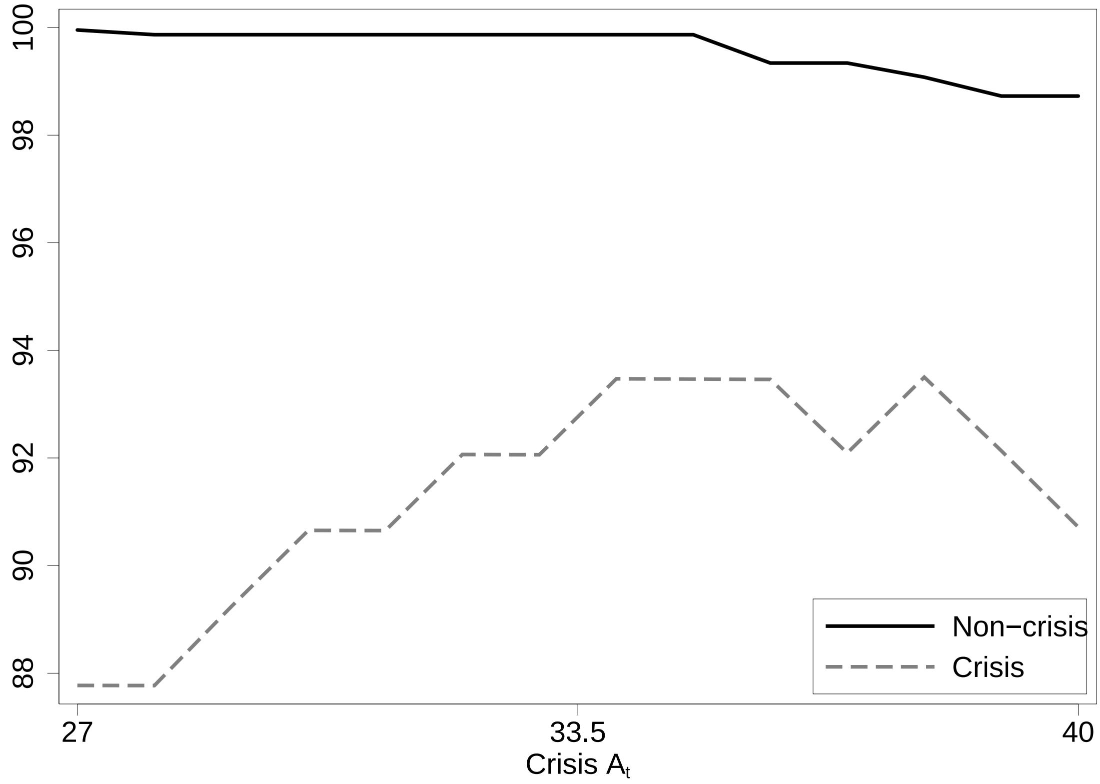
Note: Each panel plots from 5 years before a crisis that commences at T=0 through 6 years after. The conditional expected values are plotted in solid black, and the dashed lines are two standard deviation confidence intervals. The dotted lines are the actual country group averages. T=0 corresponds with 2008.

Fig. XIV: Near Country Contagion with Varying Levels of Capital Controls



Note: The x-axis measures the crisis level of A_t, ranging from the near countries' average crisis level of 26.5 up through their non-crisis average of 40.4. The "SS to SS contagion" line plots the non-crisis steady-state output to the crisis steady-state output level of contagion. The "1st crisis period contagion" line plots the decline in output from the non-crisis steady-state through the first full period of a crisis.

Fig. XV: Near Country Output with Varying Levels of Capital Controls



Note: The x-axis measures the crisis level of A_t, ranging from the near countries' average crisis level of 26.5 up through their non-crisis average of 40.4. The non-crisis line plots the non-crisis times steady-state aggregate home output, and the other line the crisis steady-state level.

Exposure to International Crises: Trade vs. Financial Contagion Online Appendix

Everett Grant
Federal Reserve Bank of Dallas

August 15, 2016

1 Country Group Maps

Figure A.1: Crisis, Near & Far Country Group Maps

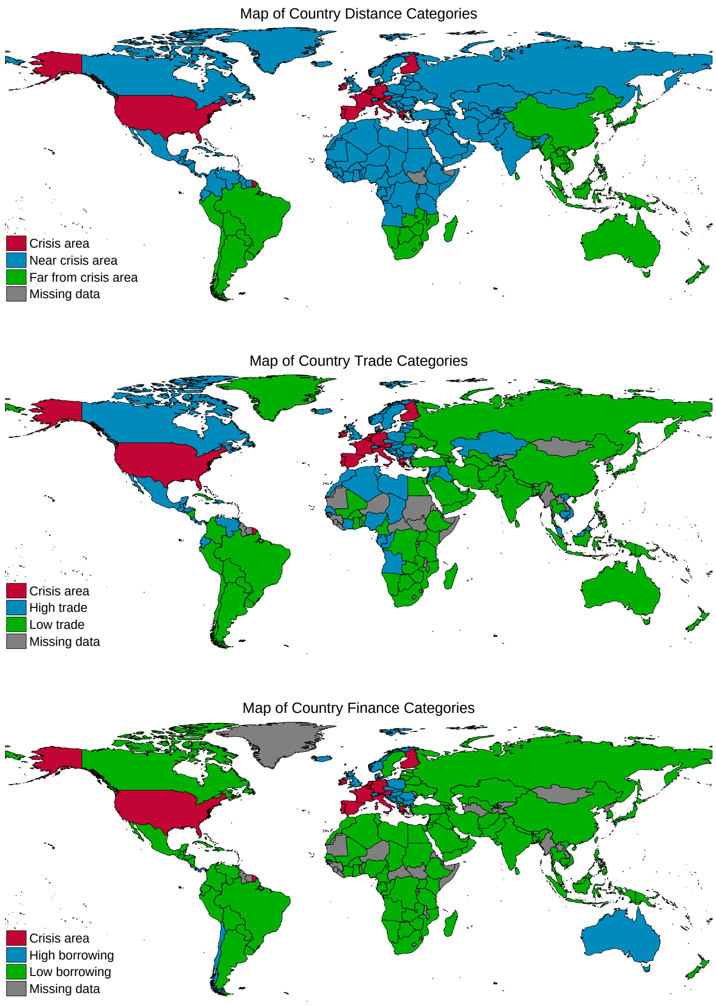
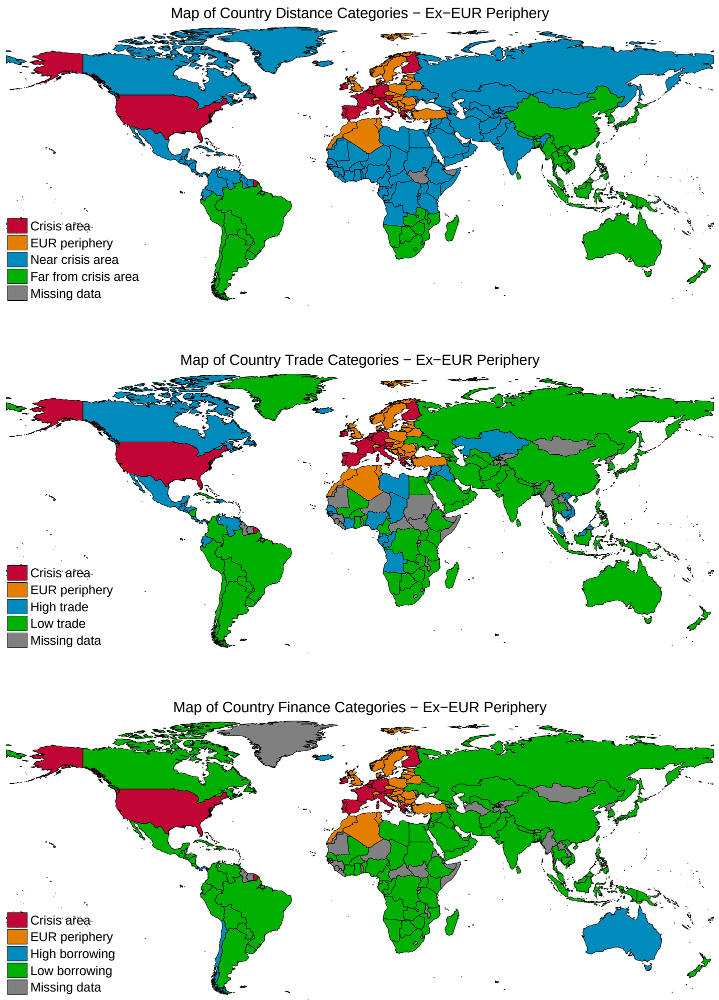


Figure A.2: Crisis, Near, Far & European Periphery Country Group Maps



2 Exogenous States

There are two exogenous state variables in the model that cause uncertainty: the Foreign endowment level (W_t); and the Home economy wide productivity shock (Z_t).

3 Aggregates to Simplify Notation

The consumer and firm problems can be greatly simplified by substitution of quantities that repeatedly show up. The following is a list of those quantities that will be used in the following sections.

3.1 Home

1) Quantity index for all goods $C_{Hts} \equiv C_{Hts}(\{c_{Hits}\}, f_{Hts}) = \left(\int_0^1 (c_{Hits})^{\frac{\sigma-1}{\sigma}} di + \alpha_H f_{Hts}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$

2) Period utility $U_{Hts}(\{c_{Hits}\}, f_{Hts}, n_{Ht}) \equiv \frac{\left(C_{Hts}(\{c_{Hits}\}, f_{Hts}) - \frac{n_{Ht}^{\omega}}{\omega}\right)^{1-\rho}}{1-\rho}$

3) Price index for home goods $P_{Hts} \equiv \left(\int_0^1 (p_{its})^{1-\sigma} di\right)^{\frac{1}{1-\sigma}}$

4) Price index for all goods $\Phi_{Hts} \equiv \left(P_{Hts}^{1-\sigma} + \alpha_H^{\sigma}\right)^{\frac{1}{1-\sigma}}$

5) Quantity index for home goods $\mathcal{Y}_{Hts} \equiv \left(\int_0^1 (c_{Hits})^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}}$

3.2 Foreign

Note: χ_t is the set of home goods exported to foreign.

1) Quantity index for all goods $C_{Fts} \equiv C_{Fts}(\{c_{Fits}\}, f_{Fts}) = \left(\int_{\chi_t} (c_{Fits})^{\frac{\sigma-1}{\sigma}} di + \alpha_F f_{Fts}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$

2) Period utility $U_{Fts}(\{c_{Fits}\}, f_{Fts}, n_{Ft}) \equiv \frac{\left(C_{Fts}(\{c_{Fits}\}, f_{Fts}) - \frac{n_{Ft}^{\omega}}{\omega}\right)^{1-\rho}}{1-\rho}$

3) Price index for home goods $P_{Fts} \equiv \left(\int_{\chi_t} (\tau p_{its})^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} = \tau\left(\int_{\chi_t} (p_{its})^{1-\sigma} di\right)^{\frac{1}{1-\sigma}}$

4) Price index for all goods $\Phi_{Fts} \equiv \left(P_{Fts}^{1-\sigma} + \alpha_F^{\sigma}\right)^{\frac{1}{1-\sigma}}$

5) Quantity index for home goods $\mathcal{Y}_{Fts} \equiv \left(\int_{\chi_t} (c_{Fits})^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}}$

4 Consumer Problem

There is a unit continuum of identical Home households, and a similar unit continuum of Foreign households. The utility functions of the two groups differ only in their relative preferences for Home and Foreign goods, α_H & α_F. See the period utilities given above for details. Home households pay the endogenously determined wage tax rate T_t and receive an exogenously determined payment of G from the government each period.

4.1 Home Solution

1) HH budget constraint $B_{Hts} \equiv (1 - T_t) n_{Ht} w_t + G + \Pi_{ts} = \int_0^1 c_{Hits} p_{its} di + f_{Hts}$

2) f_{Hts} FOC $f_{Hts} = \left(\frac{B_{Hts}}{\Phi_{Hts}}\right) (\alpha_H \Phi_{Hts})^{\sigma} = C_{Hts} (\alpha_H \Phi_{Hts})^{\sigma} = B_{Hts} \Phi_{Hts}^{\sigma-1} \alpha_H^{\sigma}$

3) c_{Hits} FOC $\forall\ i \ : \ p_{its} c_{Hits} = B_{Hts} \Phi_{Hts}^{\sigma-1} p_{its}^{1-\sigma}$

4) n_{Ht} FOC $n_{Ht}^{\omega-1} = \frac{(1-T_t) w_t E_s\left(U_{Hts}^{\frac{-\rho}{\rho-1}} \Phi_{Hts}^{-1}\right)}{E_s\left(U_{Hts}^{\frac{-\rho}{\rho-1}}\right)}$

5) Optimal aggregate consumption $C_{Hts} = \frac{B_{Hts}}{\Phi_{Hts}}$

6) Optimal home good consumption $\mathcal{Y}_{Hts} = \left(\frac{B_{Hts}}{P_{Hts}}\right)\left(\frac{1}{1+\alpha_H^\sigma P_{Hts}^{\sigma-1}}\right) = \left(\frac{B_{Hts}}{P_{Hts}}\right)\left(\frac{P_{Hts}}{\Phi_{Hts}}\right)^{1-\sigma} = C_{Hts}\left(\frac{\Phi_{Hts}}{P_{Hts}}\right)^{\sigma}$

4.2 Foreign Solution

1) HH budget constraint $B_{Fts} \equiv W_{ts} = \int_{\chi_t} \tau c_{Fits} p_{its} di + f_{Fts}$

2) f_{Fts} FOC $f_{Fts} = \left(\frac{B_{Fts}}{\Phi_{Fts}}\right)(\alpha_F \Phi_{Fts})^{\sigma} = C_{Fts}(\alpha_F \Phi_{Fts})^{\sigma} = B_{Fts}\Phi_{Fts}^{\sigma-1}\alpha_F^{\sigma}$

3) c_{Fits} FOC $\forall\ i \quad : \quad \tau p_{its} c_{Fits} = B_{Fts}\Phi_{Fts}^{\sigma-1}\tau^{1-\sigma}p_{its}^{1-\sigma}$

4) n_{Ft} FOC wages in Foreign are zero, so $n_{Ft} = 0$

5) Optimal aggregate consumption $C_{Fts} = \frac{B_{Fts}}{\Phi_{Fts}}$

6) Optimal home good consumption $\mathcal{Y}_{Fts} = \left(\frac{B_{Fts}}{P_{Fts}}\right)\left(\frac{1}{1+\alpha_F^\sigma P_{Fts}^{\sigma-1}}\right) = \left(\frac{B_{Fts}}{P_{Fts}}\right)\left(\frac{P_{Fts}}{\Phi_{Fts}}\right)^{1-\sigma} = C_{Fts}\left(\frac{\Phi_{Fts}}{P_{Fts}}\right)^{\sigma}$

4.3 Home Household Budget Check

This analyzes the home household budget by deriving the amount of foreign good remaining at home at the end of the period and showing that this is equal to f_{Hts}. This is the sum of the foreign good provided to home households and firms, which will all end up with home households because they own the firms.

$$
\begin{aligned}
&\int_{\chi_t} \tau c_{Fits} p_{its} di && \text{Home sales to Foreign} \\
&+G - T_t w_t n_{Ht} && \text{Net flows from the government (=Net government borrowing passed on to Households)} \\
&-\gamma \int_0^1 x_{it} di && \text{Fixed costs paid to exporters} \\
&+\int_0^1 h_i \kappa_i di - \int_0^1 \kappa_i di && \text{Net bank borrowing} \\
&+\int_0^1 d_{its} \kappa_i di && \text{Default gains for Home firms} \\
&= \int_{\chi_t} \tau c_{Fits} p_{its} di + G - T_t w_t n_{Ht} - \gamma \int_0^1 x_{it} di + \int_0^1 h_i \kappa_i di - \int_0^1 \kappa_i (1 - d_{its}) di \\
&= \int_{\chi_t} \tau c_{Fits} p_{its} di + G - T_t w_t n_{Ht} - \gamma \int_0^1 x_{it} di + \int_0^1 (w_t N_{it} + \gamma x_{it}) di - \int_0^1 \kappa_i (1 - d_{its}) di \\
&= \int_{\chi_t} \tau c_{Fits} p_{its} di + w_t n_{Ht} + G - T_t w_t n_{Ht} - \int_0^1 \kappa_i (1 - d_{its}) di \\
&= n_{Ht} w_t + G - T_t w_t n_{Ht} + \Pi_{ts} + \int_{\chi_t} \tau c_{Fits} p_{its} di - \int_0^1 Revenue_{its} di \\
&= (1 - T_t) n_{Ht} w_t + G + \Pi_{ts} - \int_0^1 c_{Hits} p_{its} di \\
&= f_{Hts} \text{ by the H budget constraint, as desired}
\end{aligned} \tag{1}
$$

5 Firm Problem

There is a unit continuum of home firms taking part in monopolistic competition. These firms are broadly split into two types by their labor productivity factors, a_h and a_l where $a_h > a_l$. $\mathcal{H}$ is the fraction of firms of type h.

5.1 Firm Expected Profit Function

First, some notation for two helpful price indices and firm default & export decisions:

$$\psi_{sExporter} \equiv B_{Hts}\Phi_{Hts}^{\sigma-1} + B_{Fts}\Phi_{Fts}^{\sigma-1}\tau^{1-\sigma} \tag{2}$$

$$\psi_{sNonexporter} \equiv B_{Hts}\Phi_{Hts}^{\sigma-1}, \tag{3}$$

$$d_{it} = \begin{cases} 0 & \text{if pay off debt} \\ 1 & \text{if default} \end{cases}, \tag{4}$$

and

$$x_{it} = \begin{cases} 0 & \text{if do not choose to export} \\ 1 & \text{if choose to export} \end{cases}. \tag{5}$$

The consumers' problems then give us that for all firms:

$$\psi_{sx_{it}} = Q_{its}(1-\mu d_{its})p_{its}^{\sigma} \tag{6}$$

The full form of the firm profit problem is:

$$\begin{aligned} E_s(\pi_{its}) = \max_{\{\Theta_{its}\}} & E_s\left[p_{its}Q_{its}(1-\mu d_{its}) - w_tN_{it} + h_{it}\kappa_{it} - \kappa_{it}(1-d_{its}) - \gamma x_{it}\right] \\ s.t. \quad & \tfrac{Z_{ts}}{a_i}N_{it}^{\alpha} \geq Q_{its} \\ & Q_{its}(1-\mu d_{its}) \geq m_{Hits} + \tau m_{Fits} \\ & \text{On consumer demand curves: } \begin{array}{l} m_{Hits} \geq c_{Hits} \\ m_{Fits} \geq c_{Fits} \end{array} \\ & h_{it}\kappa_{it} \geq w_tN_{it} + \gamma x_{it} \\ & m_{Fits} \begin{cases} = 0 & \text{if } x_{it} = 0 \\ \geq 0 & \text{if } x_{it} = 1 \end{cases} \end{aligned} \tag{7}$$

where $\Theta_{its} \equiv \{N_{it}, \kappa_{it}, x_{it}, \{Q_{its}\}, \{m_{Hits}\}, \{m_{Fits}\}, \{p_{its}\}, \{d_{its}\}\}$ is the choice set. After substitution of constraints that will hold strictly this becomes:

$$E_s(\pi_{its}) = \max_{\{N_{it}, x_{it}, \{d_{its}\}\}} E_s\left[\psi_{sx_{it}}^{\frac{1}{\sigma}}\left(\frac{Z_{ts}}{a_i}N_{it}^{\alpha}(1-\mu d_{its})\right)^{\frac{\sigma-1}{\sigma}} - (1-d_{its})\frac{(w_tN_{it}+\gamma x_{it})}{h_{it}}\right]. \tag{8}$$

The first order condition for N_{it} is:

$$w_t \frac{E_s(1-d_{its})}{h_{it}} = N_{it}^{\frac{\alpha(\sigma-1)}{\sigma}-1} a_i^{\frac{1-\sigma}{\sigma}} \frac{\alpha(\sigma-1)}{\sigma} E_s \left[\psi_{sx_{it}}^{\frac{1}{\sigma}} \left(Z_{ts}(1-\mu d_{its})\right)^{\frac{\sigma-1}{\sigma}} \right]. \tag{9}$$

From the firms' FOC we get the following, where the LHS is constant for all firms producing goods this period from the bank problem:

$$w_t \frac{\sigma}{\alpha(\sigma-1)} \left[\frac{E_s(1-d_{its})}{h_{it}} \right] = N_{it}^{-1} E_s \left[p_{its} Q_{its}(1-\mu d_{its}) \right] = N_{it}^{-1} E_s(Revenue_i) \tag{10}$$

$$\implies w_t n_{Ht} \frac{\sigma}{\alpha(\sigma-1)} \left[\frac{E_s(1-d_{its})}{h_{it}} \right] = \int_0^1 E_s(Revenue_i) di \tag{11}$$

The choice of whether to default is based on the aggregate price index, which firms take as given. A firm pays its debt

$$\iff \psi_{sx_{it}}^{\frac{1}{\sigma}} Q_{its}^{\frac{\sigma-1}{\sigma}} - \kappa_{it} \geq \psi_{sx_{it}}^{\frac{1}{\sigma}} \left(Q_{its}(1-\mu)\right)^{\frac{\sigma-1}{\sigma}} \tag{12}$$

$$\iff \psi_{sx_{it}}^{\frac{1}{\sigma}} Q_{its}^{\frac{\sigma-1}{\sigma}} \left(1 - (1-\mu)^{\frac{\sigma-1}{\sigma}}\right) \geq \kappa_{it} \tag{13}$$

The choice of whether to export or not is based on whether moving to the greater price index for exporters is worth the fixed cost of exporting.

5.1.1 Solving for the ψ_s Price Indices

We will solve for a system of two equations in the unknowns ψ_{sH} and ψ_{sx} given that it is currently the state s. First, the following can be derived from the definitions of the price indices and the solution to the consumers' problems:

$$B_{Hts} \Phi_{Hts}^{\sigma-1} \alpha_H^{\sigma} = \psi_{sH} \alpha_H^{\sigma} = f_{Hts} \tag{14}$$

$$B_{Fts} \Phi_{Fts}^{\sigma-1} \alpha_F^{\sigma} = (\psi_{sx} - \psi_{sH}) \tau^{\sigma-1} \alpha_F^{\sigma} = f_{Fts} \tag{15}$$

Define the available export quantity index k_{st} as:

$$k_{ts} = \int_{\chi_t} \left(\frac{Z_{ts}}{a_i} N_{it}^{\alpha} (1-\mu d_{its}) \right)^{\frac{\sigma-1}{\sigma}} di. \tag{16}$$

The following relationships then hold:

$$\begin{aligned}
&\text{Value of Home sales to Foreign} = \\
&\int_{\chi_t} \tau c_{Fits} p_{its} di = \\
&\int_0^1 Q_{its}(1-\mu d_{its}) p_{its} di - \int_0^1 c_{Hits} p_{its} di = \\
&\int_0^1 \psi_{sx_{it}}^{\frac{1}{\sigma}} \left(\frac{Z_{ts}}{a_i} N_{it}^{\alpha} (1-\mu d_{its}) \right)^{\frac{\sigma-1}{\sigma}} di - \int_0^1 c_{Hits} p_{its} di = \\
&\int_0^1 \psi_{sx_{it}}^{\frac{1}{\sigma}} \left(\frac{Z_{ts}}{a_i} N_{it}^{\alpha} (1-\mu d_{its}) \right)^{\frac{\sigma-1}{\sigma}} di - \psi_{sH} \int_0^1 \psi_{sx_{it}}^{\frac{1-\sigma}{\sigma}} \left(Q_{its}(1-\mu d_{its}) \right)^{\frac{\sigma-1}{\sigma}} di = \\
&\int_0^1 \psi_{sx_{it}}^{\frac{1}{\sigma}} \left(\frac{Z_{ts}}{a_i} N_{it}^{\alpha} (1-\mu d_{its}) \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{\psi_{sx_{it}} - \psi_{sH}}{\psi_{sx_{it}}} \right) di = \\
&\int_{\chi_t} \psi_{sX}^{\frac{1-\sigma}{\sigma}} \left(\frac{Z_{ts}}{a_i} N_{it}^{\alpha} (1-\mu d_{its}) \right)^{\frac{\sigma-1}{\sigma}} \left(\psi_{sX} - \psi_{sH} \right) di = \\
&\left(\psi_{sX} - \psi_{sH} \right) \psi_{sX}^{\frac{1-\sigma}{\sigma}} k_{ts}
\end{aligned} \tag{17}$$

$$\Pi_{ts} \equiv \int_0^1 \pi_{its} di = \int_0^1 \psi_{sx_{it}}^{\frac{1}{\sigma}} \left(\frac{Z_{ts}}{a_i} N_{it}^{\alpha} (1-\mu d_{its}) \right)^{\frac{\sigma-1}{\sigma}} di - \int_0^1 (1-d_{its}) \frac{(w_t N_{it} + \gamma x_{it})}{h_{it}} di \tag{18}$$

The home budget can then be rewritten as:

$$\begin{aligned}
\psi_{sH} \alpha_H^{\sigma} &= (1-T_t) w_t n_{Ht} + G + \Pi_{ts} - \int_0^1 c_{Hits} p_{its} di = \\
&(1-T_t) w_t n_{Ht} + G - \int_0^1 c_{Hits} p_{its} di + \\
&\int_0^1 \psi_{sx_{it}}^{\frac{1}{\sigma}} \left(\frac{Z_{ts}}{a_i} N_{it}^{\alpha} (1-\mu d_{its}) \right)^{\frac{\sigma-1}{\sigma}} di - \int_0^1 (1-d_{its}) \kappa_{it} di = \\
&(1-T_t) w_t n_{Ht} + G + \left(\psi_{sX} - \psi_{sH} \right) \psi_{sX}^{\frac{1-\sigma}{\sigma}} k_{ts} - \int_0^1 (1-d_{its}) \kappa_{it} di
\end{aligned} \tag{19}$$

The foreign budget constraint can be rewritten as:

$$\begin{aligned}
W_{ts} &= \int_{\chi_t} \tau c_{Fits} p_{its} di + (\psi_{sx} - \psi_{sH}) \tau^{\sigma-1} \alpha_F^{\sigma} = \\
&\left(\psi_{sX} - \psi_{sH} \right) \psi_{sX}^{\frac{1-\sigma}{\sigma}} k_{ts} + (\psi_{sx} - \psi_{sH}) \tau^{\sigma-1} \alpha_F^{\sigma}
\end{aligned} \tag{20}$$

These two then reduce to the following two equation system with two unknowns given the state, pre-production decisions, and current firm default choices:

$$\psi_{sX}\alpha_H^\sigma = (1-T_t)w_t n_{Ht} + G - \int_0^1 (1-d_{its})\kappa_{it} di + (\psi_{sX} - \psi_{sH})\left(\psi_{sX}^{\frac{1-\sigma}{\sigma}} k_{ts} + \alpha_H^\sigma\right) \tag{21}$$

$$\frac{W_{ts}}{\psi_{sX}^{\frac{1-\sigma}{\sigma}} k_{ts} + \tau^{\sigma-1}\alpha_F^\sigma} = (\psi_{sX} - \psi_{sH}) \tag{22}$$

This then gives the following equation to solve for ψ_{sX}:

$$\psi_{sX}\alpha_H^\sigma = (1-T_t)w_t n_{Ht} + G - \int_0^1 (1-d_{its})\kappa_{it} di + \left(\frac{W_{ts}}{\psi_{sX}^{\frac{1-\sigma}{\sigma}} k_{ts} + \tau^{\sigma-1}\alpha_F^\sigma}\right)\left(\psi_{sX}^{\frac{1-\sigma}{\sigma}} k_{ts} + \alpha_H^\sigma\right) \tag{23}$$

Let:

1) $\mathcal{S} = (1-T_t)w_t n_{Ht} + G - \int_0^1 (1-d_{its})\kappa_{it} di$

2) $\mathcal{F} = \alpha_H^\sigma \alpha_F^\sigma \tau^{\sigma-1}$

3) $\mathcal{M} = (\alpha_F^\sigma \tau^{\sigma-1}\mathcal{S} + \alpha_H^\sigma W_{ts})$

4) $\mathcal{N} = (\mathcal{S}k_{ts} + W_{ts}k_{ts})$

$$\begin{aligned} \implies 0 &= \alpha_H^\sigma \alpha_F^\sigma \tau^{\sigma-1}\psi_{sX} + \alpha_H^\sigma k_{ts}\psi_{sX}^{\frac{1}{\sigma}} - (\mathcal{S}k_{ts} + W_{ts}k_{ts})\psi_{sX}^{\frac{1-\sigma}{\sigma}} - (\alpha_F^\sigma \tau^{\sigma-1}\mathcal{S} + \alpha_H^\sigma W_{ts}) \\ \implies 0 &= \mathcal{F}\psi_{sX} + \alpha_H^\sigma k_{ts}\psi_{sX}^{\frac{1}{\sigma}} - \mathcal{N}\psi_{sX}^{\frac{1-\sigma}{\sigma}} - \mathcal{M} \equiv f(\psi_{sX}) \end{aligned} \tag{24}$$

We then want to look at the properties of $f(\psi_{sX})$ since its roots are the potential equilibrium values of the export price index. By looking at these we will find that there is an unique solution to the problem. The proof is as follows:

1) Show that $\psi_{sX} > 0$ in equilibrium.
   (a) $\psi_{sX} \neq 0$ in equilibrium because $f(\psi_{sX})$ is undefined there since $\sigma > 1$.
   (b) $\psi_{sH} \geq 0$ because $\psi_{sH}\alpha_H^\sigma = f_{Hts} \geq 0$.
   (c) We now show that $\psi_{sX} > 0$. There are two cases to consider: when there are measure zero exporters and when there aren't.
      i. With measure zero exporters we can use $\psi_{sH} \geq 0$ and $\psi_{sX} = \psi_{sH} + \frac{W_{ts}}{\alpha_F^\sigma}\tau^{1-\sigma} > 0$.
      ii. If there's a nonzero measure of exporters then it's clear that $\psi_{sX} \geq 0$ as otherwise the prices for exporters ($p_{its} = (\frac{\psi_{sx_{it}}}{Q_{its}(1-\mu d_{its})})^{\frac{1}{\sigma}}$) would be less than zero or complex, which is inconsistent with profit maximizing firms.

2) We then show there is at most one root of $f(\psi_{sX})$ for $\psi_{sX} > 0$.

   (a) We first show that $\mathcal{S} + W_{ts} \geq 0$ because f_{Hts} & f_{Fts} both are:
   $0 \leq f_{Hts} + f_{Fts} = (1-T_t)w_t n_{Ht} + G + \Pi_{ts} - \int_0^1 c_{Hits}p_{its}di + W_{ts} - \int_{\chi_t} \tau c_{Fits}p_{its}di$
   $= (1-T_t)w_t n_{Ht} + G - \int_0^1 Q_{its}(1-\mu d_{its})p_{its}di + \Pi_{ts} + W_{ts}$
   $= (1-T_t)w_t n_{Ht} + G - \int_0^1 Q_{its}(1-\mu d_{its})p_{its}di + \int_0^1 Q_{its}(1-\mu d_{its})p_{its}di - \int_0^1 (1-d_{its})\kappa_{it}di + W_{ts}$
   $= (1-T_t)w_t n_{Ht} + G - \int_0^1 (1-d_{its})\kappa_{it}di + W_{ts} = \mathcal{S} + W_{ts}$

   (b) Trivially we then get $\mathcal{N} \geq 0$.

   (c) We then have that there is at most one root of $f(\psi_{sX})$ for $\psi_{sX} > 0$ since
   $f'(\psi_{sX}) = \mathcal{F} + \frac{\alpha_H^\sigma k_{ts}}{\sigma}\psi_{sX}^{\frac{1-\sigma}{\sigma}} + \frac{\mathcal{N}(\sigma-1)}{\sigma}\psi_{sX}^{\frac{1-2\sigma}{\sigma}} > 0 \;\forall\; \psi_{sX} > 0.$

3) We next show that ψ_{sX} comes from $-\infty$ near zero and goes to $+\infty$ as ψ_{sX} increases. Combined with the fact that $f(\psi_{sX})$ is continuous for $\psi_{sX} > 0$ means that there will exist a solution to $f(\psi_{sX}) = 0$ with $\psi_{sX} > 0$:

   (a) $\lim_{\psi_{sX}\to 0} f(\psi_{sX}) = \lim_{\psi_{sX}\to 0}\left(\mathcal{F}\psi_{sX} + \alpha_H^\sigma k_{ts}\psi_{sX}^{\frac{1}{\sigma}} - \mathcal{N}\psi_{sX}^{\frac{1-\sigma}{\sigma}} - \mathcal{M}\right) = -\mathcal{N}\lim_{\psi_{sX}\to 0}\left(\psi_{sX}^{\frac{1-\sigma}{\sigma}}\right) - \mathcal{M} = -\infty$

   (b) $\lim_{\psi_{sX}\to\infty} f(\psi_{sX}) = \lim_{\psi_{sX}\to\infty}\left(\mathcal{F}\psi_{sX} + \alpha_H^\sigma k_{ts}\psi_{sX}^{\frac{1}{\sigma}} - \mathcal{N}\psi_{sX}^{\frac{1-\sigma}{\sigma}} - \mathcal{M}\right) = +\infty \;+\; +\infty - \mathcal{N}(0) - \mathcal{M} = +\infty$

Note also that the formula to find ψ_{sH} will provide a single solution given this unique answer, so there will not be multiple equilibria. Hence, there will exist an unique answer to the pricing problem.

Note also that $\psi_{sX} > \psi_{sH}$ because:

$$\psi_{sX} - \psi_{sH} = \frac{W_{ts}}{\psi_{sX}^{\frac{1-\sigma}{\sigma}} k_{ts} + \tau^{\sigma-1}\alpha_F^\sigma} > 0. \tag{25}$$

6 Bank Problem

6.1 Bank problem when the Home government defaults

In this case the Lagrangian is:

$$\mathcal{L}(B, Z_{-1}, W_{-1}) \mid_{D=1}= \max_{\{K,L,\{I_s\},\{\kappa_i\},\Omega,\{\lambda_s\},\Theta,\Gamma\}} E_s\{I_s + \delta\mathcal{L}(\phi B, Z_s, W_s) + \lambda_s(r^f K + \int_0^1 \kappa_i(1-d_{is})di - I_s - r^H L)\} \\ +\Omega K + \Theta L + \Gamma\left(A + L - \int_0^1 h_i\kappa_i di - K\right) \tag{26}$$

Resulting in the following conditions:

1) $I_s \ \forall \ s: \ \ \lambda_s = 1$ since the banker is risk neutral.

2) $K: \ \ \Gamma = r^f + \Omega$

3) $L: \ \ \Gamma = r^H - \Theta$

4) $\kappa_i \ \forall \ i: \ \ h_i = \frac{E_s(1-d_{is})}{\Gamma}$

5) $\Omega, \{\lambda_s\}, \Theta, \Gamma$ K-T conditions.

   (a) From the I_s condition we know that $\lambda_s > 0$ so those constraints will hold strictly.

   (b) From the K condition we know that $\Gamma > 0$ so that constraint will hold strictly.

   (c) From the K and L conditions we know that $\Omega + \Theta = r^H - r^f$ which is greater than zero so can't have both K and $L > 0$. This means that the bank will not both invest and borrow risk free in the uncollateralized short term market.

From these conditions we can see that Γ is the expected return of investments in the short term market for Home firms and the government. Define Home Borrowing as being the total borrowing by home firms and the government (If the Home government defaulted then this is just the total Home firm borrowing). We will also define the Bank Cash as $A + D_t B_t$, which is just A if the government defaults. There are three potential cases based on the desired total amount of Home borrowing, which directly relate to the cases for the values of K and L:

A) $K = L = 0 \implies \Omega \ \& \ \Theta \geq 0$: in this case the bank invests all of the Bank Cash in Home Borrowing, and the required rate of return is $\Gamma \in [r^f, r^H]$.

B) $K = 0, L > 0 \implies \Omega > 0, \Theta = 0$: in this case the required return will be r^H, and the bank will invest the Bank Cash $+L$ in Home Borrowing.

C) $K > 0, L = 0 \implies \Omega = 0, \Theta > 0$: in this case the required return will be r^f, and the bank will invest some fraction of the Bank Cash in Home Borrowing and the remainder in K.

6.2 Bank problem when the Home government honors its debt

The Lagrangian for this problem is:

$$\begin{aligned}
&\mathcal{L}(B, Z_{-1}, W_{-1}) \mid_{D=0}= \\
&\max_{\{B',K,L,R,\{I_s\},\{\kappa_i\},\Omega,\{\lambda_s\},\Theta,\Gamma,\Lambda,\Phi\}} E_s \left\{ I_s + \delta\mathcal{L}(B', Z_s, W_s) + \lambda_s(r^f K + \int_0^1 \kappa_i(1-d_{is})di - I_s - r^H L - r^R R) \right\} \\
&\qquad +\Gamma\left(A + B + L + R - \int_0^1 h_i\kappa_i di - K - p_B B'\right) \\
&\qquad +\Phi(p_B B' - R) + \Omega K + \Theta L + \Lambda R
\end{aligned} \tag{27}$$

Resulting in the same conditions 1)-5) as the default problem in addition to the following conditions:

6) B' : $p_B = \frac{\delta E_s\left(\Gamma'_s(1-D'_s)\right)}{\Gamma-\Phi}$ using the Envelope Theorem.

7) R : $\Gamma = r^R + \Phi - \Lambda$

From these conditions we can sift through the feasible equilibria and the required return on Home Borrowing in each case. There are four main cases to consider based on which R constraints are binding, with sub cases based on the values of K and L:

I) $\Lambda > 0$ & $\Phi > 0$. Note that for this case to hold it must be that $p_B B' = R = 0$. There are three sub cases here depending on the values of K and L, resulting in possible equilibria similar to if the bond and repo markets are closed.

(a) $K > 0$ & $L = 0 \implies \Omega = 0 \implies \Gamma = r^f \implies \Gamma - \Phi < r^f$

(b) $K = L = 0 \implies \Omega$ & $\Theta \geq 0 \implies \Gamma \in [r^f, r^H] \implies \Gamma - \Phi < r^R$

(c) $K = 0$ & $L > 0 \implies \Theta = 0 \implies \Gamma = r^H \implies \Gamma - \Phi < r^R$

II) $\Lambda > 0$ & $\Phi = 0$. This implies that $R = 0$ and $\Gamma - \Phi < r^R$.

(a) $K > 0$ & $L = 0 \implies \Omega = 0 \implies \Gamma = \Gamma - \Phi = r^f$

(b) $K = L = 0 \implies \Omega$ & $\Theta \geq 0 \implies \Gamma = \Gamma - \Phi \in [r^f, r^R)$

(c) $K = 0$ & $L > 0$ — This case is not possible because $\Lambda > 0$ & $\Phi = 0 \implies \Gamma < r^R \implies \Theta > 0 \implies L = 0$

III) $\Lambda = 0$ & $\Phi > 0$. This implies that $R = p_B B'$ and $\Gamma - \Phi = r^R$.

(a) $K > 0$ & $L = 0$ — This case is not possible because $\Gamma - \Phi = r^R$ & $\Phi > 0 \implies \Gamma > r^R \implies \Omega > 0 \implies K = 0$

(b) $K = L = 0 \implies \Omega > 0$ & $\Theta \geq 0 \implies \Gamma \in (r^R, r^H]$

(c) $K = 0$ & $L > 0 \implies \Theta = 0 \implies \Gamma = r^H$

IV) $\Lambda = \Phi = 0$. This implies that $\Gamma = \Gamma - \Phi = r^R$ and $R \in [0, p_b B']$.

(a) $K > 0$ & $L = 0$ — This case is not possible because $\Gamma = r^R \implies \Omega > 0 \implies K = 0$

(b) $K = L = 0$ — This case is feasible.

(c) $K = 0$ & $L > 0$ — This case is not possible because $\Gamma = r^R \implies \Theta > 0 \implies L = 0$

These cases provide the following interest rate schedules relating the expected rate of return in the short term market and the aggregate amount of Home Borrowing that's desired. Note that a government default shifts the threshold from $A + B$ to A, but that firms will not have to compete with the government for funds if there is a default.

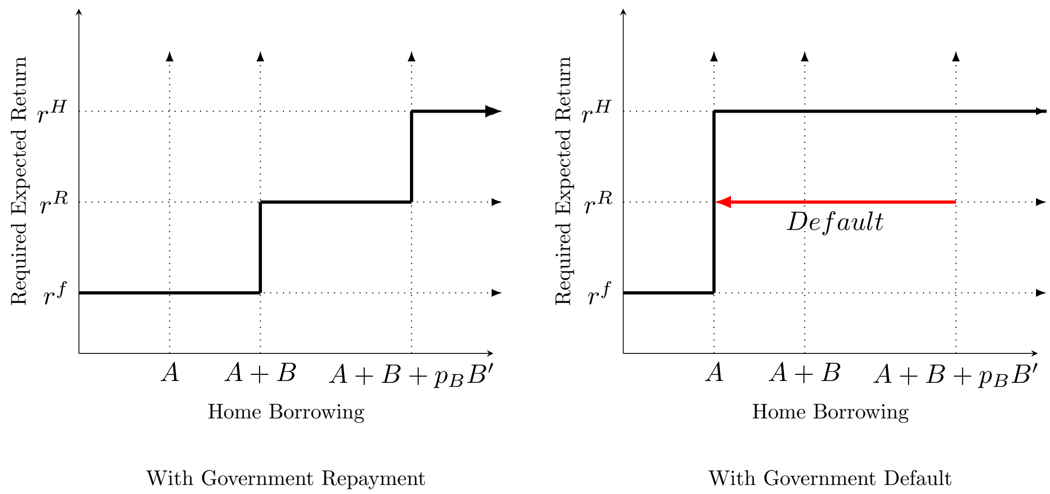
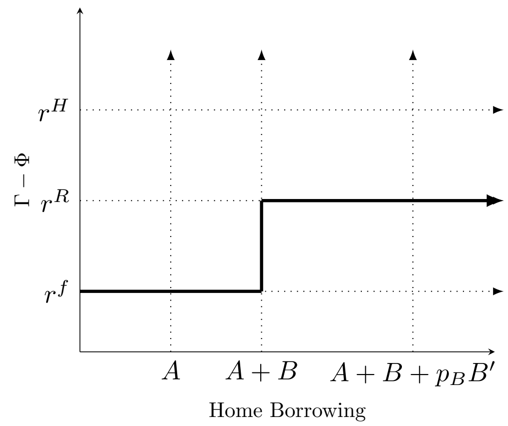
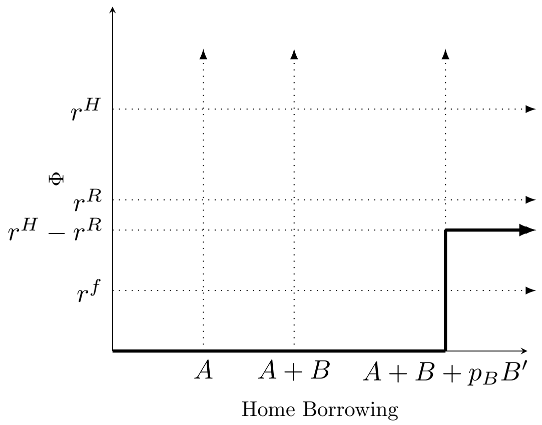


Figure A.3: Expected short term return for home firm and government borrowing v.s. the amount of money these two groups wish to borrow. r^H is the rate that the bank can borrow at uncollateralized, r^R is the collateralized borrowing rate, and r^f is the return on the risk free asset it can invest in.



$\Gamma - \Phi$ used in bond pricing formula with Government Repayment

Figure A.4: $\Gamma - \Phi$ used in bond pricing v.s. the amount of money home firms and government wish to borrow. r^H is the rate that the bank can borrow at uncollateralized, r^R is the collateralized borrowing rate, and r^f is the return on the risk free asset it can invest in.



Φ used in bond pricing formula with Government Repayment

Figure A.5: Φ used in bond pricing v.s. the amount of money home firms and government wish to borrow. r^H is the rate that the bank can borrow at uncollateralized, r^R is the collateralized borrowing rate, and r^f is the return on the risk free asset it can invest in. Φ is zero when not repoing out full position. When the full position is repoed and this constraint is binding then Φ (i.e., the benefit of further repoing) equals the required return at that point, r^H, minus the market cost of repoing, r^R.

7 Model Calibration Details

7.1 Common Parameters

The fraction of production lost when a firm defaults (μ) is set to a level such that when there are the highest draws for both the home productivity factor and foreign endowment the low cost firms do not default; there are many such levels that satisfy this condition and the results are not qualitatively changed by marginally adjusting this value. With the level chosen, my model does not produce firm defaults in any state.

Government spending (G) is set to 15 to match the median of government revenue as a percentage of GDP for the non-crisis area countries in 2007. The myopic government discount factor (ξ) is calculated as 0.80 assuming an expected government planning horizon of 5 years. Finally, the fraction of government debt remaining after a default (ϕ) is chosen as 60% after reviewing historical creditor losses during country defaults in Sturzenegger & Zettelmeyer (2008).

7.2 Firm Specific Cost Parameters

This section outlines how the cost parameters for the two types of firms (a_H & a_L) are determined. First, the value of a_L is normalized to one. This is possible because the productivity parameters will always be multiplied by another model constant, namely Z_{ts}. Next, we use the results of Del Gatto, Mion and Ottaviano (2007)'s study of the distribution of firm productivities to calculate what average costs are for the subsets high and low cost firms. We use the ratio of these mean costs as the level for a_H.

7.2.1 The Distribution of Firm Costs

Del Gatto, Mion and Ottaviano (2007) (http://crenos.unica.it/crenos/sites/default/files/wp/07-03.pdf) examine the distribution of firm costs and productivities across a number of countries. They assume constant labor returns to scale across a continuum of firms, which each firm's marginal cost drawn from the following distribution:

$$G(c) = \left(\frac{c}{c_{Max}}\right)^k \quad ; \quad c \in [0, c_{Max}] \tag{28}$$

As k increases the relative number of high cost firms increases, and the cost distribution of firms becomes more concentrated at these higher levels. This cost distribution implies the following productivity distribution:

$$F(p) = 1 - \left(\frac{1}{p\ c_{Max}}\right)^k \quad ; \quad p \in \left[\frac{1}{c_{Max}}.\infty\right] \tag{29}$$

They estimate k to be about 2 (See Table 9 of their paper), and the average productivity to be about 40 (See Table 8 of their paper). From these values we can derive c_{Max} as follows:

$$
\begin{aligned}
40 = E(p) &= \frac{k}{c_{Max}(k-1)} \\
\implies c_{Max} &= \frac{k}{40(k-1)} = \frac{1}{20}
\end{aligned}
\tag{30}
$$

Using data from Bernard et al (2012) "The Empirics of Firm Heterogeneity and International Trade" Table 1 we find that 82% of US manufacturing firms do not export. We will use this as the fraction of firms that are high cost in our model. The next step is then to find the average costs for firms in the bottom 18% and the top 82%. The cutoff between these two subsets of the firm population is:

$$
\begin{aligned}
1 - .82 = G(c^*) &= \left(\frac{c^*}{c_{Max}}\right)^k \\
\implies c^* = c_{Max}(1-.82)^{\frac{1}{k}} &= .05(1-.82)^{\frac{1}{2}}
\end{aligned}
\tag{31}
$$

The average cost of the high cost/low productivity group is:

$$
\frac{1}{.82}\int_{c^*}^{c_{Max}} \frac{kc^k}{c_{Max}^k} di = \frac{c_{Max}k\left(1-(1-.82)^{\frac{k+1}{k}}\right)}{.82(k+1)} = 0.0375 \tag{32}
$$

and the average cost of the low cost/high productivity group is:

$$
\frac{1}{1-.82}\int_{0}^{c^*} \frac{kc^k}{c_{Max}^k} di = \frac{c_{Max}k(1-.82)^{\frac{1}{k}}}{(k+1)} = 0.0141 \tag{33}
$$

Taking the ratio of these two values to find the normalized a_H value gives:

$$
a_H = \frac{.0375}{.0141} = 2.65 \tag{34}
$$

7.3 Trade Costs & Financial Flows

The target total trade cost for near countries is chosen to match the estimates for US-Canadian trade. For $\sigma = 5$, Anderson & van Wincoop (2004) estimate total US-Canadian trade costs to be 91% of the goods' value. That is, the total price of goods abroad is about 191% of the domestic price, and I use this level for the near countries.

The trade costs for far country goods are found by using the distance elasticity of trade costs found in Hummels (2001) of 0.27. Using this value, and the fact that countries in the far group are on average about 2.75 times as far from the crisis area as those in the near group, the far group's iceberg costs can be computed as $191\% * 2.75^{0.27} = 251\%$.

These target trade costs are used to determine τ and γ by selecting a fixed trade cost great enough that the high cost firms do not export, and then choosing the iceberg cost such that total trade costs as calculated in the following formula match the targets above during normal, non-crisis times:

$$\text{Trade cost} = E_s\left[\frac{(\tau-1)p_{Lts}c_{FLts}+\gamma}{p_{Lts}c_{FLts}}\right] = \tau - 1 + E_s\left[\frac{\gamma}{p_{Lts}c_{FLts}}\right]$$
$$\implies \tau = (1+\text{Trade cost}) - E_s\left[\frac{\gamma}{p_{Lts}c_{FLts}}\right].$$

Only the low cost firms' goods are considered here because of the choice of γ to suppress high cost firm exports.

7.4 Aggregate Shocks and Interest Rates

This section describes how the parameters defining the states of the world are determined. These include the good and bad Z levels, the good and bad W levels, the interest rates, and the transition probabilities between these. The Z and W production shocks follow two and four state random Markov processes independent of one another. The Z states represent high and low productivity shocks for the home economy. The W states represent high and low output levels during normal times, as well as two potential crisis situations. The first crisis state is targeted to match what occurred during the Great Depression, and the second to match what occurred over the late 2000s crisis period.

The interest rates are assumed to be determined in the foreign country so their values depend on the foreign state. This setup is inspired by risk tolerances in the major financial centers of the US and Eurozone connecting the state of the economy there with interest rates.

7.4.1 Z Levels and Transitions

To determine the good and bad levels and transition matrix for Z, I use the Barro & Ursua (2008) real GDP per-capita data from 1870-2007 for the available countries in my two non-crisis area regions.[1] I calculate the annual growth for each country in each year and aggregate the results across all countries. The sample average is taken as the trend level of growth and is subtracted from all observations to get detrended values. Whether each year's growth was above or below trend and the status in the following period is tracked to create the transition matrix between these states. As a check, I confirmed that the limiting transition matrix produced from the probabilities in Table A.1 mirrored the fractions of good and bad states observed in the data.

The relative levels of Z_G and Z_B are selected such that the following relationship holds

$$Z_B = Z_G\frac{1+\text{Avg Detrended Growth in Below Trend Years}}{1+\text{Avg Detrended Growth in Above Trend Years}},$$

and the expected value of home production is 100 when the crisis zone has a good endowment draw, as would have been the case in 2007. The calibration to 100 is performed separately for near and far countries, as the differences in parameterizations mean that the two areas cannot both have the same expected value

[1] The available countries far from the crisis area were: Argentina, Australia, Brazil, Chile, China, India, Indonesia, Japan, Korea, Malaysia, New Zealand, Peru, Philippines, South Africa, Singapore, Sri Lanka, and Uruguay. The available countries near to the crisis area were: Canada, Colombia, Egypt, Greece, Iceland, Mexico, Norway, Russia, Sweden, Switzerland, Turkey, and Venezuela.

of home production for a given set of Z_G and Z_B values and transition probabilities. These values are presented with the other simultaneously determined values below.

Table A.1: Model Calibration — State Transition Matrix

			Next Period State							
		Z Draw	High				Low			
	Z Draw	W Draw	19,512	18,625	17,650	16,080	19,512	18,625	17,650	16,080
		19,512	0.3797	0.2109	0.0059	0.0062	0.2503	0.1391	0.0039	0.0041
	High	18,625	0.2237	0.3669	0.0059	0.0062	0.1475	0.2419	0.0039	0.0041
Current		17,650	0.0457	0.0749	0.236	0.2461	0.0301	0.0494	0.1556	0.1623
Period		16,080	0.0457	0.0749	0.236	0.2461	0.0301	0.0494	0.1556	0.1623
State		19,512	0.2821	0.1567	0.0044	0.0046	0.3479	0.1933	0.0054	0.0056
	Low	18,625	0.1662	0.2726	0.0044	0.0046	0.205	0.3362	0.0054	0.0056
		17,650	0.0339	0.0556	0.1754	0.1829	0.0418	0.0686	0.2162	0.2255
		16,080	0.0339	0.0556	0.1754	0.1829	0.0418	0.0686	0.2162	0.2255

7.4.2 Non-Crisis Foreign Economy: W Levels, Interest Rates, & Transitions

The normal times W states and transition matrix are determined in a manner similar to that for Z. The difference is that rather than look at the results across many countries, I examine the results of the aggregated crisis zone from 1870-2007. The crisis zone data is aggregated across countries by using a weighted average of the Barro & Ursua (2008) real GDP per-capita data based on 1960-2012 World Bank GDP (Constant 2005 US$). These weights changed little over this period, supporting the use of the average country weights over this period for all years. The growth in this time series is used to get normal times good and bad endowment levels similarly to the individual countries in the previous section. The primary difference is that the mid-value for the foreign endowment is set to 100*(2007 aggregate crisis area GDP) / (Average 2007 non-crisis area country GDP) to get the proper relative sizes for the home and foreign economies. These results can be seen in Table A.1. Again, I confirmed that the limiting transition matrix for the foreign state mirrored the fractions of good and bad states observed in the data. The interest rates are calculated using US 1-year real US Treasury bill and LIBOR interest rates them for 1990-2007. The average levels of each of these are calculated separately for periods when there were and were not US recessions to get values for the good and bad states.

7.4.3 Crisis Foreign Economy: W Levels, Interest Rates, & Transitions

The method for determining the good and bad W levels and their transition probabilities is quite different for the crisis scenarios. In this case the worst case scenario is taken to be experiencing 1929-1934 Great Depression growth, and the best case is taken as what occurred during the late 2000s crisis period. The probability of ending up in each state is determined by taking the average initial crisis area detrended growth during each real world crisis period, and the April 2008 IMF advanced country growth expectations detrended to -5.85%.[2] This growth expectation is between the detrended growth rates of the two alternative crisis scenarios, so it is easy to impute the expectations of entering each one to derive the transition matrix. The two crisis state W levels are the normal times mid-value adjusted down by the average crisis zone growth in each crisis period.

The transition matrix given in Table A.1 is created by using the foreign endowment probabilities from this and the previous section, as well as the Z transition probabilities. It is important to reiterate that the home productivity parameter follows the same Markov process independent of whether there is a crisis or not. Additionally, it is assumed that there is a 2% chance of entering a crisis period from a non-crisis one — reflecting the two global crisis periods in the last hundred years — and crises are expected to continue into another crisis period with an 80% chance — to give a five year expected duration to match the two actual crises. The interest rates are derived from the same rates as for normal times.

[2]http://www.imf.org/external/pubs/ft/weo/2009/01/pdf/text.pdf

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